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Application Of Scheffe's Mathematical Model For The Optimization Of Compressive Strength Of Nylon Fibre Reinforced Concrete (NFRC)

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ABSTRACT
Sometimes, concrete mix design can be proved to be uneconomical and laborious due to the rigorous, time consuming nature with several trial mixes before a desired strength or quality of mixture is attained in a typical empirical method. Owing to this problem, optimization process is usually sought for.This research work is aimedat applying Scheffe's Second Degree (5,2) Mathematical Model to optimize the compressive strength of Nylon Fibre Reinforced Concrete (NFRC).In this study, Scheffe's (5,2) Mathematical Model derived byNwachukwu and others (2017) for five component mixture will be used to optimize the mix proportion that will produce the maximum strength of NFRC. Using Scheffe's Simplex method, the compressive strength of NFRC was determined for different mix ratios. Control experiments were also performed and the compressive strengths evaluated. After the tests have been conducted, the adequacy of the model was tested using Student's t-test.The test statistics confirmed the adequacy of the model. Maximum compressive strength for the NFRC using Scheffe's(5,2) model was obtained as 21.96N/mm^2. This optimum value meets the required strength since structural concrete elements are generally made with concrete having a compressive strength of 20 to 35 MPa (or 20 to35N/mm^2). Since the goal of every Engineering project is to satisfy the safety, economic and aesthetic criteria, it then means that optimized NFRC based on Scheffe's model can produce the required compressive strength needed in construction projects such as Bridge, Building pillars, Sidewalks, Building floors, Drainage pipes, Septic tanks etc., still satisfying all the required criteria because of the presence of Nylon Fibres.Therefore, major professionals in the construction industry are implored to use the optimizedNFRC, mainly for its economic, aesthetic and safety advantages.

Keywords: NFRC, Scheffe's (5,2) Mathematical Model, Optimization, Compressive strength ,Regression

I. INTRODUCTION

Optimization of the concrete mixture design is a process of search for a mixture for which the sum of the costs of the ingredients is lowest, yet satisfying the major required performance of concrete, such as workability, flexibility, homogeneity, strength and durability . Scheffe's Mathematical Models are typical examples of optimization model . In this study, Scheffe's Second Degree Mathematical Model for five components mixtures (namely cement, fine aggregate, coarse aggregate, water and nylon fibre) will be on focus.

There is no doubt that concrete has remained a very importantconstruction material widely used since ancient time. According to Neville(1990), concrete plays a crucial part in all building structures owing to its numerous advantages which ranges from low built in fire resistance, high compressive strength to low maintenance. At the same time, it also has a major limitation which is that concrete is inherently a brittle material. Also, concrete is known for its problem associated with its low tensile strength compared to its compressive strength. As a result of this, many new technologies of concrete and some modern concrete specification approach were introduced. One of the technologies introduced for concrete was the addition of steel bars to reinforce its tension zone. This enables concrete gain an amount of tensile strength and thus reducing its brittle nature.There is no doubt that concrete reinforced with steel reinforcement is a widely used construction materials. However, these types of reinforced concrete structures are experiencing deterioration when exposed to deleterious environment. These sorts of deterioration often reduce the service life of the structure. Based on several further researches over the years, the reinforcement (usually steel bars) has been replaced with other materials like fibre (glass fibre, polypropylene fibre, nylon fibre, steel fibre , plastic fibre etc.) to further increase both its tensile strength and compressive strength and also, produce light weighted reinforced concrete unlike when reinforced with steel bars.Fibre-Reinforced Concrete (FRC), in general, is concrete that has fibrous materials mixed in to increase the concrete's durability and structural integrity. Thus all fibres reduce the concrete's need for steel reinforcements. And since fibre reinforcement tends to be less expensive than steel bars (and less likely to corrode), it makes FRC more cost-effective.In a nut shell , fibres can improve the concrete's: Workability, Flexibility, Tensile strength, Durability—by controlling and reducing crack widths, Ductility, Cohesion, Freeze-thaw resistance, Abrasion- and impact-resistance, Resistance to plastic shrinkage while curing, Resistance to cracking, Shrinkage at an early age, Fire resistance, Homogeneity etc..

Nylon Fibre Reinforced Concrete (NFRC) is concrete mixture where the conventionally steel reinforcement in concrete production is replaced with nylon fibre. Nylon is a synthetic plastic material composed of polyamides of high molecular weight. Nylons are high-performance semi-crystalline thermoplastics with attractive physical and mechanical properties that provide a wide range of end-use performances important in many industrial and construction applications. Nylon Fibres (NF) are produced when nylon are drawn, cast or extruded through spinnerets from a melt or solution. A typical example of NF is shown in Figure 1. Nylon fibre has high resistance to wear, heat and chemicals and also cheaper when compared with conventional steel reinforcement. It is these basic characteristics that make NF finds extensive use as construction material in highly resistant concrete production.. Concrete's compressive strength is one of the most useful properties of concrete and in most structural applications, concrete primarily resists compressive stress.Compressive strength of concrete is the Strength of hardened concrete measured by the compression test. It is a measure of the concrete's ability to resist loads which tend to compress it. It is measured by crushing cylindrical concrete specimens in compression testing machine or universal testing machineThe compressive strength of the concrete cube test provides an idea about all the characteristics of concrete.

