International Journal of Advances in Engineering and Management (IJAEM)
Volume 4, Issue 3 Mar 2022, pp: 1249-1258 www.ijaem.net ISSN: 2395-5252

A Double New General Integral Transform

D. G. Kaklij

Faculty of K.T.H.M.College,NashikS.P. Pune University, Pune.(Maharashtra)

Submitted: 20-03-2022 Revised: 27-03-2022 Accepted: 30-03-2022

ABSTRACT
In this paper, we have introduced the definition of Double New General Integral Transform and some properties. The theme of this paper is to give introductory study of Double New General Integral transform. The Double New General integral transform is derived from New General integral transform. It is useful for finding the integral transform of functions of two variables. This work includes some basic properties like shifting property etc. of Double New General integral transform.

Keywords: Integral transforms, Gamma function, double integral transform.
Mathematics Subject Classification: 44A05, 33E50.
Computing Classification System: AMS 2010.

I. INTRODUCTION

Many researchers are attracted toward the integral transform because of its simplicity and accuracy about getting the results. The origin of the research transform can be traced back to calibrated work of P.S. Laplace and Joseph Fourier. Transforms have been successively used since 1780's. [11] Since then they play an important role in many fields of sciences and Engineering. Many integral transforms such as Laplace transform [11], Sumudu transform [21], Aboodh transform[1], Elzaki transform[19], Maghoub transform[23], Natural transform[3] and Kamal transform [8] have been developed by many academicians to solve ordinary differential equation, partial differential equations, integral equations, integro-differential equations, system of differential equations, fractional differential equations, boundary value problems.

Many researchers used these integral transforms in various fields. Recently the author Patil has used Sawi transform in Bessel's function [12]. Mahgoub transform is used by same author in parabolic boundary value problems [13]. Sawi transforms of error function is used form evaluating improper integrals [15] Laplace and Shehu transform is used in chemical sciences [16]

Mohand Omer Eshag studied double Laplace transform and double Sumudu transform [14]. Dualities between Double Laplace transform, Double Elzaki transform, Double Kamal transform, Double Mohand transform, Double Mahgoub transform, Double Sumudu transform, Double Sawi transform and Aboodh transform has been studied by D. P. Patil [9, 11] in 2020. Double Laplace and Double Sumudu transforms are used by Patil to solve the wave equation [10] Parabolic boundary value problems are solved by using Mahgoub transform by Patil [14]. Double Natural transform have been studied by Kiliçman, Adem, et al. [2].

This paper is organised as follows. In second section we state definitions of some integral transforms, definitions and formulas for general integral transform. In third section we have defined double integral transform and state and proved some properties. Fourth section is devoted to the relations of double new general integral transform with other double integral transforms. Formulae of some elementary functions are derived in fifth section. Conclusion is drawn in sixth section.

II. DEFINITIONS OFSOME INTEGRAL TRANSFORMS

If $f(t)$ be a function of exponential order α then we define the following transforms

Laplace transform [11]	$L\{f(t)\} = \int_0^\infty f(t)e^{-ut}dt$
Sumudu transform [11]	$S\{f(t)\} = \frac{1}{u}\int_0^\infty f(t)e^{-t/u}dt$
Elzaki Transform [19]	$E\{f(t)\} = u\int_0^\infty f(t)e^{-t/u}dt$

Natural Transform [3]	$N\{f(t)\} = R(s,u) = \frac{1}{u}\int_0^{\infty} f(t)e^{\frac{-st}{u}}dt$
Aboodh transform [1]	$A\{f(t)\} = \frac{1}{u}\int_0^{\infty} f(t)e^{-ut}dt$
Mohand transform [22]	$M\{f(t)\} = u^2\int_0^{\infty} f(t)e^{-ut}dt$
Mahgoub transform [23]	$M\{f(t)\} = u\int_0^{\infty} f(t)e^{-ut}dt$
Sawi transform [15]	$Sa\{f(t)\} = \frac{1}{u^2}\int_0^{\infty} f(t)e^{-t/u}dt$
Kamal transform [8]	$K\{f(t)\} = \int_0^{\infty} f(t)e^{-t/u}dt$
α-integral transform [25]	$L_{\alpha}\{f(t)\} = \int_0^{\infty} f(t)e^{-(u^{1/\alpha})t}dt, \quad \alpha > 0, u \in \mathbb{R}$
Pourreza transform [17]	$HJ\{f(t)\} = u\int_0^{\infty} f(t)e^{-u^2t}dt$
G_transform [18]	$G\{f(t)\} = u^{\alpha}\int_0^{\infty} f(t)e^{-1/u}dt$, where α is an integer.
New general integral transform [7]	$T\{f(t)\} = p(s)\int_0^{\infty} f(t)e^{-q(s)t}dt$

2.1 Definition of New General Integral Transform [7].

Let $f(t)$ be an integrable function defined for $t \geq 0, p(s) \neq 0$ and $q(s)$ are positive real valued functions then the general integrable transform $\mathcal{T}(s)$ of $f(t)$ is defined by

$$T\{f(t)\} = p(s)\int_0^{\infty} f(t)e^{-q(s)t}dt$$

Provided that the integral exists for some $q(s)$.

