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Application of Scheffe's Second Degree Mathematical Model for the Optimization of Compressive Strength of Steel Fibre Reinforced Concrete (SFRC)

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ABSTRACT
Optimization of Steel Fibre Reinforced Concrete (SFRC) mixture design is a process of search for a mixture for which the sum of the costs of the ingredients making up the SFRC mixture (that is, water, cement, fineaggregate, coarse aggregate and steel fibre) is lowest, yet satisfying the major required performance of concrete, such as workability, flexibility, homogeneity, strength and durability. This research work is aimedat applying Scheffe's Second Degree (5,2) Polynomial Model to optimize the compressive strength of Steel Fibre Reinforced Concrete (SFRC).In this study, Scheffe's (5,2) Mathematical Model which has already been developed / derived byNwachukwu and others (2017) for five component mixture will be used to optimize the mix proportions of SFRC that will produce maximum strength. UsingScheffe's Simplex method, the compressive strength of SFRC was determined for different mix ratios/ proportions. As a check, control experiments were carried out where the compressive strengths were also evaluated. The adequacy of the model was evaluated using the Student's t-test.The test statistics authenticated the adequacy of the model. Highest compressive strength for the SFRC using Scheffe's(5,2) model was obtained as 27.81N/mm^2. This optimum value is in line with the strength limit of 20 to 35 MPa (or 20 to35N/mm^2) specified by the American Concrete Institute (ACI). Thus, considering its safety and economic advantages, SFRC can easily find applications as concrete flooring for parking lots, playgrounds, airport runways, taxiways, maintenance hangars, access roads, workshops, port pavements, container storage and handling areas, bulk storage warehouses, and military warehouses.

Keywords: SFRC, Scheffe's(5,2) Model, Optimization, Compressive Strength ,Polynomial/ Mix Design/Ratio

I. INTRODUCTION

One of the fastest ways to obtain desiring mix proportion for a concrete without going through rigorous procedures is by optimization. An optimization problem is one requiring the determination of the optimal (maximum or minimum) value of a given function, called the objective function, subject to a set of stated restrictions, or constraints placed on the variables concerned. Every optimization problem requires an objective which might be to maximize profit or benefit, to minimize cost or to minimize the use of material resources. Scheffe's MathematicalModels are typical examples of optimization model. In this study, Scheffe's Second Degree Polynomial Model forfive components mixtures (viz cement, fine aggregate, coarse aggregate, water and steel fibre) will be examined.

Concrete is the most commonly used material in the construction industries over times. It is only second to water in terms of usage in the construction industry.According to Neville(1990), concrete plays a crucial part in all building structures owing to its numerous advantages which ranges from low built in fire resistance, high compressive strength to low maintenance.However conventional concrete has two major drawbacks: low tensile strength and a destructive and brittle failure .Concrete is a brittle material with low tensile strength and low strain capacity that result in low resistance to cracking. As a result of this, many new technologies of concrete and some modern concrete specification approaches have been introduced. One of the technologies introduced for concrete was the addition of steel bars to reinforce its tension zone. This enables concrete gain an amount of tensile strength and thus reducing its brittle nature. However, these types of reinforced concrete structures still experience deterioration when exposed to deleterious environment which often reduce the service life of the structure. Based on several further researches over the years, the reinforcement (usually steel bars) has been replaced with other materials like fibre (glass fibre, polypropylene fibre, nylon fibre, steel fibre , plastic fibre etc.) to further increase both its tensile strength and compressive strength and also, produce light weighted reinforced concrete unlike when reinforced with steel bars.Fibre reinforced concrete (FRC) may be defined as composite materials made with Portland cement, aggregate, and incorporation of discrete discontinuous fibres as listed above. The main objective of incorporating the fibrous materials is to increase the concrete's durability and structural integrity and at the same time save costs. This is to say that all fibres reduce the concrete's need for steel reinforcements. And since fibre reinforcement tends to be less expensive than steel bars (and less likely to corrode), it makes FRC more cost-effective.In summary , fibres can improve the concrete's: Workability, Flexibility, Tensile strength, Durability—by controlling and reducing crack widths, Ductility, Cohesion, Freeze-thaw resistance, Abrasion- and impact-resistance, Resistance to plastic shrinkage while curing, Resistance to cracking, Shrinkage at an early age, Fire resistance, Homogeneity, to mention but a few..