The present study therefore focuses on the application of scheffe's second degree mathematical model in optimizing thecompressive strength of NFRC. Few researchers have carryout investigations on either properties of NFRC or effects of NF in concrete. For instance, Ganesh Kumar and others (2019) have carried out a study on waste nylon fibre in concrete.Samrose and Mutsuddy (2019) have investigated the durability of NFRC. Hossain and others (2012) have also investigated the effect of NF in concrete rehabilitation. Ali and others (2018) have carried out a study on NFRC through partial replacement of cement with metakaolin. Song and others (2005) also investigated the strength properties of NFRC and PFRC respectively. On optimization, a lot of researchers have used Scheffe's method to carryout one form of optimization project or the other. For example, Nwakonobi and Osadebe (2008) used Scheffe's model to optimize the mix proportion of Clay- Rice Husk Cement Mixture for Animal Building. Ezeh and Ibearugbulem (2009) applied Scheffe's model to optimize the compressive cube strength of River Stone Aggregate Concrete. Scheffe's model was used by Ezeh and others (2010a) to optimize the compressive strength of cement- sawdust Ash Sandcrete Block. Again Ezeh and others (2010b) optimized the aggregate composition of laterite/ sand hollow block using Scheffe's simplex method. The work of Ibearugbulem (2006) and Okere(2006) were also based on the use of Scheffe' mathematical model in the optimization of compressive strength of Perwinkle Shell- Granite Aggregate Concrete and optimization of the Modulus of Rupture of Concrete respectively. Obam (2009) developed a mathematical model for the optimization of strength of concrete using shear modulus of Rice Husk Ash as a case study. The work of Obam (2006) was based on four component mixtures, that is Scheffe's(4,2) and Scheffe's(4,3). Nwachukwu and others (2017) developed and employed Scheffe's Second Degree Polynomial model to optimize the compressive strength of Glass Fibre Reinforced Concrete (GFRC). Also,Nwachukwu and others (2022a)

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developed and used Scheffe's Third Degree Polynomial model, abbreviated as Scheffe's (5,3) to optimize the compressive strength of GFRC and compared the results with his previous work, Nwachukwu and others (2017). Again, Nwachukwu and others (2022b) used Scheffe's (5,2) optimization model to optimize the compressive strength of Polypropylene Fibre Reinforced Concrete (PFRC). From the forgoing, it can be envisaged that no work has been done on the use of Scheffe's method to optimize the compressive strengthof NFRC .Henceforth, the need for this research work.

Fig. 1 : Typical sample of Nylon Fibre

II. BACKGROUND OFSCHEFFE'S SECOND DEGREE OPTIMIZATION MODEL

A simplex lattice is a structural representation of lines joining the atoms of a mixture, and these atoms are constituent components of the mixture. For NFRC mixture, the constituent elements are the water, cement, fineaggregate (sand), coarse aggregate and nylon fibre. That is to say that, a simplex of five-component mixture is a four-dimensional solid. See Nwachukwu and others (2017).According to Obam (2009), mixture components are subject to the constraint that the sum of all the components must be equal to 1. That is:

$$X_1 + X_2 + X_3 + \ldots + X_q = 1 \; ; \quad \Rightarrow \sum_{i=1}^{q} X_i = 1 \qquad \textbf{(1)}$$

where $X_i \geq 0$ and $i = 1, 2, 3\ldots q$, and q = the number of mixtures

2.1. THE SIMPLEX LATTICE DESIGN

The (q, m) simplex lattice design are characterized by the symmetric arrangements of points within the experimental region and a well-chosen polynomial equation to represent the response surface over the entire simplex region(Aggarwal, 2002). The (q, m) simplex lattice design given by Scheffe, according to Nwakonobi and Osadebe (2008) contains ${}^{q+m-1}C_m$ points where each components proportion takes (m+1) equally spaced values $X_i = 0, \frac{1}{m}, \frac{2}{m}, \frac{3}{m}, \ldots, 1; \quad i = 1, 2, \ldots, q$ ranging between 0 and 1 and all possible mixture with these component proportions are used, and m is scheffe's polynomial degee, which in this present study is 2.

For example a (3, 2) lattice consists of ${}^{3+2-1}C_2$ i.e. ${}^{4}C_2$ = 6 points. Each X_i can take m+1 = 3 possible values; that is $x = 0, \frac{1}{2}, 1$with which the possible design points are:

$$(1,0,0), (0,1,0), (0,0,1), \left(\tfrac{1}{2},\tfrac{1}{2},0\right), \left(0,\tfrac{1}{2},\tfrac{1}{2}\right), \left(\tfrac{1}{2},0,\tfrac{1}{2}\right)$$

.

