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Analytical Solution of an Unsteady MHD flow of Heat and Mass Transfer for Second Grade Fluid Involving Thermal Radiation and Chemical Reaction

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ABSTRACT

An analytical solution of an unsteady flow of heat and mass transferfor second grade fluid involving thermal radiation chemical reaction has been carried out with the effect of Magneto-hydro-dynamics. The dimensional governing equations are solved using Laplace transform technique. The solution for velocity, temperature and concentration field are obtained and analysed for the different physical parameters such as thermal Grashof number (Gr), Mass Grashof number (Gc),, Prandtl number (Pr),thermal radiation parameter (R), Schmidt number (Sc), chemical reaction (k) and time (t).The result obtained shows that velocity profile increases with increasing parameter such as Sc, Gc, R,t, Pr,Gr and decreases with increasing value of M. The result for temperature decreases with increasing number of Pr and R, and increases with incresing number of t, while the concentrationprofiles is increasing with increasing value of Sc, k and decreases with increasing value of t.

Keywords: Unsteady, MHD, Second Grade Fluid, Thermal Radiation and Chemical Reaction.

I. INTRODUCTION

In process of chemical engineering, chemical reaction between a foreign mass and the fluid in which the plate is moving occurs. Chemical reactions can be varied as either heterogeneous or homogeneous processes. This depends on whether they occur at an interface or as a single phase volume reaction. These processes take place in many industrial applications, such as polymer production, manufacturing of ceramics or glassware, food processing, andthe study of flow of heat and mass transfer for second grade fluid in the presence of magnetic field has attracted the interest of many investigators in view of its applications in many engineering problems such as magneto-hydro-dynamic (MHD) generator, plasma studies, nuclear reactors, oil exploration, geothermal energy extractions and the boundary layer control in the field of aerodynamics. Also, free convection flows are of great interest in a number of industrial applications such as fiber and granular insulation, geothermal etc. Magneto-hydro-dynamic has attracted the attention of a large number of scholars due to its diverse applications. In astrophysics and geophysics, it is applied to study the stellar and solar structures, interstellar matter, radio propagation through the ionosphere etc. In engineering, it finds its application in MHD pumps, MHD bearings etc. Second grade fluids can model many fluids such as dilute polymer solutions, slurry flows, and industrial oils, and many flow problems with various geometries and different mechanical and thermal boundary conditions have been studied.Convection in porous media has applications in geothermal energy recovery, oil extraction, thermal energy storage and flow through filtering devices

A study on MHD heat and mass transfer free convection flow along a vertical stretching sheet in the presence of magnetic field with heat generation was carried out by Samad and Mohebujjaman 2009. Saravana 2011 studied the mass transfer effects on MHD viscous flow past an impulsively started infinite vertical plate with constant mass flux. Singh 2001 analyzed the MHD free convection and mass transfer flow with heat source and thermal diffusion. The Paper deals with the study of free convection and mass transfer flow of an incompressible, viscous and electrically conducting fluid past a continuously moving infinite vertical plate in the presence of large suction and under the influence of uniform magnetic field considering heat source and thermal diffusion. Kim 2000 considered the unsteady magneto hydrodynamic convective heat transfer past a semi-infinite vertical porous moving plate with variable suction. However, most of the previous works assume that the plate is at rest. In the present work, we consider the case of a semi-infinite moving porous plate with a constant velocity in the longitudinal direction when the magnetic field is imposed transversely to the plate. We also consider the free stream to consist of a mean velocity and temperature with a superimposed exponentially variation with time. In view of the applications of free convective phenomenon, heat source and thermal diffusion, in the present work it is proposed to study the unsteady two- dimensional MHD free convective heat and mass transfer of polar fluids past a semi-infinite vertical moving porous plate via a porous medium taking into account the combined effect of heat source and thermal diffusion. The aim of this paper is to make a numerical calculation, on convective heat and mass transfer flow which have been of interest to the engineering community and to the investigators dealing with the problem in geophysics, astrophysics, plasma studies, nuclear reactors etc. From the technical point of view, free convective flow past an infinite or semi-infinite vertical plate is always important for many practical applications.

In general, the study of Darcian porous MHD is very complicated. It is necessary to consider in detail the distribution of velocity and temperature distributions across the boundary layer. Representative results for the velocity, angular velocity and temperature profiles are displayed graphically showing the effect of several governing parameters entering into the problem. Also, we have prepared a table of the values of skin friction displaying the effects of various material parameters. To the best of our knowledge this problem has not been studied before and the results reported here are new.

II. MATHEMATICAL FORMULATION AND ITS SOLUTIONS

An analytical solution of an unsteady MHD flow of heat and mass transfer for second grade fluid involving thermal radiation and chemical reaction with variable velocity, temperature and concentration has been considered, It is assumed that the flow and plate are at the same temperature T; the velocity of the fluid raise with respect to time. It is also assumed that all the fluid properties are constant except first normal stress module and the density in buoyancy terms.