Steel Fibre Reinforced Concrete (SFRC) is concrete mixture where the conventionally steel reinforcement in concrete production is replaced (wholly or partially) with steel fibre.Steel fibres are short discontinues strips of specially manufactured steel. A certain amount of steel fibre in concrete can cause qualitative changes in concrete's physical property, greatly increases resistance to cracking, impact, fatigue, and bending, tenacity, durability, and other properties. It is a well-established fact that one of the important properties of Steel Fibre Reinforced Concrete (SFRC) is its superior resistance to cracking. This property is likely attributed to the addition of steel fibres (SF). A typical example of SF is shown in Figure 1.Compressive strength of concrete is the Strength of hardened concrete measured by the compression test. It is a measure of the concrete's ability to resist loads which tend to compress it. It is measured by crushing cylindrical concrete specimens in a universal testing machineThe compressive strength of the concrete cube testalso provides an idea about all the characteristics of concrete in question.

The present study therefore examines the application of Scheffe's Second Degree Polynomial Model in optimizing thecompressive strength of SFRC.Many researchers have done works related to either SFRC or optimization, but none has addressed the recent subject matter, the application ofScheffe'smodel in optimizing the compressive strength of SFRC. For instance, Baros and others (2005) investigated the post – cracking behaviour of SFRC. Jean-Louis and Sana (2005) investigated the corrosion of SFRC from the crack. Lima and Oh (1999) carried out an experimental and theoretical investigation on the shear of SFRC beams. Similarly, Lau and Anson (2006) carried out research on the effect of high temperatures on high performance SFRC. The work of Lie and Kodar (1996) was on the study of thermal and mechanical properties of SFRC at elevated temperatures. Blaszczynski and Przybylska-Falek (2015) investigated the use of SFRC as a structural material. Huang and Zhao (1995) investigated the properties of SFRC containing larger coarse aggregate.Arube and others (2021) investigated the Effects of Steel Fibres in Concrete Paving Blocks. Again,Khaloo and others (2005) examined the flexural behaviour of small SFRC slabs. And Ghaffer and others (2014) investigated the use of steel fibres in structural concrete to enhance the mechanical properties of concrete.On optimization, a lot of researchers have used Scheffe's method to carryout one form of optimization project or the other. For example,Nwakonobi and Osadebe (2008) used Scheffe's model to optimize the mix proportion of Clay- Rice Husk Cement Mixture for Animal Building. Ezeh and Ibearugbulem (2009) applied Scheffe's model to optimize the compressive cube strength of River Stone Aggregate Concrete. Scheffe's model was used by Ezeh and others (2010a) to optimize the compressive strength of cement- sawdust Ash Sandcrete Block. Again Ezeh and others (2010b) optimized the aggregate composition of laterite/ sand hollow block using Scheffe's simplex method. The work of Ibearugbulem (2006) and Okere(2006) were also based on the use of Scheffe' mathematical model in the optimization of compressive strength of Perwinkle Shell- Granite Aggregate Concrete and optimization of the Modulus of Rupture of Concrete respectively.

Obam (2009) developed a mathematical model for the optimization of strength of concrete using shear modulus of Rice Husk Ash as a case study. The work of Obam (2006) was based on four component mixtures, that is Scheffe's(4,2) and Scheffe's(4,3).Nwachukwu and others (2017) developed and employed Scheffe's Second Degree Polynomial model to optimize the compressive strength of Glass Fibre Reinforced Concrete (GFRC). Also,Nwachukwu and others (2022a) developed and usedScheffe'sThird Degree Polynomial model, abbreviated as Scheffe's (5,3) to optimize the compressive strength of GFRC and compared the results with his previous work, Nwachukwu and others (2017). Nwachukwu and others (2022b) used Scheffe's (5,2) optimization model to optimize the compressive strength of Polypropylene Fibre Reinforced Concrete (PFRC). And finally,Nwachukwu and others (2022c) applied Scheffe's (5,2) mathematical model to optimize the compressive strength of Nylon Fibre Reinforced Concrete (NFRC), From the forgoing, it can be envisaged that no work has been done on the use of Scheffe's method to optimize the compressive strengthof SFRC .Henceforth, the need for this research work.