Formulae for New general integral transform

Function $f(t)$	New integral transform $T(f(t)) = \mathcal{T}(s)$
1	$\frac{p(s)}{q(s)}$
t	$\frac{p(s)}{(q(s))^2}$
t^{α}	$\frac{\Gamma(\alpha+1)p(s)}{(q(s))^{\alpha+1}}, \quad \alpha > 0$
$\sin t$	$\frac{p(s)}{(q(s))^2+1}$
$\sin \alpha t$	$\frac{\alpha p(s)}{(q(s))^2+\alpha^2}$
$\cos t$	$\frac{p(s)q(s)}{(q(s))^2+1}$
e^t	$\frac{p(s)}{q(s)-1}, q(s) > 1$
$f'(t)$	$q(s)\mathcal{T}(s) - p(s)f(0)$

III. DOUBLE NEW GENERAL INTEGRAL TRANSFORM

3.1 Definition of the Double new general integral transform

International Journal of Advances in Engineering and Management (IJAEM)
Volume 4, Issue 3 Mar 2022, pp: 1249-1258 www.ijaem.net ISSN: 2395-5252

Let $f(x,y)$ be an integrable function defined for the variables x and y in the first quadrant, $p_1(s) \neq 0$, $p_2(s) \neq 0$ and $q_1(s),\ q_2(s)$ are positive real functions; we define the Double new general integral transform $T_2\{f(x,y)\}$ by the formula

$$T_2\{f(x,y)\} = \mathcal{T}(s) = p_1(s)p_2(s)\int_0^\infty\int_0^\infty e^{-(q_1(s)x+q_2(s)y)}f(x,y)dxdy$$

Provided that the integral exists for some $q_1(s),\ q_2(s)$.

3.2 Properties of Double new general integral transform:

a) Linearity property:

$$T_2\{af(x,y)+bg(x,y)\} = aT_2\{f(x,y)\} + bT_2\{g(x,y)\}$$

$L.H.S. = T_2\{a\,f(x,y) + b\,g(x,y)\}$

$$= p_1(s)p_2(s)\int_0^\infty\int_0^\infty e^{-(q_1(s)x+q_2(s)y)}\big(a\,f(x,y)+b\,g(x,y)\big)dx\,dy$$

$$= p_1(s)p_2(s)\left(\int_0^\infty\int_0^\infty e^{-(q_1(s)x+q_2(s)y)}\,a\,f(x,y)dx\,dy + \int_0^\infty\int_0^\infty e^{-(q_1(s)x+q_2(s)y)}\,b\,g(x,y)dx\,dy\right)$$

$$= p_1(s)p_2(s)\left(a\int_0^\infty\int_0^\infty e^{-(q_1(s)x+q_2(s)y)}\,f(x,y)dx\,dy + b\int_0^\infty\int_0^\infty e^{-(q_1(s)x+q_2(s)y)}\,g(x,y)dx\,dy\right)$$

$$= a\left(p_1(s)p_2(s)\int_0^\infty\int_0^\infty e^{-(q_1(s)x+q_2(s)y)}\,f(x,y)dx\,dy\right)$$

$$+ b\left(p_1(s)p_2(s)\int_0^\infty\int_0^\infty e^{-(q_1(s)x+q_2(s)y)}\,g(x,y)dx\,dy\right)$$

$$= aT_2\{f(x,y)\} + bT_2\{g(x,y)\}$$

= R.H.S.

b) Shifting property:

If $T_2\{f(x,y)\} = \mathcal{T}(s)$ then

$$T_2\{e^{-(ax+by)}f(x,y)\} = \mathcal{T}(s,a,b)$$

That is $T_2\{e^{-(ax+by)}f(x,y)\} = p_1(s)p_2(s)\int_0^\infty\int_0^\infty e^{-[(q_1(s)+a)x+(q_2(s)+b)y]}f(x,y)dx\,dy$

$$L.H.S. = T_2\{e^{-(ax+by)}f(x,y)\}$$

$$= p_1(s)p_2(s)\int_0^\infty\int_0^\infty e^{-(q_1(s)x+q_2(s)y)}e^{-(ax+by)}f(x,y)dx\,dy$$

$$= p_1(s)p_2(s)\int_0^\infty\int_0^\infty e^{-(q_1(s)x+q_2(s)y+ax+by)}f(x,y)dx\,dy$$

$$= p_1(s)p_2(s)\int_0^\infty\int_0^\infty e^{-((q_1(s)+a)x+(q_2(s)+b)y)}f(x,y)dx\,dy$$

$$= R.H.S.$$

c) Change of scale property:

If $T_2\{f(x,y)\} = \mathcal{T}(s)$ then $T_2\{f(ax,by)\} = \frac{1}{ab}\,\mathcal{T}(s,a,b)$

$L.H.S. = T_2\{f(ax,by)\}$

$$= p_1(s)p_2(s)\int_0^\infty\int_0^\infty e^{-(q_1(s)x+q_2(s)y)}f(ax,by)dx\,dy$$

