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Accelerated degradation tests modelingofphotovoltaic modules based on the gammaprocess

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ABSTRACT : Analyzing the reliability of a product is sometimes difficult and complex. An observation is necessary if we want to know the evolution of the degradation. On this subject we have adopted the accelerated degradation tests based on the gamma process. The latter has a reputation for describing degradation processes and component cracks. The Arrhenius model is the basic model for accelerated degradation testing because temperature is a way to quickly stress a component. The estimation of the parameters on this model is quite complicated, hence the introduction of a new Bayesian approach and the Monte Carlo method. On the degradation process requires a tailored approach.

KEY WORDS :gamma, bayesian, Monte Carlo process

I. INTRODUCTION

The use of a stochastic process allows us to model and estimate the degradation of a photovoltaic module. Environmental stresses influence the degradation process. Our study on the gamma process. It is a shock process famous for describing accumulated damage and fatigue in a monotonous way, it is characterized by independent and positive increments resulting in a wide range of applications such as the calculation of remaining life. useful systems, maintenance planning, assessment of the reliability of equipment in real time. We are going to model with accelerated degradation tests by integrating the Arrhenius model. A Bayesian method and Monte Carlo method are used to estimate the various unknown parameters.

II. METHODOLOGY

1. Model based on Gamma process

suppose that $\{X(t), t \geq 0\}$ the degradation random variable and it is defined as a gamma process if $X(t) \sim Ga(\alpha\Lambda(t), \beta)$ with α the shape parameter and β scale parameter with $\Lambda(t)$ consists of the monotonically increasing function of time t. The gamma density function translates as follows[3]:

$$f_{gam}(x) = \frac{\beta^{-\alpha\Lambda(t)}}{\Gamma(\alpha\Lambda(t))} x^{\alpha\Lambda(t)-1} \exp\left[-\frac{x}{\beta}\right] \quad (1)$$

And when $\Lambda(t) = \alpha t$, the process becomes homogeneous gamma process for $\alpha \succ 0$, the expectation and the variance of the inhomogeneous gamma process $\Gamma(\Lambda(t), \beta)$ for $\forall t \geq 0$.

$$F_{Bs}(t) = \Phi\left[\frac{1}{m}\left(\sqrt{\frac{t}{n}} - \sqrt{\frac{n}{t}}\right)\right], t \succ 0 \quad (3)$$

With $\Gamma(a,z)=\int_{z}^{\infty} x^{a-1}e^{-x}dx$ is an incomplete function gamma and $\Gamma(a,z)$ the full function. According to the statistical properties of the gamma properties, we have the mathematical expectation and the variance respectively:

$$E\left[X(t)\right]=\beta.\rho\Lambda(t)\ Var\left[X(t)\right]=\beta^{2}.\rho\Lambda(t)$$
(2)

2. Lifetime distribution

It designates the inoperability of the component from which it is considered that it does not respond correctly to its operation. Consider a faulty component when its level of degradation exceeds a critical threshold ρ. That is T_ρ the distribution at which the failure occurs, it is also called the first time of reaching the level of degradation ρ. The distribution of the first passage time is:

$$T_{\rho}=\inf\left\{t:t\geq 0\middle| X(t)\geq\rho\right\}$$

The lifetime distribution T_ρ follows the inverse Gaussian distribution.

Due to the difficulty on calculating the lifetime distribution, we introduce the Birnbaum-Saunders distribution to approximate the cumulative function as follows [5]:

With $:\Phi(.)$ is a normal distribution with $m=\sqrt{\beta/\rho}$ and $n=\frac{\rho/\beta}{\alpha}$

And we get the probability density function:

$$f_{BS}\Lambda(t)=\frac{\sqrt{\Lambda(t)/n}+\sqrt{n/\Lambda(t)}}{2\sqrt{2\pi}m\Lambda(t)}\times\exp\left[-\frac{1}{2}\left(\frac{\sqrt{\Lambda(t)/n}+\sqrt{n/\Lambda(t)}}{m}\right)^{2}\right] \quad (4)$$