Within the frame work of the above assumptions, under the usual Boussinesq's approximation the unsteady flow equation are momentum equation energy equation and mass equation respectively and are govern by the following equations;

$$\frac{\partial u}{\partial t}=v\frac{\partial^2 u}{\partial y^2}+\alpha\frac{\partial^3 u}{\partial t\partial y^2}+g\beta(T-T_0)+g\beta_\infty(C-C_0)-\frac{\sigma\beta_0^2 U\bar{U}}{\rho} \quad (1)$$

$$\frac{\partial T}{\partial t}=\alpha_1\frac{\partial^2 T}{\partial y^2}-\frac{\partial^2 q_r}{\partial y} \quad (2)$$

$$\frac{\partial C}{\partial t}=D\frac{\partial^2 C}{\partial y^2}-k^*(C-C_O) \quad (3)$$

Where u is the fluid velocity in the x -direction, Tthe temperature of the fluid, g the acceleration due to gravity, β the coefficient of thermal expansion, β∗coefficient of volumetric expansion due to concentration change, ν the kinematic viscosity,ρ the fluid density, k the thermal conductivity, Cpthe specific heat at constant pressure, mass diffusivity D and qrthe radiative heat flux.

The initial and boundary conditions are

$$\begin{array}{llll} u=0 & T=T_0 & C=C_0 & for\, all\; y,t\le 0 \\ u\to 0 & T=T_1 & C=C_\infty+(C_0+C_\infty) & at\; y=0 \\ u(h,t)=UH(t)\quad t\le 0; & T\to T_\infty & C\to C_\infty & at\; y\to\infty \end{array} \quad (4)$$

Introducing non-dimensional quantities as follows;

$$u=U\bar{U},\ \theta=\frac{T-T_0}{T_1-T_0}\ \varphi=\frac{C-C_0}{C_1-C_0}\ \eta=\frac{y}{h},\tau=\frac{Vt}{h^2},l=h^{-1}\sqrt{\alpha},M^2=\frac{\sigma\beta_0^2h^2}{\rho V},,$$

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$$\frac{\partial^2 q_r}{\partial y} = -4a*\sigma(T_o^4 - T^4), \qquad T^4 = 4T_0^3 T - 3T_0^4, \tag{5}$$

$$J^* = P_r R, \qquad b* = ScK$$

Non- dimentionaling the governing equations 1 – 3, we have;

$$\frac{\partial U}{\partial \tau} = \frac{\partial^2 U}{\partial \eta^2} + l^2 \frac{\partial^3 U}{\partial \tau \partial \eta^2} + \theta GrJ^* + \varphi Gcb^* - M^2 U \tag{6}$$

$$\frac{\partial \theta}{\partial \tau} = \frac{1}{\Pr}\frac{\partial^2 \theta}{\partial \eta^2} + R\theta \tag{7}$$

$$\frac{\partial \varphi}{\partial \tau} = \frac{1}{Sc}\frac{\partial^2 \varphi}{\partial \eta^2} - k\varphi \tag{8}$$

Where $GrJ^* = \frac{g\beta(\mathrm{T}_1 - \mathrm{T}_0)h^2}{\bar{U}V}$, $Gcb^* = \frac{g\beta(\mathrm{C}_1 - \mathrm{C}_0)h^2}{\bar{U}V}$ $\Pr = \frac{\mu}{\alpha}, V = \frac{\mu}{\rho}, R = \frac{16b^* T_0^3 h^2}{V}$ $\alpha_1 = \frac{\alpha}{\rho}$

Solving equation 6 – 8 using Laplace transform techniques subject to the boundary conditions in equation 4, the solution of velocity equation is obtained as;

$$\bar{\mathrm{U}} = \frac{e^{-\eta m}}{s} + \frac{G_r}{S^2(P_r-1) - S(R+M^2)} e^{-\eta m} + \frac{G_c}{S^2(Sc-1) - S(Sck+M^2)} e^{-\eta m} - \frac{G_r}{S^2(P_r-1) - S(R+M^2)} e^{-\eta\sqrt{(SPr-R)}} - \frac{G_c}{S^2(Sc-1) - S(Sck+M^2)} e^{-\eta\sqrt{sc(S+k)}} \tag{9}$$

The solution of temperature equation is;

$$\bar{\theta}(\eta,\tau) = \frac{e^{-\eta\sqrt{(SP_r-R)}}}{S} \tag{10}$$

The solution of concentration equation is;

$$\bar{\varphi}(\eta,\tau) = \frac{1}{s} e^{(-\eta\sqrt{(S_c(s+k))})} \tag{11}$$

The solutions of equations are further obtained by applying the invers Laplace transform give the expression for the velocity, temperature and concentration distributions as in equatins 12 – 14 respectively.