According to Obam (2009), a Scheffe's polynomial function of degree, m in the q variable X_1, X_2, X_3, X_4 … X_q is given in form of:

$$Y = b_0 + \sum bi\; xi + \sum bij xj + \sum bi\, jxjxk + + \sum bi^{j}{}_2 + \ldots i_n x i_2 x i_n \qquad \textbf{(2)}$$

where ($1 \leq i \leq q$, $1 \leq i \leq j \leq k \leq q$, $1 \leq i_1 \leq i_2 \leq \ldots \leq i_n \leq q$ respectively) , b = constant coefficients and Y is the response(the response is a polynomial function of pseudo component of the mix) which represents the property under study, which ,in this case is the compressive strength.

This research work is based on the Scheffe's (5, 2) simplex.. The actual form of Eqn. (2) has already been developed for five component mixture, based on Scheffe's second degree polynomial by Nwachukwu and others (2017) and will be applied subsequently in this work.

2.2. RELATIONSHIP BETWEEN PSEUDO AND ACTUAL COMPONENTS.

In Scheffe's mix design, the relationship between the pseudo components and the actual components has been established as:

$$Z = A * X \qquad \textbf{(3)}$$

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where Z is the actual component; X is the pseudo component and A is the coefficient of the relationship

Re-arranging the equation

$$X = A^{-1} * Z \quad \textbf{(4)}$$

2.3. POLYNOMIAL EQUATION FOR SCHEFFE'S (5, 2) LATTICE

The regression or polynomial equation by Scheffe(1958), otherwise known as response is given in Eqn.(2) .Hence, for Scheffe's (5,2) simplex lattice, the regression equation for five component mixtures has been derived from Eqn.(2) by Nwachukwu and others (2017) and is given as follows:

$$Y = b_0 + b_1X_1 + b_2X_2 + b_3X_3 + b_4X_4 + b_5X_5 + b_{11}X_1^2 + b_{12}X_1X_2 + b_{13}X_1X_3 + b_{14}X_1X_4 + b_{15}X_1X_5 + b_{22}X_2^2 + b_{23}X_2X_3 + b_{24}X_2X_4 + b_{25}X_2X_5 + b_{33}X_3^3 + b_{34}X_3X_4 + b_{35}X_3X_5 + b_{44}X_4^4 + b_{45}X_4X_5 + b_{55}X_5^5 \quad \textbf{(5)}$$

$$= ß_1X_1 + ß_2X_2 + ß_3X_3 + ß_4X_4 + ß_5X_5 + ß_{12}X_1X_2 + ß_{13}X_1X_3 + ß_{14}X_1X_4 + ß_{15}X_1X_5 + ß_{23}X_2X_3 + ß_{24}X_2X_4 + ß_{25}X_2X_5 + ß_{34}X_3X_4 + ß_{35}X_3X_5 + ß_{45}X_4X_5 \quad \textbf{(6)}$$

Where,

$ß_1 = b_0 + b_1 + b_{11}$; $ß_2 = b_0 + b_2 + b_{22}$; $ß_3 = b_0 + b_3 + b_{33}$; $ß_4 = b_0 + b_4 + b_{44}$; $ß_5 = b_0 + b_5 + b_{55}$; $ß_{12} = b_{12} – b_{11} – b_{22}$; $ß_{13} = b_{13} – b_{11} – b_{33}$; $ß_{14} = b_{14} – b_{11} – b_{44}$; $ß_{15} = b_{15} – b_{11} – b_{55}$; $ß_{23} = b_{23} – b_{22} – b_{33}$; $ß_{24} = b_{24} – b_{22} – b_{44}$; $ß_{25} = b_{25} – b_{22} – b_{55}$; $ß_{34} = b_{34} – b_{35} – b_{44}$; $ß_{35} = b_{35} – b_{33} – b_{55}$;
$ß_{45} = b_{45} – b_{44} – b_{55}$. **(7)**

2.4 . MIXTURE DESIGN MODEL

The procedure for the determination of the coefficient of Scheffe's (5,2) regression model has been explained by Nwachukwu and others (2017). After coefficients evaluation, the equation for the mixture design model is as shown in Eqn.(8).

$$Y = X_1(2X_1 – 1)Y_1 + X_2(2X_2 – 1)Y_2 + X_3(2X_3 – 1)Y_3 + X_4(2X_4 – 1)Y_4 + X_5(2X_5 – 1)Y_5 + 4Y_{12}X_1X_2 + 4Y_{13}X_1X_3 + 4Y_{14}X_1X_4 + 4Y_{15}X_1X_5 + 4Y_{23}X_2X_3 + 4Y_{24}X_2X_4 + 4Y_{25}X_2X_5 + 4Y_{34}X_3X_4 + 4Y_{35}X_3X_5 + 4Y_{45}X_4X_5 \quad \textbf{(8)}$$

Eqn. (8) is the second degree based mix design model for the optimization of a concrete mix that comprises five components, such as NFRC. Y_1, Y_2 …….. $Y_{45}$are determined through laboratory test.