Fig. 1 :A Typical Sample of Steel Fibre

II. SCHEFFE'S SECOND DEGREE OPTIMIZATION FUNDAMENTALS

A simplex lattice is a structural representation of lines joining the atoms of a mixture, and these atoms are constituent components of the mixture. For SFRC mixture, the constituent elements are the water, cement, fineaggregate (sand), coarse aggregate and steel fibre. That is to say that, a simplex of five-component mixture is a four-dimensional solid. See Nwachukwu and others (2017).According to Obam (2009), mixture components are subject to the constraint that the sum of all the components must be equal to 1. That is:

$$X_1 + X_2 + X_3 + \dots + X_q = 1 \; ; \quad \Rightarrow \sum_{i=1}^{q} X_i = 1 \quad \textbf{(1)}$$

where $X_i \geq 0$ and $i = 1, 2, 3\dots q$, and q = the number of mixtures

2.1.SIMPLEX LATTICE DESIGN BASICS

The (q, m) simplex lattice design are characterized by the symmetric arrangements of points within the experimental region and a well-chosen polynomial equation to represent the response surface over the entire simplex region(Aggarwal, 2002). The (q, m) simplex lattice design given by Scheffe, according to Nwakonobi and Osadebe (2008) contains ${}^{q+m-1}C_m$ points where each components proportion takes (m+1) equally spaced values $X_i = 0, \frac{1}{m}, \frac{2}{m}, \frac{3}{m}, \dots, 1; \quad i = 1, 2, \dots, q$ ranging between 0 and 1 and all possible mixture with these component proportions are used, and m is scheffe's polynomial degee, which in this present study is 2.

For example a (3, 2) lattice consists of ${}^{3+2-1}C_2$ i.e. ${}^{4}C_2 = 6$ points. Each X_i can take m+1 = 3 possible values; that is $x = 0, \frac{1}{2}, 1$with which the possible design points are:

$$(1,0,0), (0,1,0), (0,0,1), \left(\frac{1}{2},\frac{1}{2},0\right), \left(0,\frac{1}{2},\frac{1}{2}\right), \left(\frac{1}{2},0,\frac{1}{2}\right)$$

.

According to Obam (2009), a Scheffe's polynomial function of degree, m in the q variable X_1, X_2, X_3,X_4 …X_q is given in form of:

$$Y = b_0 + \sum bi\, xi + \sum bij x j + \sum bi\, j x j x k + + \sum bi^{j}_{2} + \dots i_n x i_2 x i_n \quad \textbf{(2)}$$

where ($1 \leq i \leq q$, $1 \leq i \leq j \leq k \leq q$, $1 \leq i_1 \leq i_2 \leq \dots \leq i_n \leq q$ respectively) , b = constant coefficients and Y is the response(the response is a polynomial function of pseudo component of the mix) which represents the property under study, which ,in this case is the compressive strength.

This research work is based on the Scheffe's(5, 2) simplex..The actual form of Eqn. (2) has already beendeveloped for five component mixture, based on Scheffe'ssecond degree polynomial by Nwachukwu and others (2017) and will be applied subsequently in this work.