$$\mathrm{U} = \frac{1}{2}\{e^{\eta M}\mathrm{erfc}(M\sqrt{t} + \eta M) + e^{-\eta M}\mathrm{erfc}(-M\sqrt{t} + \eta M)\}$$
$$+ \frac{Gr}{2a}\{e^{\eta M}\mathrm{erfc}(M\sqrt{t} + \eta M) + e^{-\eta M}\mathrm{erfc}(-M\sqrt{t} + \eta M)\} - \frac{Gr}{b}[\mathrm{erf}(\eta)\, e^{-\eta M}]$$
$$+ \frac{G_c}{2f}\{e^{\eta M}\mathrm{erfc}(M\sqrt{t} + \eta M) + e^{-\eta M}\mathrm{erfc}(-M\sqrt{t} + \eta M)\} - \frac{G_c}{g}[\mathrm{erf}(\eta)\, e^{-\eta M}]$$
$$- \frac{G_r}{2a}\{e^{\eta\sqrt{P_r}}\mathrm{erfc}(\sqrt{Rt} + \eta\sqrt{P_r}) + e^{-\eta\sqrt{P_r}}\mathrm{erfc}(-\sqrt{Rt} + \eta\sqrt{P_r})\} + \frac{G_r e^{-\eta\sqrt{R}}}{2b}\{e^{\eta\sqrt{P_r R}}\mathrm{erfc}(\sqrt{Rt} + \eta\sqrt{P_r}) +$$

e−ηPrRerfc−Rt+ηPr−Gctf1+2η2ScerfcηSc−2ηScπe−η2Sc+erfc(η)SGcgc+

$$\frac{Gc}{2g}\left\{e^{\eta\sqrt{b}}erfc\left(\sqrt{b\,t} + \frac{\eta}{2\sqrt{t}}\right) + e^{\eta\sqrt{b}}erfc\left(-\sqrt{bt} + \frac{\eta}{2\sqrt{t}}\right)\right\} \tag{12}$$

$$\theta = \frac{1}{2}\{e^{2n\sqrt{Rt}}\mathrm{erfc}(\eta\sqrt{P_r} + \sqrt{Rt}) + e^{-2\eta\sqrt{Rt}}\mathrm{erfc}(\eta\sqrt{P_r} - \sqrt{Rt})\} \tag{13}$$

$$\varphi(\eta,\tau) = \frac{1}{2}\left\{e^{\eta\sqrt{Scb}}erfc\left(\sqrt{bt} + \frac{\eta}{2}\sqrt{\frac{sc}{t}}\right) + e^{\eta\sqrt{Scb}}erfc\left(-\sqrt{bt} + \frac{\eta}{2}\sqrt{\frac{Sc}{t}}\right)\right\} \tag{14}$$

III. RESULT AND DISCUSSION

An analytical solution of an unsteady MHD flow of heat and mass transfer for second grade flow involving thermal radiation and chemical reaction has been formulated, analyzed and solved.The effects of physical parameters

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namely thermal Grashof number Gr, mass Grashof number Gc, Prandtl number Pr, Schmidt number Sc, time t, radiation parameter R, chemical reaction parameter K are studiedand the computation of the flow fields are carried out using MATLAB software. The value of the Prandtl number Pr is chosen to represent air (Pr = 0.71). The value of Schmidt number is chosen to represent water vapour(Sc = 0.57). The value of velocity, temperature and concentration are obtained for the physical parameters as mentioned.

Figure 4.1 to 4.13 present the flow profiles of velocity, temperature and concentration, and investigated for physical parameters namely thermal Grashof number Gr, mass Grashof number Gc, Schmidt number Sc, Prandtl number Pr, Chemical reaction and time t. The effect' of velocity for different values of Schmidt number (Sc = 0.45, 0.62, 0.78, 2.01) is presented in Figure 4.1. It observed that velocity increases with the increasing Schmidt number. The effect of velocity for different values of time (t = 0.3, 0.4, 0.5, 0.6, 7) is presented in Figure 4.2. It shows that velocity increases with increasing values of t.

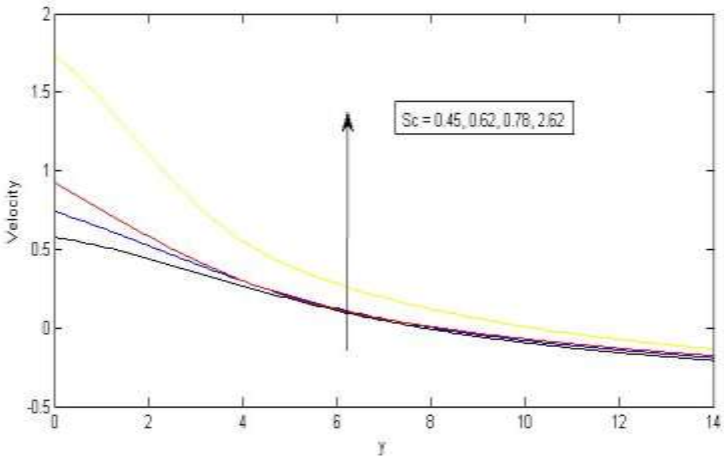

Figure 4.1 velocity profiles for different value of Sc.