2.5. ACTUAL AND PSEUDO MIX RATIO

The requirement of simplex lattice design based on Eqn. (1) criteria makes it impossible to use the conventional mix ratios such as 1:2:4, 1:3:6, etc., at a given water/cement ratio for the actual mix ratio. This necessitates the transformation of the actual components proportions to meet the above criterion. Such transformed ratios, $x_1^{(i)}$, $x_2^{(i)}$, $x_3^{(i)}$, for the ith experimental points are called pseudo – components (or coded components). Based on experience and previous knowledge from literature, the following arbitrary prescribed mix proportions are always chosen for the five points/vertices. See the works of Nwachukwu and others (2017), for different vertices.

A_1 (0.67:1: 1.7: 2:0.5); A_2 (0.56:1:1.6:1.8:0.8); A_3 (0.5:1:1.2:1.7:1); A_4 (0.7:1:1:1.8:1.2) and A_5 (0.75:1:1.3:1.2:1.5), which represent water/cement ratio, cement, fine aggregate, coarse aggregate and nylon fibre.

For the pseudo mix ratio, the following corresponding mix ratios at the vertixes are always chosen: A_1(1:0:0:0:0), A_2(0:1:0:0: 0), A_3(0:0:1:0:0), A_4(0:0:0:1:0), and A_5(0:0:0:0:1)

For the transformation of the actual component, z to pseudo component, x, and vice versa ,Eqns.(3)and (4) are used.

Substituting the mix ratios from point A_1 into Eqn. (3) gives:

$$\begin{Bmatrix} 0.67 \\ 1 \\ 1.7 \\ 2 \\ 0.5 \end{Bmatrix} = \begin{pmatrix} A_{11} & A_{12} & A_{13} & A_{14} & A_{15} \\ A_{21} & A_{22} & A_{23} & A_{24} & A_{25} \\ A_{31} & A_{32} & A_{33} & A_{34} & A_{35} \\ A_{41} & A_{42} & A_{43} & A_{44} & A_{45} \\ A_{51} & A_{52} & A_{53} & A_{54} & A_{55} \end{pmatrix} \begin{Bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} \quad \textbf{(9)}$$

Solving, we obtain :

. $A_{11} = 0.67$, $A_{21} = 1$, $A_{31} = 1.7$, $A_{41} = 2$, and $A_{51} = 0.5$

The same goes for point 2 through point 5 and the overallresults are depicted in Eqn. (10)

$$\begin{Bmatrix} Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \\ Z_5 \end{Bmatrix} = \begin{pmatrix} 0.67 & 0.56 & 0.5 & 0.7 & 0.75 \\ 1 & 1 & 1 & 1 & 1 \\ 1.7 & 1.6 & 1.2 & 1 & 1.3 \\ 2 & 1.8 & 1.7 & 1.8 & 1.2 \\ 0.5 & 0.8 & 1 & 1.2 & 1.5 \end{pmatrix} \begin{Bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \\ X_5 \end{Bmatrix} \quad \textbf{(10)}$$

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Therefore , from Eqn.(4), we obtain :

$$\begin{Bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \\ X_5 \end{Bmatrix} = \begin{pmatrix} 0.67 & 0.56 & 0.5 & 0.7 & 0.75 \\ 1 & 1 & 1 & 1 & 1 \\ 1.7 & 1.6 & 1.2 & 1 & 1.3 \\ 2 & 1.8 & 1.7 & 1.8 & 1.2 \\ 0.5 & 0.8 & 1 & 1.2 & 1.5 \end{pmatrix}^{-1} \begin{Bmatrix} Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \\ Z_5 \end{Bmatrix} \quad (11)$$

Thus

$$\begin{Bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \\ X_5 \end{Bmatrix} = \begin{pmatrix} 3.99 & 10.37 & -2.14 & -3.05 & -4.62 \\ -4.88 & -21.46 & 5.40 & 5.95 & 7.31 \\ -1.78 & 17.83 & -3.49 & -4.20 & -4.62 \\ 1.04 & -9.24 & 0.37 & 3.28 & 2.69 \\ 1.63 & 3.49 & -0.13 & -1.98 & -0.77 \end{pmatrix} \begin{Bmatrix} Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \\ Z_5 \end{Bmatrix} \quad \mathbf{(12)}$$

Considering the mix ratios at the midpoints, we have:
A_{12} (0.5, 0.5, 0, 0, 0); A_{13} (0.5, 0, 0.5, 0, 0); A_{14} (0.5, 0, 0, 0.5, 0); A_{15} (0.5, 0, 0, 0, 0.5); A_{23} (0, 0.5, 0.5, 0,0); A_{24} (0, 0.5, 0, 0.5, 0); A_{25} (0, 0.5, 0, 0, 0.5); A_{34} (0, 0, 0.5, 0.5, 0); A_{35} (0, 0, 0.5, 0, 0.5) and A_{45} (0, 0, 0, 0.5, 0.5)
Substituting these pseudo mix ratios in turn into Eqn. (10) will give the corresponding actual mix ratio