2.2. PSEUDO AND ACTUAL COMPONENTS.

In Scheffe's mix design, there exist a relationship between the pseudo components and the actual components. It has been established as Eqn.(3):

$$Z = A * X \quad \textbf{(3)}$$

where Z is the actual component; X is the pseudo component and A is the coefficient of the relationship

Re-arranging the equation

$$X = A^{-1} * Z \quad \textbf{(4)}$$

2.3. DEVELOPED POLYNOMIAL EQUATION FOR SCHEFFE'S (5, 2) LATTICE

The regression or polynomial equation by Scheffe(1958), otherwise known as response is given in Eqn.(2) .Hence, for Scheffe's (5,2) simplex lattice, the regression equation for five component mixtures has been derived from Eqn.(2) by Nwachukwu and others (2017) and is given as follows:

$$Y = b_0 + b_1X_1 + b_2X_2 + b_3X_3 + b_4X_4 + b_5X_5 + b_{11}X_1^2 + b_{12}X_1X_2 + b_{13}X_1X_3 + b_{14}X_1X_4 + b_{15}X_1X_5 + b_{22}X_2^2 + b_{23}X_2X_3 + b_{24}X_2X_4 + b_{25}X_2X_5 + b_{33}X_3^3 + b_{34}X_3X_4 + b_{35}X_3X_5 + b_{44}X_4^4 + b_{45}X_4X_5 + b_{55}X_5^5 \quad \textbf{(5)}$$

$$= ß_1X_1 + ß_2X_2 + ß_3X_3 + ß_4X_4 + ß_5X_5 + ß_{12}X_1X_2 + ß_{13}X_1X_3 + ß_{14}X_1X_4 + ß_{15}X_1X_5 + ß_{23}X_2X_3 + ß_{24}X_2X_4 + ß_{25}X_2X_5 + ß_{34}X_3X_4 + ß_{35}X_3X_5 + ß_{45}X_4X_5 \quad \textbf{(6)}$$

Where,

$ß_1 = b_0 + b_1 + b_{11}$; $ß_2 = b_0 + b_2 + b_{22}$; $ß_3 = b_0 + b_3 + b_{33}$; $ß_4 = b_0 + b_4 + b_{44}$; $ß_5 = b_0 + b_5 + b_{55}$; $ß_{12} = b_{12} – b_{11} – b_{22}$; $ß_{13} = b_{13} – b_{11} – b_{33}$; $ß_{14} = b_{14} – b_{11} – b_{44}$; $ß_{15} = b_{15} – b_{11} – b_{55}$; $ß_{23} = b_{23} – b_{22} – b_{33}$; $ß_{24} = b_{24} – b_{22} – b_{44}$; $ß_{25} = b_{25} – b_{22} – b_{55}$; $ß_{34} = b_{34} – b_{35} – b_{44}$; $ß_{35} = b_{35} – b_{33} – b_{55}$; $ß_{45} = b_{45} – b_{44} – b_{55}$. **(7)**

2.4 . MIXTURE DESIGN MODEL

The procedure for the determination of the coefficient of Scheffe's (5,2) regression model has been explained byNwachukwu and others (2017). After coefficients evaluation, the equation for the mixture design model is as shown in Eqn.(8).

$$Y = X_1(2X_1 – 1)Y_1 + X_2(2X_2 – 1)Y_2 + X_3(2X_3 – 1)Y_3 + X_4(2X_4 – 1)Y_4 + X_5(2X_5 – 1)Y_5 + 4Y_{12}X_1X_2 + 4Y_{13}X_1X_3 + 4Y_{14}X_1X_4 + 4Y_{15}X_1X_5 + 4Y_{23}X_2X_3 + 4Y_{24}X_2X_4 + 4Y_{25}X_2X_5 + 4Y_{34}X_3X_4 + 4Y_{35}X_3X_5 + 4Y_{45}X_4X_5 \quad \textbf{(8)}$$

Eqn. (8) is the second degree based mix design model for the optimization of a concrete mix that comprises five components, such as SFRC.Y_1 ,Y_2 …….. $Y_{45}$are determined through laboratory test.

2.5. ACTUAL AND PSEUDO MIX RATIO

The requirement of simplex lattice design based on Eqn. (1) criteria makes it impossible to use the conventional mix ratios such as 1:2:4, 1:3:6, etc., at a given water/cement ratio for the actual mix ratio. This necessitates the transformation of the actual components proportions to meet the above criterion. Such transformed ratios, $x_1^{(i)}$, $x_2^{(i)}$, $x_3^{(i)}$, for the ith experimental points are called pseudo – components (or coded components). Based on experience and previous knowledge from literature, the following arbitrary prescribed mix proportions are always chosen for the five points/vertices. See the works of Nwachukwu and others (2017), for different vertices.