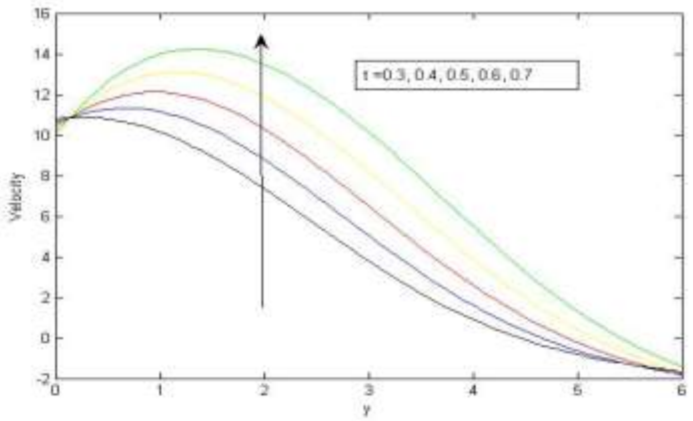

Figure 4.2 velocity profiles for different value of t.

The effect of velocity for different values of Thermal radiation R (R = 0.3, 1, 1.5, 2) is presented in Figure 4.3. It observed that velocity increases with the increasing values of thermal radiation. The effect of velocity for different values of M (M = 0.5, 1.0, 3.0) is presented in Figure 4.4. It shows that velocity increases with increasing values of M. The effect of velocity for different values of Pr(Pr = 0.71, 2, 5,7) is presented in Figure 4.15. It shows that velocity increases with increasing values of Pr. The effect of velocity for different values of Gr (Gr = 0.2, 0.4, 0.6, 0.8) is presented in Figure 4.17. It shows that velocity increases with increasing values of Gr. The velocity profile for different values of Gc(Gc = 0.2, 0.4, 0.6, 0.8) is presented in Figure 4.7. It shows that velocity increases with increasing values of Gc.

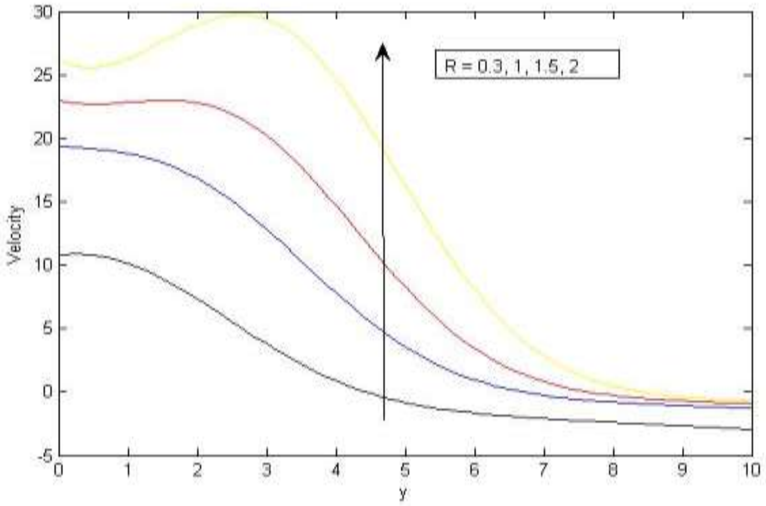

Figure 4.3 velocity profiles for different value of R.

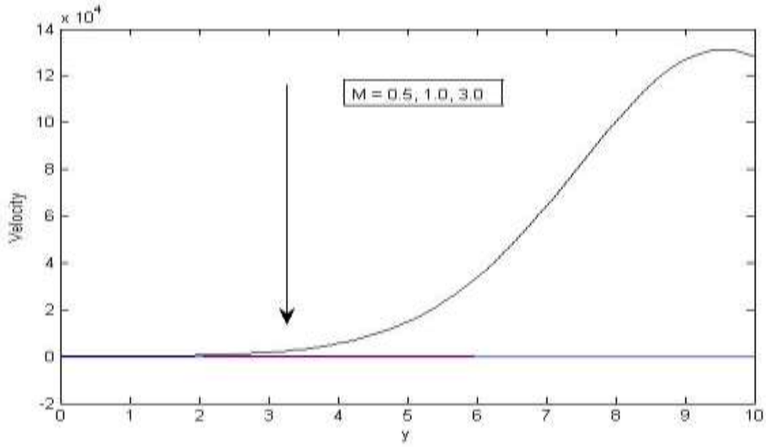

Figure 4.4 velocity profiles for different value of M.