Substituting $ax = u$ and $by = v$ in the above equation

Hence as $x \to 0,\ u \to 0$ and $y \to 0, v \to 0$ also as $x \to \infty,\ u \to \infty$ and $y \to \infty,\ v \to \infty$

$adx = du \Rightarrow dx = \frac{du}{a}$ and $bdy = dv \Rightarrow dy = \frac{dv}{b}$

$$\therefore L.H.S. = p_1(s)p_2(s)\int_0^\infty\int_0^\infty e^{-\left(q_1(s)\frac{u}{a}+q_2(s)\frac{v}{b}\right)}f(u,v)\frac{du}{a}\frac{dv}{b}$$

$$= \frac{1}{ab}p_1(s)p_2(s)\int_0^\infty\int_0^\infty e^{-\left(\left(\frac{q_1(s)}{a}\right)u+\left(\frac{q_2(s)}{b}\right)v\right)}f(u,v)du\,dv$$

$$= \frac{1}{ab}\left[p_1(s)p_2(s)\int_0^\infty\int_0^\infty e^{-\left(\left(\frac{q_1(s)}{a}\right)u+\left(\frac{q_2(s)}{b}\right)v\right)}f(u,v)du\,dv\right]$$

$= \frac{1}{ab}\left[p_1(s)p_2(s)\int_0^\infty\int_0^\infty e^{-(r_1(s)u+r_2(s)v)}f(u,v)du\,dv\right]$, where $r_1(s) = \frac{q_1(s)}{a}$ and $r_2(s) = \frac{q_2(s)}{b}$

$= \frac{1}{ab} \mathcal{T}(s,a,b)$
$= R.H.S.$

IV. RELATION BETWEEN THE DOUBLE NEW GENERAL INTEGRALTRANS FORM AND OTHER DOUBLE INTEGRALTRANSFORMS.

i) If $p_1(s) = 1, p_2(s) = 1, q_1(s) = u$ and $q_2(s) = v$ then the Double new general transform gives the Double Laplace transform [4].

ii) If $p_1(s) = \frac{1}{u}, p_2(s) = \frac{1}{v}, q_1(s) = \frac{1}{u}$ and $q_2(s) = \frac{1}{v}$ then the Double new general transform gives the Double Sumudu transform [5, 20].

iii) If $p_1(s) = u, p_2(s) = v, q_1(s) = \frac{1}{u}$ and $q_2(s) = \frac{1}{v}$ then the Double new general transform gives the Double Elzaki transform [6].

iv) If $p_1(s) = \frac{1}{u}, p_2(s) = \frac{1}{v}, q_1(s) = \frac{p}{u}$ and $q_2(s) = \frac{q}{v}$ then the Double new general transform gives the Double Natural transform [2].

v) If $p_1(s) = \frac{1}{u}, p_2(s) = \frac{1}{v}, q_1(s) = u$ and $q_2(s) = v$ then the Double new general transform gives the Double Aboodh transform [9].

vi) If $p_1(s) = u^2, p_2(s) = v^2, q_1(s) = u$ and $q_2(s) = v$ then the Double new general transform gives the Double Mohand transform [9].

vii) If $p_1(s) = u, p_2(s) = v, q_1(s) = u$ and $q_2(s) = v$ then the Double new general transform gives the Double Mahgoub transform [9].

viii) If $p_1(s) = \frac{1}{u^2}, p_2(s) = \frac{1}{v^2}, q_1(s) = \frac{1}{u}$ and $q_2(s) = \frac{1}{v}$ then the Double new general transform gives the Double Sawi transform [24].

ix) If $p_1(s) = 1, p_2(s) = 1, q_1(s) = \frac{1}{u}$ and $q_2(s) = \frac{1}{v}$ then the Double new general transform gives the Double Kamal transform [9].

V. FORMULAE OF SOME ELEMENTARY FUNCTIONS

In this section we shall derive some formulae for some elementary functions by using double new general integral transform.

Formula5.1

If $f(x,y) = 1$ for $x > 0$ and $y > 0$

$$T_2\{1\} = p_1(s)p_2(s) \int_0^\infty \int_0^\infty e^{-(q_1(s)x+q_2(s)y)} dx\, dy$$

$= p_1(s)p_2(s) \int_0^\infty \int_0^\infty e^{-q_1(s)x} e^{-q_2(s)y} dx\, dy$

$= p_1(s)p_2(s) \left(\int_0^\infty e^{-q_1(s)x}\, dx \int_0^\infty e^{-q_2(s)y}\, dy \right)$

$= p_1(s)p_2(s) \left(\frac{e^{-q_1(s)x}}{-q_1(s)} \right)_{x=0}^{\infty} \left(\frac{e^{-q_2(s)y}}{-q_2(s)} \right)_{y=0}^{\infty}$

$= p_1(s)p_2(s) \left(\frac{1}{q_1(s)} \right) \left(\frac{1}{q_2(s)} \right)$

$= \frac{p_1(s)p_2(s)}{q_1(s)q_2(s)}$

Formula 5.2

If $f(x,y) = exp(ax + by)$

$T_2\{exp(ax + by)\} = p_1(s)p_2(s) \int_0^\infty \int_0^\infty e^{-(q_1(s)x+q_2(s)y)} e^{ax+by}\, dxdy$