We presented in the previous section the definition of the lifetime, and we have the following different functions:

- **Cumulative function**:

$$F_{T_G}(t)=P\left(T_G\leq t\right)=P\left(x(t)\geq\rho\right) \quad (5)$$
$$=1-\int_{0}^{\rho}\frac{\beta^{-\alpha\Lambda(t)}}{\Gamma\left(\alpha\Lambda(t)\right)}x^{\alpha\Lambda(t)-1}\exp\left[-\frac{x}{\beta}\right]dx$$

$$=1-\frac{\beta^{-\alpha\Lambda(t)}}{\Gamma\left(\alpha\Lambda(t)\right)}\int_{0}^{\frac{\rho}{\beta}}x^{\alpha\Lambda(t)-1}\exp(-x)dx$$

$$=\frac{1}{\Gamma\left(\alpha\Lambda(t)\right)}\int_{\frac{\rho}{\beta}}^{\infty}x^{\alpha\Lambda(t)-1}\exp(-x)dx=\frac{\Gamma\left(\alpha\Lambda(t),\frac{\rho}{\beta}\right)}{\Gamma\left(\alpha\Lambda(t)\right)}$$

- **Reliability**:

$$R_{BS}\left(\Lambda(t)\right)=1-F_{Bs}(t)$$

$$R_T(t) = 1 - \frac{\Gamma\left(\alpha\Lambda(t), \frac{\rho}{\beta}\right)}{\Gamma\left(\alpha\Lambda(t)\right)} \tag{6}$$

- **Mean time to failure**:

$$MTTF_{BS} = \int_0^{\infty} \Lambda(t).f_{BS}\left(\Lambda(t)\right)d\Lambda(t) \tag{7}$$

$$\approx \left[\frac{\rho/\beta}{\alpha} + \frac{1}{2\alpha}\right]^c$$

By adopting the function on time transformation $\Lambda(t) = t^c$, we get the mean time to failure

3. Accelerated degradation model under single stress

In this section, we introduce the accelerate model on the performance degradation that is obtained by the traditional accelerated life test model. The Arrhenius model is renowned for describing the evolution of degradation by the effect of temperature.

The Arrhenius model is defined by [3]:

$$A(T) = a\exp\left(-\frac{E_a}{KT}\right)$$

With : $A(T)$ represents the rate of the reaction, T absolute temperature, and E_a activation energy, K represents the Boltzmann constant.

On the accelerated degradation analysis, the function $A(.)$ and degradation rate are parameter indicators associated with product performance degradation. The shape parameter $\alpha\Lambda(t)$ for the gamma degradation model. The Arrhenius model transforms as follows:

$$A(S) = a\exp\left(-\frac{b}{S}\right) \tag{8}$$

With S is the stress accelerator characterized by the phenomenon of thermodynamics which is the temperature. By expressing the Arrhenius model with the shape parameter $\alpha(t)$ of the degradation model

III. BAYESIAN ESTIMATION APPROACH AND MARKOV CHAINS AND MONTE CARLO ALGORITHM

1. Approach model

There is a classical estimation method like maximum likelihood that is used to estimate unknown parameters. But when the numerical solving is complicated and complex, the usual method is no longer enough, we need to add another approach like Bayesian inference. The latter is able to estimate parameters based on Bayes' theorem. It consists in observing the prior distribution with the probability of the data observed by the likelihood function in order to obtain the posterior distribution.

This method focuses upstream by the distribution parameter the prior distribution and the maximum likelihood. And downstream by the distribution of a posteriori law. [1]

The prior distribution contains different information about the useful data and we can classify the parameters as conjugate prior, informative prior, non-informative. If it is noninformative on the parameters, there is no conjugate a priori.

This approach is depicted in Figure 1.