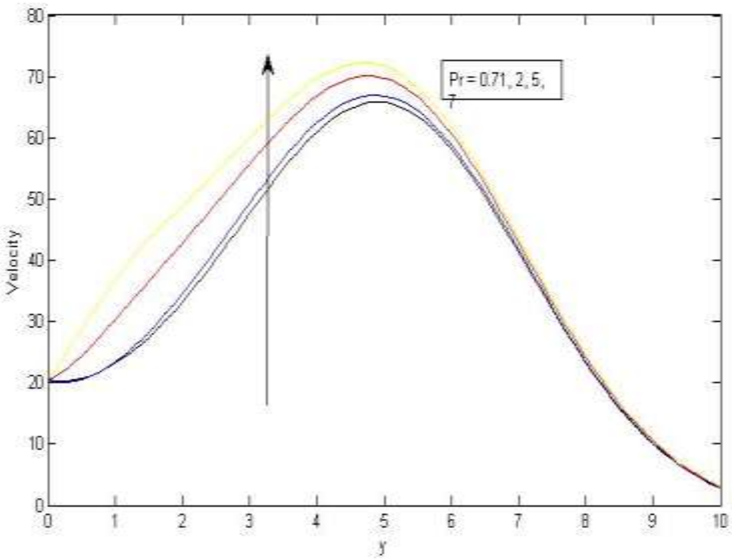

Figure 4.5 velocity profiles for different value of Pr.

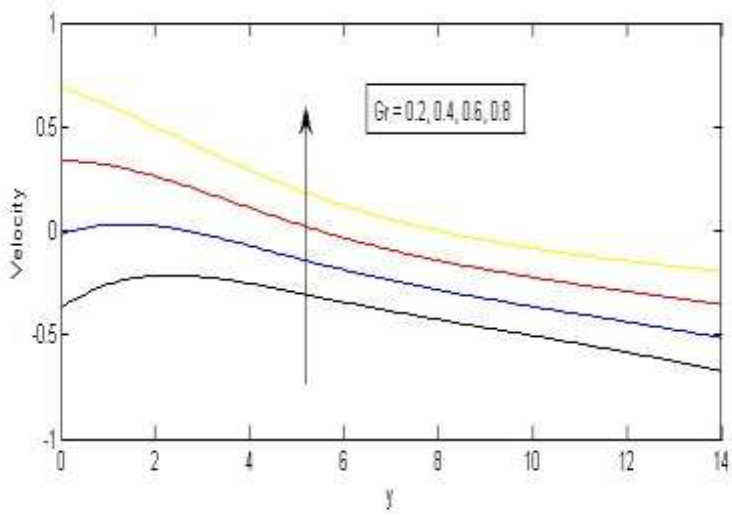

Figure 4.6 velocity profiles for different value of Gr.

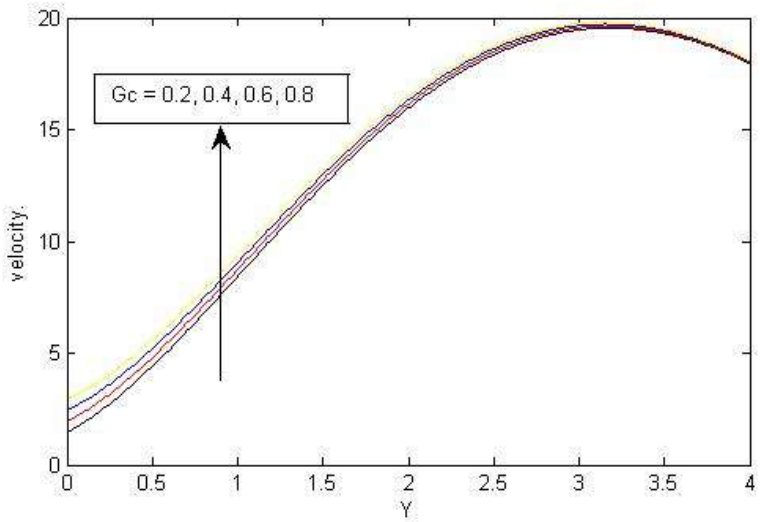

Figure 4.7 velocity profiles for different value of Gc.

The effect of temperature for different values of t. is presented in Figure 4.8 and it shows that the temperature decreases with increasing values t. The effect of temperature for different values of t. is presented in Figure 4.9 and it shows that the temperature decreases with increasing values R. The effect of temperature for different values of Pr. is presented in Figure 4.11 and it shows that the temperature decreases with increasing values Pr.

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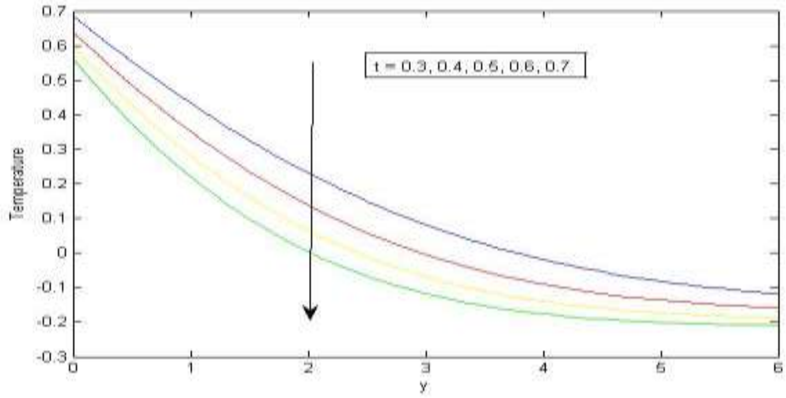

Figure 4.8 Temperature profiles for different value of t.