$= p_1(s)p_2(s) \int_0^\infty \int_0^\infty e^{-(q_1(s)x+q_2(s)y-ax-by)} dx\, dy$

$$= p_1(s)p_2(s)\int_0^\infty\int_0^\infty e^{-(q_1(s)-a)x-(q_2(s)-b)y}dx\,dy$$
$$= p_1(s)p_2(s)\left(\int_0^\infty e^{-(q_1(s)-a)x}\,dx\int_0^\infty e^{-(q_2(s)-b)y}\,dy\right)$$
$$= p_1(s)p_2(s)\left(\frac{e^{-(q_1(s)-a)x}}{-(q_1(s)-a)}\right)_{x=0}^{\infty}\left(\frac{e^{-(q_2(s)-b)y}}{-(q_2(s)-b)}\right)_{y=0}^{\infty}$$
$$= p_1(s)p_2(s)\left(\frac{1}{q_1(s)-a}\right)\left(\frac{1}{q_2(s)-b}\right)$$
$$= \frac{p_1(s)p_2(s)}{(q_1(s)-a)(q_2(s)-b)}$$

Formula5.3

If $f(x,y) = exp(i(ax+by))$

$$T_2\{exp(i(ax+by))\} = p_1(s)p_2(s)\int_0^\infty\int_0^\infty e^{-(q_1(s)x+q_2(s)y)}e^{i(ax+by)}dxdy$$
$$= p_1(s)p_2(s)\int_0^\infty\int_0^\infty e^{-(q_1(s)x+q_2(s)y-iax-iby)}dx\,dy$$
$$= p_1(s)p_2(s)\int_0^\infty\int_0^\infty e^{-(q_1(s)-ia)x-(q_2(s)-ib)y}dx\,dy$$
$$= p_1(s)p_2(s)\left(\int_0^\infty e^{-(q_1(s)-ia)x}\,dx\int_0^\infty e^{-(q_2(s)-ib)y}\,dy\right)$$
$$= p_1(s)p_2(s)\left(\frac{e^{-(q_1(s)-ia)x}}{-(q_1(s)-ia)}\right)_{x=0}^{\infty}\left(\frac{e^{-(q_2(s)-ib)y}}{-(q_2(s)-ib)}\right)_{y=0}^{\infty}$$
$$= p_1(s)p_2(s)\left(\frac{1}{q_1(s)-ia}\right)\left(\frac{1}{q_2(s)-ib}\right)$$
$$= \frac{p_1(s)p_2(s)}{(q_1(s)-ia)(q_2(s)-ib)}$$

Formula5.4

If $f(x,y) = cosh(ax+by)$

$$T_2\{cosh(ax+by\} = p_1(s)p_2(s)\int_0^\infty\int_0^\infty e^{-(q_1(s)x+q_2(s)y)}cosh(ax+by)dxdy$$
$$= p_1(s)p_2(s)\int_0^\infty\int_0^\infty e^{-(q_1(s)x+q_2(s)y)}\left(\frac{e^{(ax+by)}+e^{-(ax+by)}}{2}\right)dx\,dy$$
$$= \frac{1}{2}p_1(s)p_2(s)\int_0^\infty\int_0^\infty\left(e^{-(q_1(s)x+q_2(s)y)}e^{(ax+by)}+e^{-(q_1(s)x+q_2(s)y)}e^{-(ax+by)}\right)dx\,dy$$
$$= \frac{1}{2}\left(p_1(s)p_2(s)\int_0^\infty\int_0^\infty e^{-(q_1(s)x+q_2(s)y)}e^{(ax+by)}dx\,dy + p_1(s)p_2(s)\int_0^\infty\int_0^\infty e^{-(q_1(s)x+q_2(s)y)}e^{-(ax+by)}dx\,dy\right)$$
$$= \frac{1}{2}(T_2\{exp(ax+by)\} + T_2\{exp(-(ax+by))\})$$
$$= \frac{1}{2}\left(\frac{p_1(s)p_2(s)}{(q_1(s)-a)(q_2(s)-b)} + \frac{p_1(s)p_2(s)}{(q_1(s)+a)(q_2(s)+b)}\right)$$

Formula5.5

If $f(x,y) = sinh(ax+by)$

$$T_2\{sinh(ax+by\} = p_1(s)p_2(s)\int_0^\infty\int_0^\infty e^{-(q_1(s)x+q_2(s)y)}sinh(ax+by)dxdy$$
$$= p_1(s)p_2(s)\int_0^\infty\int_0^\infty e^{-(q_1(s)x+q_2(s)y)}\left(\frac{e^{(ax+by)}-e^{-(ax+by)}}{2}\right)dx\,dy$$

$$= \frac{1}{2} p_1(s)p_2(s) \int_0^\infty \int_0^\infty (e^{-(q_1(s)x+q_2(s)y)} e^{(ax+by)} - e^{-(q_1(s)x+q_2(s)y)} e^{-(ax+by)}) dx\, dy$$

$$= \frac{1}{2} \left(p_1(s)p_2(s) \int_0^\infty \int_0^\infty e^{-(q_1(s)x+q_2(s)y)} e^{(ax+by)} dx\, dy - p_1(s)p_2(s) \int_0^\infty \int_0^\infty e^{-(q_1(s)x+q_2(s)y)} e^{-(ax+by)} dx\, dy \right)$$

$$= \frac{1}{2} (T_2\{exp(ax+by)\} - T_2\{exp(-(ax+by))\})$$

$$= \frac{1}{2} \left(\frac{p_1(s)p_2(s)}{(q_1(s)-a)(q_2(s)-b)} - \frac{p_1(s)p_2(s)}{(q_1(s)+a)(q_2(s)+b)} \right)$$