For point A_{12}

$$\begin{Bmatrix} Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \\ Z_5 \end{Bmatrix} = \begin{pmatrix} 0.67 & 0.56 & 0.5 & 0.7 & 0.75 \\ 1 & 1 & 1 & 1 & 1 \\ 1.7 & 1.6 & 1.2 & 1 & 1.3 \\ 2 & 1.8 & 1.7 & 1.8 & 1.2 \\ 0.5 & 0.8 & 1 & 1.2 & 1.5 \end{pmatrix} \begin{pmatrix} 0.5 \\ 0.5 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{Bmatrix} 0.62 \\ 1 \\ 1.65 \\ 1.90 \\ 0.65 \end{Bmatrix} \quad \mathbf{(13)}$$

Solving ,
$Z_1 = 0.62$, $Z_2 = 1$, $Z_3 = 1.65$, $Z_4 = 1.9$, $Z_5 = 0.65$
The rest results are depicted in Table 1
In order to generate the regression coefficients, fifteen experimental tests are carried out and the corresponding mix ratio are as shown in Table 1 .

Table 1: Actual Mix Ratios for theScheffe's (5, 2) Lattice at initial experimental point

Points	Water/cement ratio	Cement	Fine Aggregate	Coarse Aggregate	Nylon fibre	Response
1	0.67	1	1.7	2	0.5	Y_1
2	0.56	1	1.6	1.8	0.8	Y_2
3	0.5	1	1.2	1.7	1	Y_3
4	0.7	1	1	1.8	1.2	Y_4
5	0.75	1	1.3	1.2	1.5	Y_5
12	0.62	1	1.65	1.9	0.65	Y_{12}
13	0.59	1	1.45	1.85	0.75	Y_{13}
14	0.69	1	1.35	1.9	0.85	Y_{14}
15	0.71	1	1.5	1.6	1	Y_{15}
23	0.53	1	1.4	1.75	0.9	Y_{23}
24	0.63	1	1.3	1.8	1	Y_{24}
25	0.66	1	1.45	1.5	1.15	Y_{25}
34	0.6	1	1.1	1.75	1.1	Y_{34}
35	0.63	1	1.25	1.45	1.25	Y_{35}
45	0.73	1	1.15	1.5	1.5	Y_{45}

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2.7.CONTROL POINTS

For the purpose of this research, fifteen different controls were predicted which according to Scheffe, their summation must conform with Eqn.(1) . They are as follows:

C_1 = (0.25, 0.25, 0.25, 0.25, 0), C_2 = (0.25, 0.25, 0.25, 0, 0.25), C_3 = (0.25, 0.25, 0, 0.25, 0.25), C_4 = (0.25, 0, 0.25, 0.25, 0.25), C_5 = (0, 0.25, 0.25, 0.25, 0.25), C_{12} = (0.20, 0.20, 0.20, 0.20, 0.20), C_{13} = (0.30, 0.30, 0.30, 0.10, 0), C_{14} = (0.30, 0.30, 0.30, 0, 0.10), C_{15} = (0.30, 0.30, 0, 0.30, 0.1), C_{23} = (0.30, 0, 0.30, 0.30, 0.1), C_{24} = (0, 0.30, 0.30, 0.30, 0.10), C_{25} = (0.10, 0.30, 0.30, 0.30, 0), C_{34} = (0.30, 0.10, 0.30, 0.30, 0), C_{35} = (0.30, 0.30, 0.10, 0.30, 0), C_{45} = (0.10, 0.20, 0.30, 0.40, 0),

Substituting into Eqn.(10) , we obtain the values of the actual mixes as follows:

Control 1 C_1

$$\begin{Bmatrix} Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \\ Z_5 \end{Bmatrix} = \begin{pmatrix} 0.67 & 0.56 & 0.5 & 0.7 & 0.75 \\ 1 & 1 & 1 & 1 & 1 \\ 1.7 & 1.6 & 1.2 & 1 & 1.3 \\ 2 & 1.8 & 1.7 & 1.8 & 1.2 \\ 0.5 & 0.8 & 1 & 1.2 & 1.5 \end{pmatrix} \begin{pmatrix} 0.25 \\ 0.25 \\ 0.25 \\ 0.25 \\ 0 \end{pmatrix} = \begin{Bmatrix} 0.61 \\ 1 \\ 1.38 \\ 1.83 \\ 0.5 \end{Bmatrix} \quad (14)$$

The rest of the results are represented in Table 2

Table 2: Actual (Z_i) and Pseudo (X_i) component of Scheffe's (5, 2) simplex lattice control point