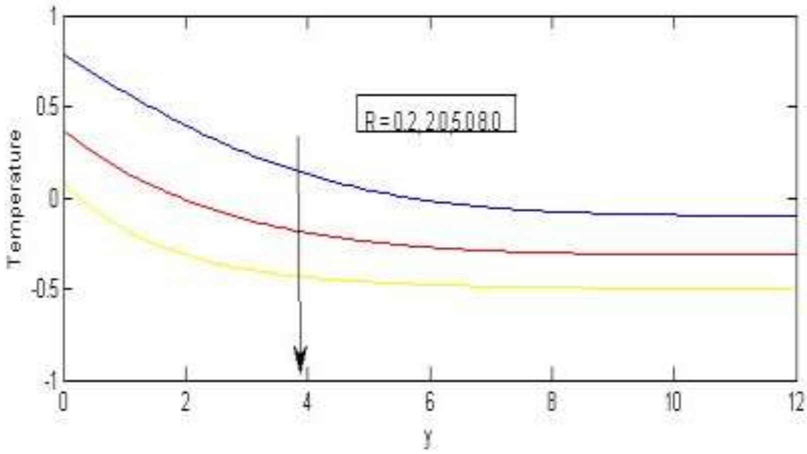

Figure 4.9 Temperature profiles for different value of R.

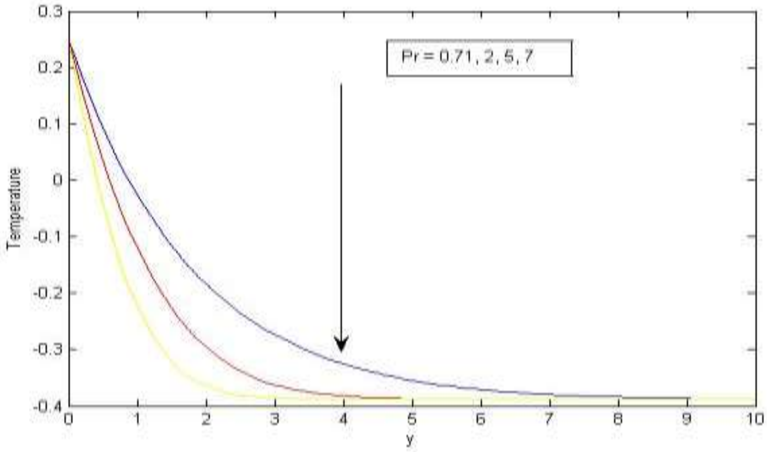

Figure 4.10 Temperature profiles for different value of Pr.

The effect of concentration for different values of t. is presented in Figure 4.11 and it shows that the temperature decreases with increasing values Pr. The effect of concentration for different values of Sc. is presented in Figure 4.12 and it shows that the concentration decreases with increasing values Sc.The effect of concentration for different values of k. is presented in Figure 4.13 and it shows that the concentration decreases with increasing values Sc.

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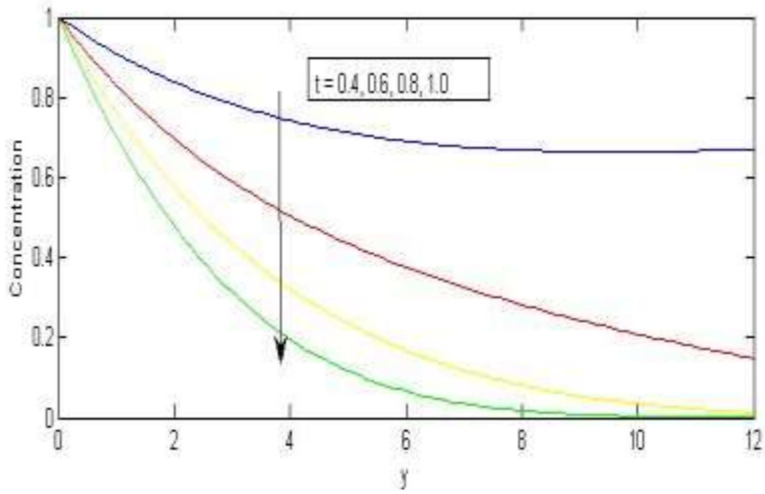

Figure 4.11 concentration profiles for different value of t.

