

# A Novel Transform Based Image Compressive Sensing Reconstruction

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**ABSTRACT:** Based on the sparse characteristics of image signal in Contourlet transform domain, the basic principle of Contourlet transform is analyzed, and a compression perception reconstruction method based on Contourlet transform is proposed. The basis function of Contourlet transform is not strictly orthogonal to the orthogonal transformation matrix. The improved gradient projection algorithm is used to recover the sparse processing coefficients, and the low rate reconstruction of the image is realized under the condition of guaranteeing the image quality. Experimental results show that the robustness of the algorithm is better.

## I. INTRODUCTION

In recent years, the image compression method based on transform has received more and more attention. The JPEG compression standard based on Discrete Cosine Transform (DCT) and the JPEG2000 compression standard based on wavelet transform are widely used [1]. But DCT time-frequency analysis performance is poor, and wavelet transform does not take into account the geometric regularity of the image itself, and can't capture the smoothness of the contour, and this will affect the image compression performance. In [2], a new multi-scale geometric analysis, contourlet transform is proposed, which uses a small number of coefficients to express smooth contours with better sparseness. Therefore, the image processing method based on Contourlet transform has been extensively studied in recent years.

Although the contourlet transform improves the compression effect of the image, the existing compression method requires the nyquist sampling theorem to complete the analog-to-digital conversion, and then discard the large number of unimportant data by compression, which leads to the waste of resources [3]. In this paper, the theory of Compressive Sensing (CS) is proposed in [4],

which effectively solves this problem and points out a new direction for image compression. At present, the image compression sensing method combined with wavelet transform has already been studied, and contourlet transform with higher sparse characteristic can obviously be applied in this direction. According to the above ideas, this paper analyzes the sparse characteristics of the image signal in the contourlet domain, and proposes an image compression sensing sampling and reconstruction method based on Contourlet transform.

## II. CONTOURLET-DOMAIN SPARSITY ANALYSIS OF IMAGE SIGNALS

the high-pass subband at each scale with the direction filter bank on the basis of the LP decomposition Contourlet transform is a multi-resolution, local, multi-directional image representation method. The transformation process mainly includes two steps. The first step is to use Laplacian Pyramid (LP) Multi-resolution decomposition and singular point acquisition. Compared with wavelet decomposition, the advantage of LP decomposition is that only one band-pass image is generated for each layer in the case of high-dimensional, which avoids scrambling. In addition, the LP decomposition can get a tight frame with frame boundary 1, which is one of the important basis for the combination of Contourlet transform and compression perception theory. Since the LP decomposition is oversampled, the second step of the Contourlet transform is to decompose. The initial direction filter bank is to adjust the input image with a plum-shaped filter and a diamond-shaped filter, but the algorithm is more complex. Therefore, the proposed method has a new direction of a filter group structure based on the literature [5]. The structure uses two modules to simplify the filter

bank direction:

1. Module 1 is a dual-channel plum filter bank consisting of a filter that breaks a two-dimensional spectrum into two directions horizontally and vertically.
2. Module 2 is a shear operator that rearranges the image samples by adding a shear operator and its inverse process before and after the two-channel filter bank, respectively, to obtain a different direction frequency Split, while still maintaining full rebuild.

The combination of LP and directional filter banks forms a double-layer filter bank structure called a pyramid-direction filter bank, and the process of this transformation is called a contourlet transform. The contourlet transform uses a base structure similar to the contour segment to approximate the image. The support range of the base is a 'long strip' structure with an aspect ratio of the scale, with directionality and anisotropy and sparse representation of the edge of the image contour. For the  $C^2$  function  $f$  supported on  $[0,1]^2$  and the curve is singular, the error approximation of the  $M$  term of the contourlet transform is:

$$\varepsilon(M) = \|f - f_M\|^2 \leq C.M^{-2} \cdot (1bM)^3$$

Contourlet transform approximation errors of the decay rate is far better than wavelet transform  $O(M^{-2a})$ . Therefore, the use of contourlet transform can provide more sparse image representation than wavelet transform.