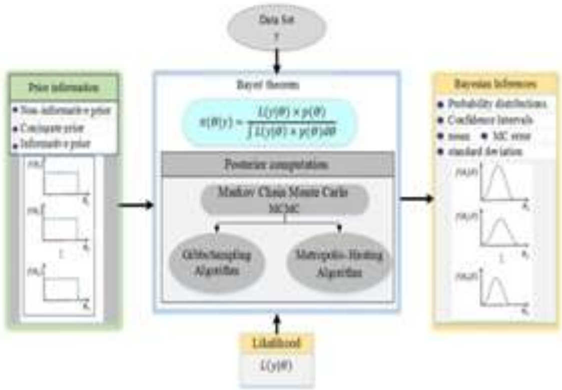

Fig1: diagram of the Bayesian model and Monte Carlo method [1]

According to the Bayesian theorem, we have the posterior distribution as follows:

$$\pi(\Phi|z)=\frac{L(z|\Phi)\times p(\Phi)}{\int L(z|\Phi)\times p(\Phi)\,d\Phi}\propto L(z|\Phi)\times p(\Phi)$$

With Φ is an unknown vector of the parameter model; $\pi(\Phi|z)$ a posteriori distribution; $p(\Phi)$ prior distribution; $L(z|\Phi)$ likelihood function; z observational data.

Due to the complexity on the resolution of the numerical calculation concerning the parameter estimation, we combine with a method of Markov chain and Monte Carlo (MCMC). This involves generating a vector algorithm with the Markov chain in order to find the posterior distribution.
In MCMC, there are two categories of algorithm: Metropolis-Hasting algorithm and Gibbs sample algorithm. It is with the latter that we work because it corresponds well to our study. [1]

2. Model selection criteria

To better compare on the proposed models, we will make a selection based on Akaike's information criterion. (AIC) and the Bayesian information criterion (BIC). These two criteria are all based on information theory.

$$AIC=-2\log\text{likelihood}+2(\text{number of parametrs})$$

Bayesian models and the Markov method have their own selection criterion which is the deviance information criterion (DIC). [1]

3. Estimation by likelihoodmethod

The likelihood estimation method is the method proposed to estimate unknown parameters based on accelerated degradation tests. Consider z_{ijk} the characteristic degradation of stress by noting j^{th} the measure of sampling at the level i^{th} on the sampling value and k the number of measures. We can then write t_{ijk} measuring time with Notons $\Delta z_{ijk}=z_{ijk}-z_{i(j-1)k}$ the increment of degradation and $\Delta t_{ijk}=t_{ijk}-t_{i(j-1)k}$ the time increment function after transformation ($\Lambda(t)=t^q$ the function is nonlinear and if $q=1$ the function will be linear). And following the gamma process
We then obtain the log likelihood function of gamma process, we can write:

$$\Delta z_{ijk}\sim Ga(\alpha\Delta t_{ijk},\beta)$$

The likelihood function is obtained by the performance degradation data

$$L(\alpha(S),\beta)=\prod_{k=1}^{N_1}\prod_{i=1}^{N_2}\prod_{j=1}^{N_3}f_G(\Delta z_{ijk};\alpha(t,S)\beta)$$

$$=\prod_{k=1}^{N_1}\prod_{i=1}^{N_2}\prod_{j=1}^{N_3}\left\{\frac{\Delta z_{ijk}{}^{\alpha(t,S_k)t_{ijk}-1}}{\Gamma(\alpha(S_k)\Delta t_{ijk})\beta^{(\alpha(S_k)\Delta t_{ijk})}}-e^{\frac{\Delta z_{ijk}}{\beta}}\right\}$$

When $\alpha(S_k)=a\exp\left(-\frac{b}{S_k}\right)$, with N_1, N_2, N_3 represent acceleration stress level numbers respectively.