Points	Pseudo					Actual				
	water	cement	Fine agg	Coarse agg	Nylon fibre	water	cement	Fine agg	Coarse agg	Nylon fibre
C_1	0.25	0.25	0.25	0.25	0	0.61	1	1.38	1.83	0.5
C_2	0.25	0.25	0.25	0	0.25	0.62	1	1.45	1.68	0.8
C_3	0.25	0.25	0	0.25	0.25	0.67	1	1.4	1.7	1
C_4	0.25	0	0.25	0.25	0.25	0.66	1	1.3	1.68	1.2
C_5	0	0.25	0.25	0.25	0.25	0.63	1	1.28	1.63	1.5
C_{12}	0.2	0.2	0.2	0.2	0.2	0.64	1	1.36	1.7	0.65
C_{13}	0.3	0.3	0.3	0.1	0	0.59	1	1.45	1.83	0.75
C_{14}	0.3	0.3	0.3	0	0.1	0.59	1	1.48	1.77	0.85
C_{15}	0.3	0.3	0	0.3	0.1	0.65	1	1.42	1.8	1
C_{23}	0.3	0	0.3	0.3	0.1	0.64	1	1.3	1.77	0.9
C_{24}	0	0.3	0.3	0.3	0.1	0.6	1	1.27	1.71	1
C_{25}	0.1	0.3	0.3	0.3	0	0.6	1	1.31	1.79	1.15
C_{34}	0.3	0.1	0.3	0.3	0	0.62	1	1.33	1.83	1.1
C_{35}	0.3	0.3	0.1	0.3	0	0.63	1	1.41	1.85	1.25
C_{45}	0.1	0.2	0.3	0.4	0	0.61	1	1.25	1.79	1.35

III. MATERIALS AND METHODS

3.1 MATERIALS

The materials investigated are the mixture of cement, water, fine and coarse aggregate and nylon fibre. The cement is Dangote cement, a brand of Ordinary Portland Cement, conforming to British Standard Institution BS 12 (1978). The fine aggregate, whose size ranges from 0.05 - 4.5mm was procured from the local river. Crushed granite of 20mm size downgraded to 4.75mm obtained from a local stone market was used in the experimental investigation. It should be noted that when mixing fibre-reinforced concrete, the maximum size of the coarse aggregates should not be more than 10 mm to avoid reducing the strength of the concrete. NylonFibre used is of 50mm in length and 0.35 mm in diameter as shown in Figure 1. Also, water drawn from the clean water source was used in the experimental investigation .

3.2. METHOD

3.2.1. SPECIMEN PREPARATION / BATCHING/ CURING

The specimens for the compressive strength were concrete cubes. They were cast in steel mould measuring 150mm*150mm*150mm. The mould and its base were damped together during concrete casting to prevent leakage of mortar. Thin engine oil was applied to the inner surface of the moulds to make for easy removal of the cubes. Batching of all the constituent material was done by weight using a weighing balance of 50kg capacity based on the adapted mix ratios and water cement ratios. A total number of 30 mix ratios were to be used to produce 60 prototype concrete cubes. Fifteen (15) out of the 30 mix ratios were as control mix ratios to produce 30 cubes for the conformation of the adequacy of the mixture design given by the Eqn. (8).. Curing commenced 24hours after moulding. The specimens were removed from the moulds and were placed in clean water for curing. After 28days of curing the specimens were taken out of the curing tank.

3.2.2. COMPRESSIVE STRENGTH TEST

Procedure for compressive strength testing was done in accordance to BS 1881 – part 116 (1983) - Method of determination of compressive strength of concrete cube .Testing was conducted immediately after the specimen was removed from the curing process and dried. Smooth surface metal plate (serving as base plate) was placed at the bottom and top of each of the specimen cube so as to ensure uniform distribution of load for accurate crushing. Two samples were crushed for each mix ratio. The compressive strength was then calculated using the formula below:

Compressive Strength = Average failure Load (N) P **(15)**

Cross- sectional Area (mm^2) A

IV. RESULTS AND DISCUSSION

4.1 COMPRESSIVE STRENGTH TEST RESULTS FOR NFRC BASED ON SCHEFFE'S (5,2) SIMPLEX LATTICE

4.1.1 EXPERIMENTAL TEST RESULTS

The results of compressive strength test based on Eqn. (15) are shown in Table 3

Table 3: Compressive Strength Test Results for NFRC Based on Eqn.(15)

Points	Experiment no	Response Y_i, N/mm^2	Response symbol	$\sum Y_i$	Average response Y, N/mm^2
1	1A 1B	19.52 20.73	Y_1	40.25	20.13
2	2A 2B	15.12 16.35	Y_2	31.47	15.74
3	3A 3B	18.72 19.81	Y_3	38.53	19.27
4	4A 4B	17.43 16.55	Y_4	33.98	16.99
5	5A 5B	20.11 19.34	Y_5	39.45	19.73
12	6A 6B	18.40 17.67	Y_{12}	36.07	18.04
13	7A 7B	16.78 17.32	Y_{13}	34.10	17.05
14	8A 8B	19.32 18.56	Y_{14}	37.88	18.94
15	9A 9B	19.62 19.22	Y_{15}	38.84	19.42
23	10A 10B	21.71 22.21	Y_{23}	43.92	21.96
24	11A 11B	15.84 17.37	Y_{24}	33.21	16.61

25	12A 12B	19.66 18.19	Y_{25}	37.85	18.93
34	13A 13B	17.93 18.74	Y_{34}	36.67	18.34
35	14A 14B	19.41 18.71	Y_{35}	38.12	19.06
45	15A 15B	14.64 12.88	Y_{45}	27.52	13.76

4.1.2SCHEFFE'S (5,2) MATHEMATICAL MODEL EQUATION FOR OPTIMIZATION OF COMPRESSIVE STRENGTH OF NFRC.