### 3. Compressed Sensing Image Reconstruction 3.1. Compressed sensing principle

The perceptual theory of compression is proposed by Donoho et al [4] in 2004 as follows: Assuming the image signal  $X \in R^{N \times 1}$ , its sparse basis is  $T \in R^{N \times N}$ ,  $TT^K = T^K T = I$ , where  $I$  is the unit matrix. The  $\Theta = T^K X$  can be obtained by sparse transformation of  $\Psi$  to  $X$ , and  $K$  contains a large number of components.  $\Theta$  and  $X$  are the equivalent representations of the same signal,  $X$  is the representation of the signal in the time domain, and  $\Theta$  is the representation of the signal in the  $\Psi$  domain. After sparse transformation of  $X$ , it is necessary to design a reasonable observation matrix  $\Phi$  to realize non-adaptive observation of high-dimensional signal  $X$ , and obtain  $M$  observations of low dimension. which is:

$$Y = \Phi X = \Phi \Psi \Theta \tag{2}$$

The observation matrix is designed to obtain  $M$  observations and to ensure that the high-dimensional signal  $X$  of length  $N$  or its equivalent representation of the sparse vector values under the  $\Psi$  domain is reconstructed from a small number of observations. Since the number of observation sets  $Y$  is smaller than the dimension of the original signal  $X$ , the reconstructed signal  $X$  becomes a problem for solving the undetermined equations and is not easy to solve. However, when  $\Phi \times \Psi$  satisfies the finite restraining property (RIP) [6], the problem of solving the system of undetermined equations can be transformed into the optimal solution problem under norm:

$$\hat{\Theta} = \arg \min_{\Theta} \|\Theta\|_1, \text{ s.t. } Y = \Phi \Psi \Theta \tag{3}$$

RIP-high probability of reconstruction in a given condition. Therefore,  $\phi \psi$  RIP properties that must be met in order to achieve an effective reconstruction of the signal. And  $\phi \psi$  have the RIP properties by  $\phi$  and  $\psi$  of the coherence between decisions, if irrelevant to  $\phi$  and  $\psi$ ,  $\phi \psi$  RIP nature of probability is high.

#### Based on Contourlet transform image compressed sensing reconstruction

Compared with the ordinary wavelet transform, contourlet transform uses a base structure similar to the line segment to approximate the image, and more inline with the smooth contour feature of the natural image, and obtain the sparse representation of the image.

Therefore, theoretically, the image compression perception reconstruction method based on contourlet transform can obtain better compression ratio under the same image quality condition. However, the compression theory requires that the sparse base  $\Psi$  is an orthogonal basis or has a tight frame property, and the discrete functions of the discrete Contourlet transform are not strictly normalized. Only when the LP and DFB adopt orthogonal reconstruction filters, the discrete contourlet transform  $T$  to provide a tight frame with a border of 1. Corresponding to the sparse base  $\Psi$  of contourlet transform is difficult to obtain, so the traditional CS reconstruction algorithm such as orthogonal matching tracking algorithm, MP algorithm e.t.c., can't be applied. However, the author finds that, although  $\Psi$  can not be determined,  $\Theta$  can be obtained by contourlet transform. Therefore, we use the gradient projection algorithm [7] to transform the explicit expression of  $\Psi$  in the optimization process into

implicit expression.

$\Theta$  is denoted by  $\Theta = u - v, u \geq 0, v \geq 0$ , then  $\|\Theta\| = \mathbf{1}^T u + \mathbf{1}^T v$ , where  $\mathbf{1}^T = [1, 1, \dots, 1]^T$ . Can

be solved by solving the following formula  $\Theta$ :

$$\min_{u,v} \frac{1}{2} \|Y - \Phi\Psi(u-v)\|_2^2 + \tau \mathbf{1}^T u + \tau \mathbf{1}^T v, \text{ s.t. } u \geq 0, v \geq 0 \quad (4)$$

Eq. (4) can be further written as a standard quadratic programming problem:

$$\min c^T z + \frac{1}{2} z^T B z = F(Z), \text{ s.t. } z \geq 0 \quad (5)$$

where,  $z = \begin{bmatrix} u \\ v \end{bmatrix}$ ,  $A = \Phi\Psi$ ,  $b = A^T Y$ ,  $c = \tau \mathbf{1}$ ,  $\begin{bmatrix} -b \\ b \end{bmatrix}$

$$B = \begin{pmatrix} A^T A & -A^T A \\ -A^T A & A^T A \end{pmatrix}$$

Initialization: Given  $z^{(0)}$ , select the parameters  $\beta_{\min}$ ,  $\beta_{\max}$ ,  $\beta(0) \in [\beta_{\min}, \beta_{\max}]$  and let  $k=0$ .