By deriving the previous function from equation we get the log-likelihood function:

$$\ln L\left(a(S_k),\beta\right)=\sum_{k=1}^{N_1}\sum_{i=1}^{N_2}\sum_{j=1}^{N_3}\left[\begin{array}{l}\left(\alpha(S_k)\Delta t_{ijk}-1\right)\ln\Delta z_{ijk}-\dfrac{\Delta z_{ijk}}{\beta}\\ -\ln\Gamma\left(\alpha(S_k)\Delta t_{ijk}\right)-\alpha(S_k)\Delta t_{ijk}-\ln\beta\dfrac{\Delta z_{ijk}}{\beta}\end{array}\right]$$

By substituting the data of Δz_{ijk}, Δt_{ijk} and S_k, we adopt the maximum likelihood estimation in order to estimate the parameters a, b, β without forgetting the parameter calculation α on the different levels of acceleration stress.

IV. APPLICATION

1. Simulation data

Our field of application relates to the degradation data of the photovoltaic modules, it is about degradation of the power supplied by the module. The observation begins at the start of the start-up. The goal is to assess degradation in order to predict maintenance.

Table 1: parameters and lifetime estimation for a photovoltaic module [2]

Module	Equation	at	b	ct	k	Initial power
S70L45	1	-	-	-	-	100
	2	100	-	4299	-9.54.10-4	98.4
	3	43	55.5	2875.4	-0.0019	98.6

The model will describe the relationship between performance degradation and usage time whether it is a linear or non-linear regression. The model is quite simple, it is obtained by estimating the remaining life of the module.

The following relationship describes the degradation [2]:

$$P(t)=P_0-D(t)$$

With :

- P: output power at time t, P_0 : rated output power; $D(t)$ the degradation random variable.

We assume that the shape of the degradation modeling curve is similar to the sigmoid function:

$$P(t)=\frac{a}{\left(1+\exp\left(-k*t\left(t-t_c\right)\right)\right)}$$

With : a , k and t_c are regression coefficients. The rated power which is exposed outdoors to $t=0$ which is not greater than 100. So we take $P_0=100$ and $a=100$

We add another regression coefficient b in order to have a lifetime between 6 to 10 years with a power degradation no more than 80% of its initial value.

$$P(t)=b+\frac{a}{\left(1+\exp\left(-k*t\left(t-t_c\right)\right)\right)}$$

We illustrate in the following table, the different data, parameters and estimates of the photovoltaic modules that we use during our studies.

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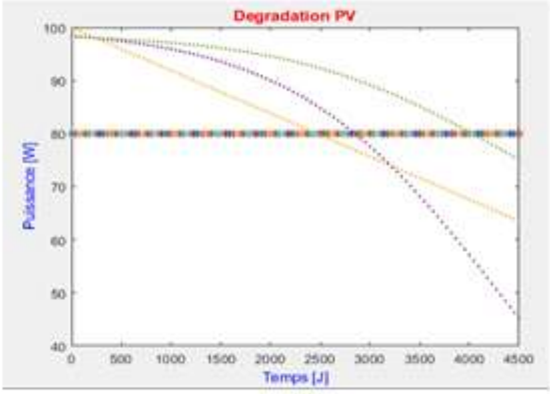

Figure 2: S70L45 Photovoltaic Module Lifetime Fig2:Estimation Model and Degradation

The photovoltaic module power degradation data is obtained by the three equations of the deterministic model.

2. Bayesian estimation and Monte Carlo method

According to the equation, we can write the following relation:

$$\pi_G\left(a,b,\beta,\alpha|\Delta z\right) \propto L_G\left(\Delta z|a,b,\beta,\alpha\right)$$
$$\cdot p\left(a\right)\cdot p\left(b\right)\cdot p\left(\beta\right)\cdot p\left(\alpha\right)$$

By using the Monte Carlo method and the Gibbs algorithm, and the open software BUGS, the unknown parameters are obtained from 100000 iterations. The estimate is composed of the posterior distribution mean, standard error, standard mounted Carlo.

And we have in Table 2, the value of the deviance information criterion:

Table 2:

	dbar	Dhat	DIC	pD
y	767.9	767.3	768.5	0.6027

For the estimation of the parameter model, the gamma process by the Bayesian and Monte Carlo method, we have in table 3, the different parameters.