Formula 5.6

If $f(x,y) = cos(ax+by)$

$$T_2\{cos(ax+by\} = p_1(s)p_2(s) \int_0^\infty \int_0^\infty e^{-(q_1(s)x+q_2(s)y)} cos(ax+by) dxdy$$

$$= p_1(s)p_2(s) \int_0^\infty \int_0^\infty e^{-(q_1(s)x+q_2(s)y)} \left(\frac{e^{i(ax+by)} + e^{-i(ax+by)}}{2} \right) dx\, dy$$

$$= \frac{1}{2} p_1(s)p_2(s) \int_0^\infty \int_0^\infty (e^{-(q_1(s)x+q_2(s)y)} e^{i(ax+by)} + e^{-(q_1(s)x+q_2(s)y)} e^{-i(ax+by)}) dx\, dy$$

$$= \frac{1}{2} \left(p_1(s)p_2(s) \int_0^\infty \int_0^\infty e^{-(q_1(s)x+q_2(s)y)} e^{i(ax+by)} dx\, dy + p_1(s)p_2(s) \int_0^\infty \int_0^\infty e^{-(q_1(s)x+q_2(s)y)} e^{-i(ax+by)} dx\, dy \right)$$

$$= \frac{1}{2} (T_2\{exp(i(ax+by))\} + T_2\{exp(-i(ax+by))\})$$

$$= \frac{1}{2} \left(\frac{p_1(s)p_2(s)}{(q_1(s)-ia)(q_2(s)-ib)} + \frac{p_1(s)p_2(s)}{(q_1(s)+ia)(q_2(s)+ib)} \right)$$

Formula 5.7

If $f(x,y) = sin(ax+by)$

$$T_2\{sin(ax+by\} = p_1(s)p_2(s) \int_0^\infty \int_0^\infty e^{-(q_1(s)x+q_2(s)y)} sin(ax+by) dxdy$$

$$= p_1(s)p_2(s) \int_0^\infty \int_0^\infty e^{-(q_1(s)x+q_2(s)y)} \left(\frac{e^{i(ax+by)} - e^{-i(ax+by)}}{2i} \right) dx\, dy$$

$$= \frac{1}{2i} p_1(s)p_2(s) \int_0^\infty \int_0^\infty (e^{-(q_1(s)x+q_2(s)y)} e^{i(ax+by)} - e^{-(q_1(s)x+q_2(s)y)} e^{-i(ax+by)}) dx\, dy$$

$$= \frac{1}{2i} \left(p_1(s)p_2(s) \int_0^\infty \int_0^\infty e^{-(q_1(s)x+q_2(s)y)} e^{i(ax+by)} dx\, dy - p_1(s)p_2(s) \int_0^\infty \int_0^\infty e^{-(q_1(s)x+q_2(s)y)} e^{-i(ax+by)} dx\, dy \right)$$

$$= \frac{1}{2i} (T_2\{exp(i(ax+by))\} - T_2\{exp(-i(ax+by))\})$$

$$= \frac{1}{2i} \left(\frac{p_1(s)p_2(s)}{(q_1(s)-ia)(q_2(s)-ib)} - \frac{p_1(s)p_2(s)}{(q_1(s)+ia)(q_2(s)+ib)} \right)$$

Formula 5.8

If $f(x,y) = (xy)^n \quad ,n > 0$

$$T_2\{(xy)^n\} = p_1(s)p_2(s) \int_0^\infty \int_0^\infty e^{-(q_1(s)x+q_2(s)y)} (xy)^n dxdy$$

$$= p_1(s)p_2(s) \int_0^\infty \int_0^\infty e^{-q_1(s)x} e^{q_2(s)y} x^n y^n dx\, dy$$

$$= \left(p_1(s) \int_0^\infty e^{-q_1(s)x} x^n\, dx \right) \left(p_2(s) \int_0^\infty e^{-q_2(s)y} y^n\, dy \right)$$

$$= T\{x^n\} T\{y^n\}$$

International Journal of Advances in Engineering and Management (IJAEM)
Volume 4, Issue 3 Mar 2022, pp: 1249-1258 www.ijaem.net ISSN: 2395-5252

$$= \left(\frac{\Gamma(n+1)p_1(s)}{q_1(s)^{n+1}}\right)\left(\frac{\Gamma(n+1)p_2(s)}{q_2(s)^{n+1}}\right) \quad \because n > 0$$
$$= \frac{\left(\Gamma(n+1)\right)^2 p_1(s)p_2(s)}{\left(q_1(s)q_2(s)\right)^{n+1}}$$