(1) Calculating the following formula

$$\delta^{(k)} = \left( z^{(k)} - \beta^{(k)} \nabla F \left( z^{(k)} \right) \right)_+ - z^{(k)} \quad (6)$$

(3) A linear search,  $\lambda^{(k)} \in [0, 1]$ , looking for  $F(z^{(k)} + \lambda^{(k)} \delta^{(k)})$  smallest  $\lambda^{(k)}$  and

$$z^{(k+1)} = z^{(k)} + \lambda^{(k)} \delta^{(k)}$$

(4) Calculation to update Beta:

$$\gamma^{(k)} = \left( \delta^{(k)} \right)^T B \delta^{(k)} \quad (7)$$

$$\gamma^{(k)} = 0, \beta^{(k+1)} = \beta_{\max}$$

$$\beta^{(k+1)} = \text{mid} \left\{ \beta_{\min}, \frac{\|\delta^{(k)}\|_2^2}{\gamma^{(k)}}, \beta_{\max} \right\} \quad (8)$$

(5) Set the termination condition as:

$$\|Y - A\Theta\|_2 \leq \sigma \|Y - A\Theta_{GP}\|_2 \quad (9)$$

where  $\sigma$  is a small constant. Under  $z^{k+1}$  does meet (9) for the termination criteria, if they meet the iteration is stopped; if it does not meet the transfer step (2).

Using an improved gradient projection algorithm for image reconstruction,  $\lambda(k)$  search is the key, in order to increase the efficiency of searches, we further improved the algorithm, step (3)  $\lambda(k)$  search the steps wise.

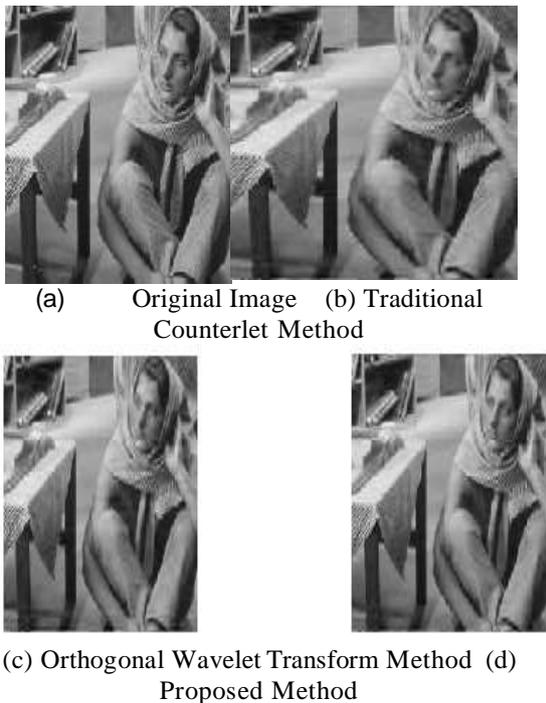
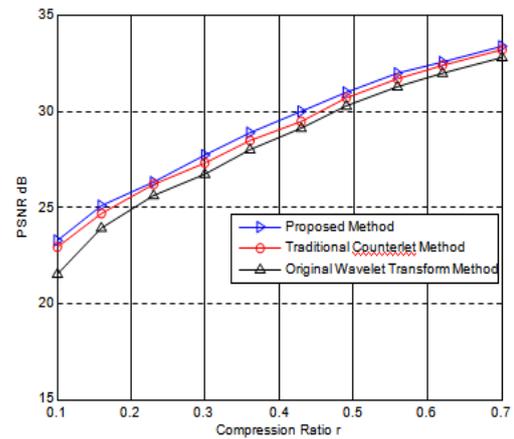