By substituting the values of Y_1, Y_2, …. Y_{45} from Table 3 into Eqn. (8) yields:

$$Y = 20.13X_1(2X_1 - 1) + 15.74X_2(2X_2 - 1) + 19.27X_3(2X_3 - 1) + 16.99X_4(2X_4 - 1) + 19.73X_5(2X_5 - 1) + 4(18.04)X_1X_2 + 4(17.05)X_1X_3 + 4(18.94)X_1X_4 + 4(19.42)X_1X_5 + 4(21.96)X_2X_3 + 4(16.61)X_2X_4 + 4(18.93)X_2X_5 + 4(18.34)X_3X_4 + 4(19.06)X_3X_5 + 4(13.76)X_4X_5 \quad \mathbf{(16)}$$

Equation (16) is the Scheffe's (5,2) mathematical model equation from which the optimization of Compressive Strength of NFRC is based.

4.1.3. EXPERIMENTAL (CONTROL) TEST RESULTS

The response (compressive strength) of control points from experimental tests is shown in Table 4

Table 4: Response of Control Points from Experimental Tests for NFRC

Points	Experiment no	Response N/mm^2	Z_1	Z_2	Z_3	Z_4	Z_5	Average response		
C1	1A 1B	18.35 19.07	0.61	1	1.38	1.83	0.5	18.71	10.42	
C2	2A 2B	15.11 14.96	0.62	1	1.45	1.68	0.8	15.02	9.04	
C3	3A 3B	18.76 18.39	0.67	1	1.4	1.7	1	18.58	7.33	
C4	4A 4B	16.44 17.91	0.66	1	1.3	1.68	1.2	17.18	7.89	
C5	5A 5B	19.35 19.82	0.63	1	1.28	1.63	1.5	19.59	12.81	
C12	6A 6B	18.48 17.43	0.64	1	1.36	1.7	0.65	17.96	10.77	
C13	7A 7B	16.89. 17.77	0.59	1	1.45	1.83	0.75	17.33	7.6	
C14	8A 8B	19.73 17.99	0.59	1	1.48	1.77	0.85	18.86	8.1	
C15	9A 9B	18.55 19.43	0.65	1	1.42	1.8	1	18.99	7.05	
C23	10A	20.67							7.25	

	10B	21.18	0.64	1	1.3	1.77	0.9	20.93		
C24	11A 11B	16.68 17.84	0.6	1	1.27	1.71	1	17.26	8.04	
C25	12A 12B	18.59 17.83	0.6	1	1.31	1.79	1.15	18.21	7.96	
C34	13A 13B	19.44 18.77	0.62	1	1.33	1.83	1.1	19.11	8.14	
C35	14A 14B	19.42 19.02	0.63	1	1.41	1.85	1.25	19.22	10.54	
C45	15A 15B	13.25 12.78	0.61	1	1.25	1.79	1.35	13.02	11.02	

4.2SCHEFFE'S (5,2) SIMPLEX MODEL RESULTS FOR NFRC

4.2.1. RESPONSE OF EXPERIMENTAL POINTS FROM SCHEFFE'S (5, 2) SIMPLEX MODEL RESULTS

By substituting the pseudo mix ratio points of the initial experiment A_1, A_2, A_3, A_4, A_5, A_{12}, A_{13}, A_{14}, A_{15}, A_{23}, A_{24}, A_{25}, A_{34}, A_{35}, and A_{45} of Table 1 into Eqn. (16), we obtain the second model response as shown in Table 5 below.

Table 5: Response of Experimental Points fromScheffe's (5, 2)Model inEqn. (16) for NFRC

points	X_1	X_2	X_3	X_4	X_5	Response N/mm^2
1	1	0	0	0	0	20.13
2	0	1	0	0	0	15.74
3	0	0	1	0	0	19.27
4	0	0	0	1	0	16.99
5	0	0	0	0	1	19.73
12	0.5	0.5	0	0	0	18.04
13	0.5	0	0.5	0	0	17.05
14	0.5	0	0	0.5	0	18.94
15	0.5	0	0	0	0.5	19.42
23	0	0.5	0.5	0	0	21.96
24	0	0.5	0	0.5	0	16.61
25	0	0.5	0	0	0.5	18.93
34	0	0	0.5	0.5	0	18.34
35	0	0	0.5	0	0.5	19.06