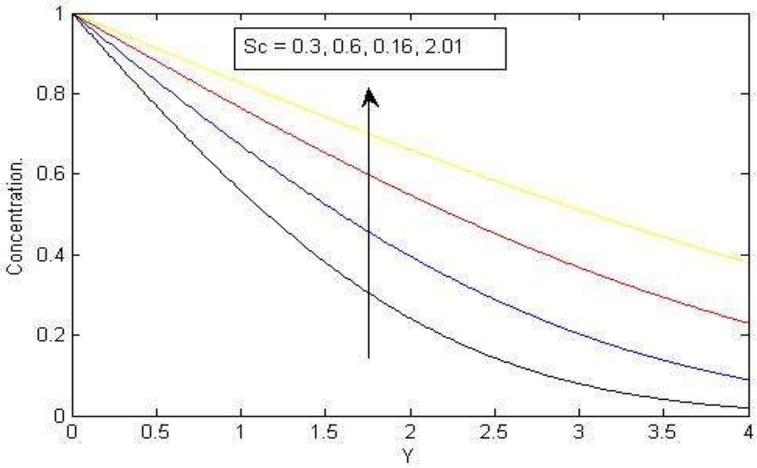

Figure 4.12 concentration profiles for different value of Sc.

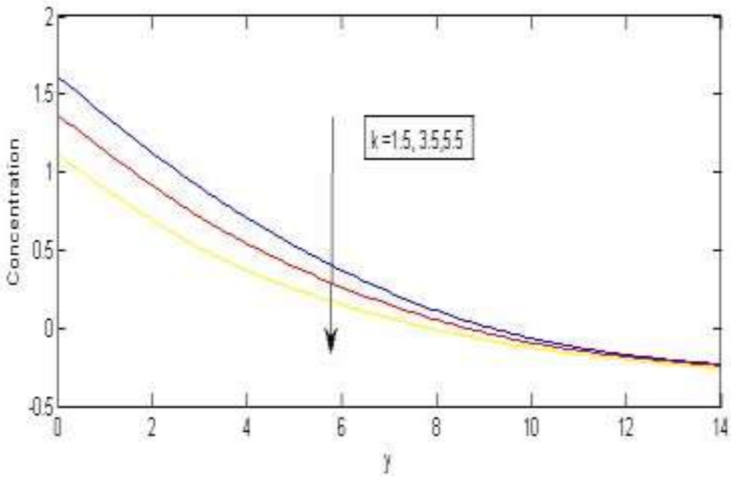

Figure 4.13 concentration profiles for different value of k.

IV. CONCLUSIONS

Analytical solution of an unsteady MHD flow of heat and mass transfer for second grade fluid involving thermal radiation and Chemical Reaction has been carried out. The dimensional governing equations are solved and analyze by Laplace transform technique and then computed for different parameters using MATLAB software. The effect of different parameters such as Schmidt number, prandtl number, mass Grashof number, thermal Grashof number and time are studied the results for flow profile of velocity, temperature and concentration were obtained.

I observed that the velocity profile increases with increasing value ofSc, Gc, R, t,Pr,Gr and decrease in M. The temperature profile depicsthe decrease with increase in R, t and Pr, And while the concentration profile showsthe increase in Sc and decreases in t and k,

REFERENCES

[1]. Aziz, A. (2012). Examined the analytical solution for unsteady magneto-hydro-dynamic Flow of third grade fluid past a porous plate with a porous medium due to an Arbitrary well velocity. Archives of Mechanics, 57(5), 405–416.

[2]. Afify, A.A. (2004) studied the effect of radiation on free convective flow and mass transfer Past a Vertical isothermal cone surface with chemical reaction in the presence of a Transverse magnetic field." Int. J. Therm. Sci. **42**(1), 23–31 .

[3]. Chamkha, A.J.(2012) investigated radiation effects on mixed convection flow over a wedge Embedded in a porous medium filled with Nano fluid." Int. J. Eng. Sci. **40**(13), 1511–1527 (2002).

[4]. Chaudhary,R.C. and Preeti (2007). Investigated the solution for the transient free convention flow Of a viscous fluid between two infinite vertical parallel plates in the presence of Radiation. " Acta Mech.**138**, 13–19.

A. R. Vijayalakshmi and Mr. J. Ravikumar (2016)Analytical Solution of Vertical Plate in a rotating fluid with variableTemperature, Uniform Mass Diffusion in the Presence of MHD andThermal RadiationInternational Journal of Applied Engineering Research ISSN 0973-4562 Volume 11, Number 4 (2016) pp 2343-2348

[5]. Garg. B. P. (2014) examined an unsteady MHD free convective flow of an electrically Conducting, viscos-elastic, Incompressible fluid through a porous medium bounded Between two insulated infinite vertical plates in the presence of hall current and Thermal radiation. ," Theor. Appl. Mech.**35**(4), 333–346

Ibrahim,F.S.(2008) .analyzed the effect of chemical reaction and radiation absorption on the Unsteady MHD free convection flow past a semi-infinite vertical permeable moving Plate with heat source and suction," J. Phys. Soc. Jpn. **52**, 470–477.

[6]. J. Uwanta, MurtalaSani (2014) Heat Mass Transfer Flow past an Infinite Vertical Plate with
Variable Thermal Conductivity, Heat Source and ChemicalReactionTheInternational Journal Of Engineering And Science (IJES) || Volume || 3 || Issue || 5 || Pages || 77-89 || 2014 ||ISSN (e): 2319 – 1813 ISSN (p): 2319 – 1805

Jordan P.M. (2004) note on startup, plane coquette flow involving second grade fluids Code: 7181, Naval Research Laboratory, Stennis Space Center, MS 39529

[7]. Kishore (2011) studied effects of thermal radiation and viscous dissipation on MHD heat and Mass diffusion flow past an oscillating vertical plate embedded in a porous medium With variable surface conditions," Int. J. Eng. Sci. **32**, 133–139.