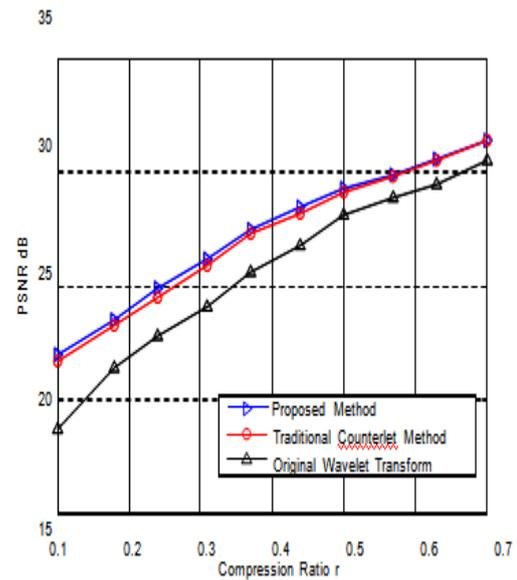
Figure 1. Barbara Image Results

### III. SIMULATION RESULTS

In this paper, two Lena and Barbara images with  $256 \times 256$  pixels in size are selected for simulation experiments. The Contourlet transform is used in the 9/7 biorthogonal filter group. The wavelet basis is orthogonal to the db4 wavelet. When the dimension of the random Gaussian observation matrix is  $128 \times 256$  and the compression ratio is  $128/256$  0.5, Barbara the experimental results are shown in Figure 1



(a) Lena Image



(b) Barbara images

Fig.2. PSNR variation curve of compressed reconstructed Image under different compression Ratio

Table 1. Comparison of PSNR Reconstruction of 3 kinds of Compression Methods

Method	Lena Image	Barbara Image
Original Wavelet Transform Method	29.6994	28.2021
Traditional Counterlet Method	30.0996	29.5690
Proposed Method	30.2035	29.5820

It can be seen that this method can obtain the compression effect equivalent to the traditional compression method based on contourlet transform. Compared with the image compression-aware reconstruction method based on orthogonal wavelet transform, the main advantages are reflected in the contours of the image, such as pants, tablecloths and other parts, because the contourlet transform can effectively express the smoothness with fewer coefficients contour.

In order to objectively reflect the advantages of this algorithm, this paper further compares the peak signal to noise ratio (PSNR) of reconstructed images. Table 1 shows the PSNR comparison results for reconstructed images with three methods with a compression ratio of 0.5. Because the sparsity of contourlet transform is better than that of wavelet transform, the result of image compression sensing reconstruction based on contourlet transform is better than that of image compression perception based on wavelet transform. When the image of Barbara is more obvious, reflected more clearly. In addition, although the compression effect of this method is slightly better than that of the traditional contourlet image compression method, the contourlet transform is not strictly orthogonal transform, which leads to the influence of image reconstruction precision. However, due to the compression-aware reconstruction method can simultaneously complete the sampling and compression, this method is more suitable for high-speed transmission of a large number of image signals.

Figure 2 shows the relationship between the PSNR of the two images and the compression ratio  $r$ . It can be seen that in the processing of Barbara images with more texture features, the PSNR results of this method and traditional Contourlet image compression are about 1 dB higher than that of orthogonal wavelet compression. The higher the compression ratio, more obvious the advantages, this shows that contourlet transform has better sparse characteristics, and is particularly suitable for dealing with rich texture details of the image. The results of this method are equivalent to or better than the traditional contourlet image compression results, which shows that this method not only has the advantages of efficiency, but also to ensure the quality of image compression.

#### IV. CONCLUSION

Aiming at the problem that the traditional image compression method is too wasteful of system resources, this paper studies the image

compression sensing reconstruction method based on Contourlet transform. The image is transformed into Contourlet domain by using the sparse characteristic of contourlet transform domain. Through the improved gradient projection algorithm, the contourlet coefficients of the sparse processing are restored to realize the compression perception reconstruction of the image. This method provides a new way for image compression. On this basis, the next step will improve the computational efficiency of the reconstruction algorithm to improve the real-time performance of the algorithm and expand the application range of the algorithm.

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