45	0	0	0	0.5	0.5	13.76

4.2.2. RESPONSE OF CONTROL POINTS FROM SCHEFFE'S (5,2) SIMPLEX MODEL RESULTS

By substituting the pseudo mix ratio into points c_1, c_2, c_3, c_4, c_5, c_{12}, c_{13}, c_{14}, c_{15}, c_{23}, c_{24}, c_{25}, c_{34}, c_{35}, and c_{45} of Table 2 into Eqn.(16) , we obtain the second order model response as shown in Table 6

Table 6: Response of Control points from Scheffe's (5, 2) Model in Eqn. (16)

Points	X_1	X_2	X_3	X_4	X_5	Response, N/mm^2
C_1	0.25	0.25	0.25	0.25	0	18.72
C_2	0.25	0.25	0.25	0	0.25	15.07
C_3	0.25	0.25	0	0.25	0.25	18.60
C_4	0.25	0	0.25	0.25	0.25	17.22
C_5	0	0.25	0.25	0.25	0.25	19.63
C_{12}	0.2	0.2	0.2	0.2	0.2	17.99
C_{13}	0.3	0.3	0.3	0.1	0	17.65
C_{14}	0.3	0.3	0.3	0	0.1	18.89
C_{15}	0.3	0.3	0	0.3	0.1	19.11
C_{23}	0.3	0	0.3	0.3	0.1	20.47
C_{24}	0	0.3	0.3	0.3	0.1	17.54
C_{25}	0.1	0.3	0.3	0.3	0	18.33
C_{34}	0.3	0.1	0.3	0.3	0	19.65
C_{35}	0.3	0.3	0.1	0.3	0	19.43
C_{45}	0.1	0.2	0.3	0.4	0	13.19

4.3. SUMMARY OF RESPONSES FORNFRC FROMSCHEFFE'S (5,2) SIMPLEX

Table 7 shows the summary of responses from Scheffe's (5, 2) simplex

Table 7: Summary of Responses of Scheffe's (5, 2) Simplex for NFRC

S/No	Experimental Test Results	Scheffe Model Results	Control Points	Experimental Test Results	Scheffe Model Results
1	20.13	20.13	C_1	18.71	18.72
2	15.74	15.74	C_2	15.02	15.07

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3	19.27	19.27	C_3	18.58	18.60
4	16.99	16.99	C_4	17.18	17.22
5	19.73	19.73	C_5	19.59	19.63
12	18.04	18.04	C_{12}	17.96	17.99
13	17.05	17.05	C_{13}	17.33	17.65
14	18.94	18.94	C_{14}	18.86	18.89
15	19.42	19.42	C_{15}	18.99	19.11
23	21.96	21.96	C_{23}	20.93	20.47
24	16.61	16.61	C_{24}	17.26	17.54
25	18.93	18.93	C_{25}	18.21	18.33
34	18.34	18.34	C_{34}	19.11	19.65
35	19.06	19.06	C_{35}	19.22	19.43
45	13.76	13.76	C_{45}	13.02	13.19

4.4: TEST OF THE ADEQUACY OF THE MODEL USING STUDENT'S – T - TEST

The main focus here is to determine if there is any significant difference between the lab responses (results) given in Table 4 and model responses given in Table 5. The procedure of theStudent's – T - test has been clearly explained by Nwachukwu and others (2022 b). The test shows that there is no significant difference between the experimental results and model results.Thus, the model is adequate for predicting the compressive strength of NFRC.

4.5. DISCUSSION OF RESULTS

Using Scheffe's (5,2) simplex model the values of the compressive strength were obtained for NFRC. The model gave highest compressive strength of 21.96 Nmm^{-2} corresponding to mix ratio of 0.53:1:1:4:1.75:0.9 for water, cement, fine and coarse aggregate and nylon fibre respectively. The lowest strength was found to be 13.76Nmm^{-2} corresponding to mix ratio of 0.73:1:1.15:1.5:1.5. The maximum strength value from themodel wasgreater than the minimum value specified by the American Concrete Institute for the compressive strength of good concrete.Using the model, compressive strength of all points in the simplex can be determined.

V. CONCLUSION

Scheffe's second degree polynomial (5,2) was used to formulate a model for predicting the compressive strength of NFRC cubes. This model has numerous advantages, one of which is that it can be used predict the compressive strength of the NFRC concrete cubes if the mix ratios are known and vice versa. The strengths predicted by the models are in good agreement with the corresponding experimentally observed results. As confirmed through student's t-test.The optimum attainable compressive strength predicted by the Scheffe's (5,2) model at the 28th day was 21.96N/mm^{2}. This meets the minimum standard requirement stipulated by American Concrete Institute (ACI) of 20N/mm^2 for the compressive strength.. With the model, any desired strength of NylonFibre Reinforced Concrete, given any mix proportions can be easily evaluated.Thus the problem of having to go through a vigorous mix-design procedure for a desiring strength has been reduced by utilizing this model.

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