Table 3: Estimation of model parameters:

	mean	sd	MC_error	val2.5pc	median	val97.5pc	start art	sample
a	36.34	4.987	0.2231	27.39	36.11	46.73	8001	50000
b	0.4469	0.06359	0.002875	0.3325	0.444	0.5848	8001	50000

The results on the analysis of the degradation of the model is observed anis that the diagnosis using figure 3 and figure 4

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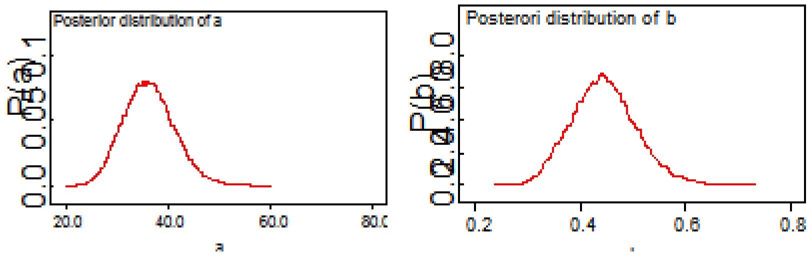

Fig3: A posteriori distribution of parameters a and b

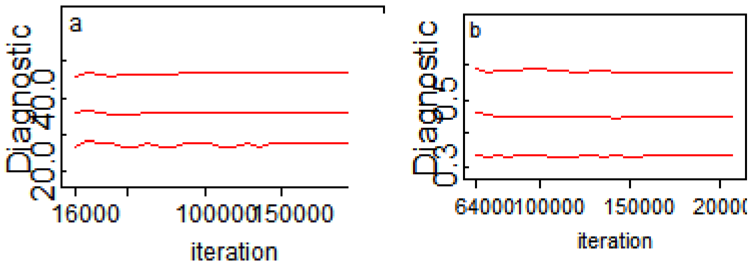

Fig 4: Diagnosis on parameters a and b

We find in figure 5 and figure 6, the shape of the quantities on the analysis of the accelerated degradation tests. These are the probability density, the cumulative function and the reliability of photovoltaic modules.

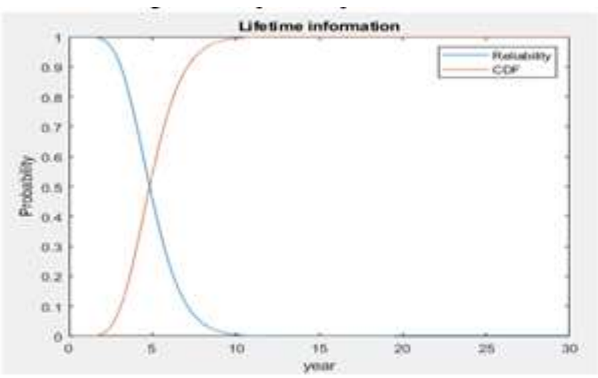

Fig 5: Reliability and cumulative function

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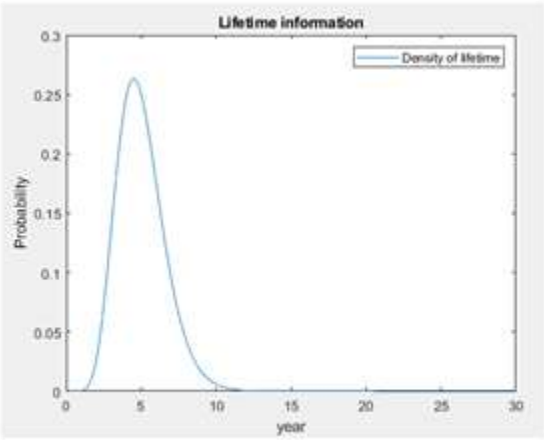

Fig6 : Probabilitydensity

V. CONCLUSION

Theaccelerated degradation test method performs well on product degradation analysis. A lot of information was found on the evaluation of reliability and residual life. The gamma stochastic process describes well the pace of the degradation process. The accompaniment with other Bayesian method and Monte Carlo method, we obtain good results on the estimate.

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