Formula 5.9

If $f(x,y) = x^m y^n$, $m > 0, n > 0$

$$T_2\{x^m y^n\} = p_1(s)p_2(s)\int_0^\infty\int_0^\infty e^{-(q_1(s)x+q_2(s)y)} x^m y^n\,dxdy$$
$$= \left(p_1(s)\int_0^\infty e^{-q_1(s)x} x^m\,dx\right)\left(p_2(s)\int_0^\infty e^{-q_2(s)y} y^n\,dy\right)$$
$$= \left(\frac{\Gamma(m+1)p_1(s)}{q_1(s)^{m+1}}\right)\left(\frac{\Gamma(n+1)p_2(s)}{q_2(s)^{n+1}}\right) \quad \because m,n > 0$$
$$= \frac{\Gamma(m+1)\Gamma(n+1)p_1(s)p_2(s)}{\left(q_1(s)\right)^{m+1}\left(q_2(s)\right)^{n+1}}$$

The above formulae are summarised in the following table.

Function $f(x,y)$	Double New integral transform $T_2\{f(x,y)\}$
1	$\frac{p_1(s)p_2(s)}{q_1(s)q_2(s)}$
$exp(ax+by)$	$\frac{p_1(s)p_2(s)}{(q_1(s)-a)(q_2(s)-b)}$
$exp(i(ax+by))$	$\frac{p_1(s)p_2(s)}{(q_1(s)-ia)(q_2(s)-ib)}$
$cosh(ax+by)$	$\frac{1}{2}\left(\frac{p_1(s)p_2(s)}{(q_1(s)-a)(q_2(s)-b)} + \frac{p_1(s)p_2(s)}{(q_1(s)+a)(q_2(s)+b)}\right)$
$sinh(ax+by)$	$\frac{1}{2}\left(\frac{p_1(s)p_2(s)}{(q_1(s)-a)(q_2(s)-b)} - \frac{p_1(s)p_2(s)}{(q_1(s)+a)(q_2(s)+b)}\right)$
$cos(ax+by)$	$\frac{1}{2}\left(\frac{p_1(s)p_2(s)}{(q_1(s)-ia)(q_2(s)-ib)} + \frac{p_1(s)p_2(s)}{(q_1(s)+ia)(q_2(s)+ib)}\right)$
$sin(ax+by)$	$\frac{1}{2i}\left(\frac{p_1(s)p_2(s)}{(q_1(s)-ia)(q_2(s)-ib)} - \frac{p_1(s)p_2(s)}{(q_1(s)+ia)(q_2(s)+ib)}\right)$
$(xy)^n$, $n > 0$	$\frac{\left(\Gamma(n+1)\right)^2 p_1(s)p_2(s)}{\left(q_1(s)q_2(s)\right)^{n+1}}$
$x^m y^n$, $m > 0, n > 0$	$\frac{\Gamma(m+1)\Gamma(n+1)p_1(s)p_2(s)}{\left(q_1(s)\right)^{m+1}\left(q_2(s)\right)^{n+1}}$

VI. APPLICATION

Theorem 1 Let $f(x,y)$ be a function of two variables. If the first ordered partial derivative $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ exists and $f(0,y)$ be given. $p_1(s), p_2(s), q_1(s)$ *and* $q_2(s)$ are positive real functions then

$$T_2\left\{\frac{\partial f}{\partial x}(x,y)\right\} = -p_1(s)T\{f(0,y)\} + q_1(s)T_2\{f(x,y)\}$$

where $T\{f(0,y)\}$ is the new general integral transform of the $f(0,y)$

Proof: $T_2\left\{\frac{\partial f}{\partial x}(x,y)\right\} = p_1(s)p_2(s)\int_0^\infty\int_0^\infty \frac{\partial f}{\partial x} e^{-(q_1(s)x+q_2(s)y)}dx\,dy$

$$= p_1(s)p_2(s)\int_0^\infty\left(\int_0^\infty \frac{\partial f}{\partial x}e^{-q_1(s)x}dx\right)e^{-q_2(s)y}dy$$

$$= p_1(s)p_2(s)\int_0^\infty\left(e^{-q_1(s)x}\int_0^\infty \frac{\partial f}{\partial x}\,dx - \int_0^\infty\left(-q_1(s)e^{-q_1(s)x}\int_0^\infty\frac{\partial f}{\partial x}\,dx\right)dx\right)e^{-q_2(s)y}dy$$

$$= p_1(s)p_2(s)\int_0^\infty\left((-f(0,y)) + q_1(s)\int_0^\infty e^{-q_1(s)x}f(x,y)dx\right)e^{-q_2(s)y}dy$$

$$= -p_1(s)p_2(s)\int_0^\infty f(0,y)\,e^{-q_2(s)y}dy + p_1(s)p_2(s)q_1(s)\int_0^\infty\int_0^\infty e^{-(q_1(s)x+q_2(s)y)}f(x,y)dx\,dy$$

$$= -p_1(s)\left(p_2(s)\int_0^\infty f(0,y)\,e^{-q_2(s)y}dy\right) + q_1(s)\left(p_1(s)p_2(s)\int_0^\infty\int_0^\infty f(x,y)e^{-(q_1(s)x+q_2(s)y)}dx\,dy\right)$$