A_1 (0.67:1: 1.7: 2:0.5); A_2 (0.56:1:1.6:1.8:0.8); A_3 (0.5:1:1.2:1.7:1); A_4 (0.7:1:1:1.8:1.2) and A_5 (0.75:1:1.3:1.2:1.5), which represent water/cement ratio, cement, fine aggregate, coarse aggregate and steel fibre.

For the pseudo mix ratio, the following corresponding mix ratios at the vertixes for five component mixtures are always chosen: A_1(1:0:0:0:0), A_2(0:1:0:0: 0), A_3(0:0:1:0:0), A_4(0:0:0:1:0), and A_5(0:0:0:0:1)

For the transformation of the actual component, Z to pseudo component, X, and vice versa ,Eqns.(3)and (4) are applied.

Substituting the mix ratios from point A_1 into Eqn. (3) gives:

$$\begin{Bmatrix} 0.67 \\ 1 \\ 1.7 \\ 2 \\ 0.5 \end{Bmatrix} = \begin{pmatrix} A_{11} & A_{12} & A_{13} & A_{14} & A_{15} \\ A_{21} & A_{22} & A_{23} & A_{24} & A_{25}0 \\ A_{31} & A_{32} & A_{33} & A_{34} & A_{35} \\ A_{41} & A_{42} & A_{43} & A_{44} & A_{45} \\ A_{51} & A_{52} & A_{53} & A_{54} & A_{55} \end{pmatrix} \begin{Bmatrix} 1 \\ \\ 0 \\ 0 \\ 0 \end{Bmatrix} \quad \textbf{(9)}$$

Solving, we obtain :

. $A_{11} = 0.67$, $A_{21} = 1$, $A_{31} = 1.7$, $A_{41} = 2$, and $A_{51} = 0.5$

The same goes for point 2 through point 5 and the overallresults are depicted in Eqn. (10)

$$\begin{Bmatrix} Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \\ Z_5 \end{Bmatrix} = \begin{pmatrix} 0.67 & 0.56 & 0.5 & 0.7 & 0.75 \\ 1 & 1 & 1 & 1 & 1 \\ 1.7 & 1.6 & 1.2 & 1 & 1.3 \\ 2 & 1.8 & 1.7 & 1.8 & 1.2 \\ 0.5 & 0.8 & 1 & 1.2 & 1.5 \end{pmatrix} \begin{Bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \\ X_5 \end{Bmatrix} \quad \textbf{(10)}$$

Therefore, from Eqn.(4), we obtain :

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$$\begin{Bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \\ X_5 \end{Bmatrix} = \begin{pmatrix} 0.67 & 0.56 & 0.5 & 0.7 & 0.75 \\ 1 & 1 & 1 & 1 & 1 \\ 1.7 & 1.6 & 1.2 & 1 & 1.3 \\ 2 & 1.8 & 1.7 & 1.8 & 1.2 \\ 0.5 & 0.8 & 1 & 1.2 & 1.5 \end{pmatrix}^{-1} \begin{Bmatrix} Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \\ Z_5 \end{Bmatrix} \quad (11)$$

Thus

$$\begin{Bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \\ X_5 \end{Bmatrix} = \begin{pmatrix} 3.99 & 10.37 & -2.14 & -3.05 & -4.62 \\ -4.88 & -21.46 & 5.40 & 5.95 & 7.31 \\ -1.78 & 17.83 & -3.49 & -4.20 & -4.62 \\ 1.04 & -9.24 & 0.37 & 3.28 & 2.69 \\ 1.63 & 3.49 & -0.13 & -1.98 & -0.77 \end{pmatrix} \begin{Bmatrix} Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \\ Z_5 \end{Bmatrix} \quad (12)$$