[8]. Kim,Y.J (2000) considered the unsteady magneto hydrodynamic convective heat transfer past A Semi-infinite vertical porous moving plate with variable suction.

[9]. Khan, A., I. Khan, F. Ali, S. Ulhaq and S. Shafie(2014). Effects of wall shear stress on unsteady
MHD conjugate flow in a porous medium with ramped wall temperature. PLOS ONE, 9(3), 1-12.

[10]. Muthucumaraswamy, R. and Chandrakala (2006) investigated radiative heat and mass Transfer Effects on a moving isothermal vertical plate in the presence of chemical Reaction," Alexandria Eng. J.**55**, 2125–2137.

[11]. Muthucumaraswamy, R. (2011) considered thermal radiation effects on hydro magnetic free Convection and mass transfer flow past an infinite oscillating isothermal plate in The presence of chemical reaction of first order. International. Journal of Thermal Sciences 47 (7), 848-857

[12]. Nandkeolyar (2013) .investigated unsteady hydromagnetic natural convection flow of a dusty Fluid past an impulsively moving vertical plate with ramped temperature in

the. Presence of thermal radiation Mass Transfer/WaermeundStoffuebertragung 41(12), 1095-1102.

[13]. Nandkeolyar (2013) considered unsteady hydro magnetic heat and mass transfer flow of a heat Radiating and chemically reactive fluid past a flat porous plate with ramped wall Temperature.Chem.Eng.Comm.199 (5), 628-643.

[14]. Ogulu and Makinde (2009) considered unsteady hydromagnetic free convection flow of a Dissipative and radiative fluid past a vertical plate with constant heat flux.Ind.J.Sci.Tech. 3, 398-401.

[15]. P.T.Hemamalini, N.Suresh Kumar (2015) An Unsteady Flow past an Accelerated Infinite Vertical Plate withthe Variable Temperature and Uniform Mass Diffusion throughthe Porous MediumJournal of Mathematics (IOSR-JM)e-ISSN: 2278-5728, p-ISSN: 2319-765X. Volume 11, Issue 3 Ver. III (May - Jun. 2015), PP 78-85

[16]. Rajesh (2011) investigated the effects of thermal radiation and first order chemical reaction on Unsteady MHD free convection and mass transfer flow of a dissipative fluid past an Infinite vertical porous plate with ramped wall temperature. J. Appl. Sci. 11,1096-1104.
Saravana, R.(2011).studied the mass transfer effects on MHD viscous flow past an impulsively Started infinite vertical plate with constant mass flux.” Appl. Math. Comput. **217**(2), 685–688.

[17]. Sudheer B.(2011) studied radiation and chemical reaction effects on unsteady MHD convection Flow past a vertically moving porous plate embedded in a porous medium with Viscousdissipation.TurkishJ.Eng,Env.Sci.35, 187-198.

[18]. Singh,J. (2001) analyzed the MHD free convection and mass transfer flow with heat source And Thermal diffusion. The International Journal of Engineering Science 37 (13), 1723-1736.

[19]. S.M.M.EL- kabir, M.Modather and A.M. Rashad (2015) Heat and Mass Transfer by Unsteady Natural Convectionover a Moving Vertical Plate Embedded in a SaturatedPorous Medium with Chemical Reaction, Soret andDufour EffectsJournal of Applied Fluid Mechanics, Vol. 8, No. 3, pp. 453-463,

[20]. Samad,M.A and Mohebujjaman (2009).A study on MHD heat and mass transfer free Convection Flow along a vertical stretching sheet in the presence of magnetic field with heat Generation Thermal Science 13(2), 171-181.

[21]. Siddiqui. (2014) study two dimensional flow of an incompressible second grade fluid isInvestigated in a symmetrically divergent channel of varying width in the presence Of a transverse magnetic field.Chem. Ind. Chem. Eng. Quar. 17, 189–198.

[22]. Uwanta, I. J., & Yale, I. D. (2014). Analytical Solution Of Unsteady Flow Past An Accelerated Vertical Plate With Constant Temperature and Variable Mass Transfer. International Journal of Computational Engineering Research||Vol, 4, 50–55.

[23]. Uwanta, I. J. and M. Sani (2012). Transient convection fluid flow with heat flux in an infinite vertical plate with chemical mass transfer. Int. J. Appl. Inf. Sys. 4(7), 21-27.

[24]. Veerakish M. and A. J.Chamkha (2018) in which they carry out on Hall Effect for unsteady MHD flow of second grade fluid through porous medium with ramp well temperature And ramp surface concentration.Chamchuri J. Mathematics 1(2), 79-92.