$$= -p_1(s)T\{f(0,y)\} + q_1(s)T_2\{f(x,y)\}$$

Theorem 2 Let $f(x,y)$ be a function of two variables. If the first ordered partial derivative $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ exists and $f(x,0)$ be given. $p_1(s), p_2(s), q_1(s)$ *and* $q_2(s)$ are positive real functions then

$$T_2\left\{\frac{\partial f}{\partial y}(x,y)\right\} = -p_2(s)T\{f(x,0)\} + q_2(s)T_2\{f(x,y)\}$$

where $T\{f(x,0)\}$ is the new general integral transform of the $f(x,0)$

Proof: $T_2\left\{\frac{\partial f}{\partial y}(x,y)\right\} = p_1(s)p_2(s)\int_0^\infty\int_0^\infty \frac{\partial f}{\partial y} e^{-(q_1(s)x+q_2(s)y)}dx\,dy$

$$= p_1(s)p_2(s)\int_0^\infty\left(\int_0^\infty \frac{\partial f}{\partial y}e^{-q_2(s)y}dy\right)e^{-q_1(s)x}dx$$

$$= p_1(s)p_2(s)\int_0^\infty\left(e^{-q_2(s)y}\int_0^\infty \frac{\partial f}{\partial y}\,dy - \int_0^\infty\left(-q_2(s)e^{-q_2(s)y}\int_0^\infty\frac{\partial f}{\partial y}\,dy\right)y\right)e^{-q_1(s)x}dx$$

$$= p_1(s)p_2(s)\int_0^\infty\left((-f(x,0)) + q_2(s)\int_0^\infty e^{-q_2(s)y}f(x,y)dy\right)e^{-q_1(s)x}dx$$

$$= -p_1(s)p_2(s)\int_0^\infty f(x,0)\,e^{-q_1(s)x}dx + p_1(s)p_2(s)q_2(s)\int_0^\infty\int_0^\infty e^{-(q_1(s)x+q_2(s)y)}f(x,y)dx\,dy$$

$$= -p_2(s)\left(p_1(s)\int_0^\infty f(x,0)\,e^{-q_1(s)x}dx\right) + q_2(s)\left(p_1(s)p_2(s)\int_0^\infty\int_0^\infty f(x,y)e^{-(q_1(s)x+q_2(s)y)}dx\,dy\right)$$

$$= -p_2(s)T\{f(x,0)\} + q_2(s)T_2\{f(x,y)\}$$

Example Solve the partial differential equation $\frac{\partial f}{\partial x} = \frac{\partial f}{\partial y}$ with the initial conditions $f(0,y) = y, f(x,0) = x.$

Solution: Let $\frac{\partial f}{\partial x} = \frac{\partial f}{\partial y}$ Applying double new general integral transform we get

$$T_2\left\{\frac{\partial f}{\partial x}(x,y)\right\} = T_2\left\{\frac{\partial f}{\partial y}(x,y)\right\}$$

$$\Rightarrow\ -p_1(s)T\{f(0,y)\} + q_1(s)T_2\{f(x,y)\} = -p_2(s)T\{f(x,0)\} + q_2(s)T_2\{f(x,y)\}$$

$$\Rightarrow\ -p_1(s)T\{y\} + q_1(s)T_2\{f(x,y)\} = -p_2(s)T\{x\} + q_2(s)T_2\{f(x,y)\}$$

$$\Rightarrow\ -p_1(s)\frac{p_2(s)}{q_2(s)^2} + q_1(s)T_2\{f(x,y)\} = -p_2(s)\frac{p_1(s)}{q_1(s)^2} + q_2(s)T_2\{f(x,y)\}$$

$$\Rightarrow T_2\{f(x,y)\}\big(q_1(s) - q_2(s)\big) = \frac{p_1(s)p_2(s)}{q_2(s)^2} - \frac{p_1(s)p_2(s)}{q_1(s)^2}$$

$$\Rightarrow T_2\{f(x,y)\}\big(q_1(s) - q_2(s)\big) = p_1(s)p_2(s)\left(\frac{1}{q_2(s)^2} - \frac{1}{q_1(s)^2}\right)$$

$$\Rightarrow T_2\{f(x,y)\}(q_1(s)-q_2(s)) = p_1(s)p_2(s)\left(\frac{q_1(s)^2-q_2(s)^2}{q_1(s)^2 q_2(s)^2}\right)$$
$$\Rightarrow T_2\{f(x,y)\} = \frac{p_1(s)p_2(s)}{q_1(s)^2 q_2(s)^2}(q_1(s)+q_2(s))$$
$$\Rightarrow T_2\{f(x,y)\} = \frac{p_1(s)p_2(s)}{q_1(s)q_2(s)^2} + \frac{p_1(s)p_2(s)}{q_1(s)^2 q_2(s)}$$
$$\Rightarrow f(x,y) = x + y$$

VII. CONCLUSION:

In this paper we have defined the double general integral transform and related it with other general integral transform successfully. We also applied double general integral transform to some problems and obtained the results successfully.

