

A study to find the optimal number or samples/persons pooled to minimise expected number of tests—a study by Chandan Chakraborty .kolkata ,india,

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In the alarming situation of COVID 19, some countries have a lesser number of test kits. For them “Pooled Test” can come into work. But the question arises here is, how many samples should be pooled together so that the expected number of kits per person is minimized?

Let n be the number of samples pooled and p be the probability that the sample gives a positive report whereas q is the probability of it being negative so that $p + q = 1$.

Let X is a random variable denoting the number of persons or samples having a positive report out of n samples pooled. Under some usual assumptions X follows Binomial distribution with parameter n and p .

If Y denote the number of tests required then

$Y = 1$ if all n persons have a negative report and only one test required i.e $X = 0$ with $P[X = 0] = P[Y = 1] = q^n$

$Y = n + 1$ if at least one of them have a positive report because they will be tested individually again, this has probability $1 - q^n$ i.e $P[Y = n + 1] = P[X \neq 0] = 1 - q^n$

So expected number of Tests required is $E[Y] = 1 * q^n + (n + 1) * (1 - q^n)$

So expected number of tests required per person is $g(n) = E[Y]/n = 1 - q^n + 1/n$ (after simplifications)

If q is estimated from previous knowledge or somehow, then $g(n)$ can be minimized.

A study on different values of q to get the minimum value of $g(n)$ is done and attached herewith.

The study reveals the following



Value of p	Values of q	Optimum value o n
.001	.999	32
.005	.995	14
.01	.99	11
.02	.98	8
.05	.95	5
.1	.9	4
.2	.8	3

See attachment