VIII. ACKNOWLEDGEMENT:

I am thankful to Department of Mathematics and library, K.R.T. Arts, B.H. Commerce and A.M. Science College, Nashik for providing me book and internet facility.

REFERENCES

[1]. Aboodh K.S., 2013, The new integral transform "Aboodh Transform", Global Journal of Pure and Applied Mathematics, 9(1), 35-43.
[2]. AdemKiliçman, Maryam Omran, 2017, On double Natural transform and its applications.Journal of Nonlinear Sciences & Applications, Vol. 10 Issue 4, 1744-1754.
[3]. Fethi Bin Muhammed Belgacem, SilambarasanR.,2012, Theory of natural transform, Mathematics In Engineering, Science and Aerospace, Vol. 3, No. 1, 105-135.
[4]. Debnath Lokenath, 2016, The Double Laplace transforms and their Properties with Applications to Functional, Integral and Partial Differential Equations, International Journal of Applied and Computer Mathematics, Vol. 2, 223-241.
[5]. Hassan Eltayeb and AdemKilicman, 2010, double Sumudu and double Laplace transform, Malaysian Journal of Mathematical Sciences, Vol. 4, Issue 1, 17-30.
[6]. Hassan M. A. and Elzaki T. M., 2020, Double Elzaki Transform Decomposition Method for Solving Non-Linear Partial Differential Equations, Journal of Applied Mathematics and Physics, Vol. 8, 1463-1471.
[7]. Jafari Hossein, 2021 A new general integral transform for solving integral equations, Volume 32, 133-138.
[8]. KamaL A., Sedeeg H.,2016, "The New Integral Transform 'Kamal Transform'," Advances in Theoretical and Applied Mathematics, Vol. 11, 451-458.
[9]. Patil D. P.,2020, Dualities between Double Integral Transforms, International Advanced research Journal in Science, Engineering and Technology, Vol. 7, issue.6, 74-82.
[10]. PatilD. P., 2021, Solution of Wave equation By Double Laplace and Double Sumudu Transform, Vidyabharati. Interdisciplinary Research Journal,135-138.
[11]. Patil D. P., 2018, Comparative study of Laplace, Sumudu, Aboodh, Elzaki andMahgoub transforms and applications in boundary value problems. International Journal of Research and Analytical Reviews, Vol.5, issue.4, 22-26.
[12]. Patil D. P., 2021, Application of Sawi transform in Bessel function, Ayushi International Interdisciplinary Research Journal, Special Issue No. 86, 171-175.
[13]. Patil D. P., 2020, Application of Mahgoub transform in parabolic boundary value problem, International Journal of Current Advanced Research, Vol.9 Issue 4(C),21949-21952.
[14]. Patil D. P., 2021, Double Mahgoub transform for the solution of parabolic boundary value problems, Journal of Engineering Mathematics and Statistics, Vol.4, Issue-2, 28-36.
[15]. Patil D. P., 2020, Application of Sawi transform of error function for evaluating improper integral, Journal of Research and Development Vol.11, Issue 20, 41-45.
[16]. Patil D. P., 2021, Application of Laplace and Shehu transform in chemical sciences, Ayushi, International Interdiciplinary Research Journal, Issue 88, 437-441.
[17]. Ahmadi S.A., HosseinzadehH. and Yazdani CheratiA., 2019, A New Integral Transform for Solving Higher Order Ordinary Differential Equations, Nonlinear Dynamics and Systems Theory, 19 (2), 243–252.
[18]. SamkoS.G., KilbasA. A. and MarichevO.I.,1993, Definition of the

G_transform, the spaces $M_{c,\gamma}^{-1}(L)$ and $L_2^{(c,\gamma)}$ and their characterization, Fractional integrals and derivatives Yverdon, Switzerland 704-709.
[19]. TarigElzaki, 2011, The new integral transform “Elzaki transform”, Global Journal of Pure and Applied Mathematics ISSN 0973-1768 Volume 7, Number 1, 57–64.
[20]. TchuencheJ. M. and MbareN. S., 2007, An Application of the double Sumudu transform, Applied Mathematical Sciences, Vol.1, No.1, 31 – 39.
[21]. WatugalaG. K., 2006,Sumudu transform: a new integral transform to solve differential equations and control engineering problems, International Journal of Mathematical Education in Science and Technology, Vol. 24, Issue 1, 35-43.
[22]. Mohand M.Mahgoub A, 2017, The New Integral Transform "Mohand Transform", Advances in Theoretical and Applied Mathematics ISSN 0973-4554 Volume 12, No. 2, 113-120
[23]. Mohand M. Mahgoub A, 2016, The New Integral Transform "Mahgoub Transform" Advances in Theoretical and Applied Mathematics Volume 11, No. 4, 391-398.
[24]. Mohand M. Mahgoub A, 2019, The New Integral Transform "Sawi Transform", Advances in Theoretical and Applied Mathematics ISSN 0973-4554 Volume 14, No. 1, 81-87.
[25]. Medina, Gustavo D. Ojeda, Pereira N. R., Romero J. H., Luis G, 2016, A New $\boldsymbol{\alpha}$ -Integral Laplace Transform, Asian Journal of Current Engineering and Maths 5:5, 59-62.