Considering the mix ratios at the midpoints, we have:
A_{12} (0.5, 0.5, 0, 0, 0); A_{13} (0.5, 0, 0.5, 0, 0); A_{14} (0.5, 0, 0, 0.5, 0); A_{15} (0.5, 0, 0, 0, 0.5); A_{23} (0, 0.5, 0.5, 0,0); A_{24} (0, 0.5, 0, 0.5, 0); A_{25} (0, 0.5, 0, 0, 0.5); A_{34} (0, 0, 0.5, 0.5, 0); A_{35} (0, 0, 0.5, 0, 0.5) and A_{45} (0, 0, 0, 0.5, 0.5)
Substituting these pseudo mix ratios in turn into Eqn. (10) will give the corresponding actual mix ratio as follows:

For point A_{12}

$$\begin{Bmatrix} Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \\ Z_5 \end{Bmatrix} = \begin{pmatrix} 0.67 & 0.56 & 0.5 & 0.7 & 0.75 \\ 1 & 1 & 1 & 1 & 1\ 0.5 \\ 1.7 & 1.6 & 1.2 & 1 & 1.3 \\ 2 & 1.8 & 1.7 & 1.8 & 1.2 \\ 0.5 & 0.8 & 1 & 1.2 & 1.5 \end{pmatrix} \begin{pmatrix} 0.5 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{Bmatrix} 0.62 \\ \\ 1.65 \\ 1.90 \\ 0.65 \end{Bmatrix} \quad (13)$$

Solving ,
$Z_1 = 0.62$, $Z_2 = 1$, $Z_3 = 1.65$, $Z_4 = 1.9$, $Z_5 = 0.65$
The rest of the results are represented in Table 1.
In order to generate the regression coefficients, fifteen experimental tests are carried out and the corresponding mix ratio are as shownin Table 1 .

Table 1: Actual Mix Ratios for theScheffe's (5, 2) Lattice at Initial Experimental Point for SFRC

Points	Water/Cement Ratio(**Z_1**)	Cement (**Z_2**)	Fine Aggregate(**Z_3**)	Coarse Aggregate(**Z_4**)	Steel Fibre(**Z_5**)	Response
1	0.67	1	1.70	2.00	0.50	Y_1
2	0.56	1	1.60	1.80	0.80	Y_2
3	0.50	1	1.20	1.70	1.00	Y_3
4	0.70	1	1.00	1.80	1.20	Y_4
5	0.75	1	1.30	1.20	1.50	Y_5
12	0.62	1	1.65	1.90	0.65	Y_{12}
13	0.59	1	1.45	1.85	0.75	Y_{13}
14	0.69	1	1.35	1.90	0.85	Y_{14}
15	0.71	1	1.50	1.60	1.00	Y_{15}
23	0.53	1	1.40	1.75	0.90	Y_{23}
24	0.63	1	1.30	1.80	1.00	Y_{24}
25	0.66	1	1.45	1.50	1.15	Y_{25}
34	0.60	1	1.10	1.75	1.10	Y_{34}
35	0.63	1	1.25	1.45	1.25	Y_{35}
45	0.73	1	1.15	1.50	1.50	Y_{45}

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2.7.CONTROL POINTS

For the purpose of this research, fifteen different controls were predicted which according to Scheffe, their summation must conform with Eqn.(1) . They are as follows:

C_1 = (0.25, 0.25, 0.25, 0.25, 0), C_2 = (0.25, 0.25, 0.25, 0, 0.25), C_3 = (0.25, 0.25, 0, 0.25, 0.25), C_4 = (0.25, 0, 0.25, 0.25, 0.25), C_5 = (0, 0.25, 0.25, 0.25, 0.25), C_{12} = (0.20, 0.20, 0.20, 0.20, 0.20), C_{13} = (0.30, 0.30, 0.30, 0.10, 0), C_{14} = (0.30, 0.30, 0.30, 0, 0.10), C_{15} = (0.30, 0.30, 0, 0.30, 0.1), C_{23} = (0.30, 0, 0.30, 0.30, 0.1), C_{24} = (0, 0.30, 0.30, 0.30, 0.10), C_{25} = (0.10, 0.30, 0.30, 0.30, 0), C_{34} = (0.30, 0.10, 0.30, 0.30, 0), C_{35} = (0.30, 0.30, 0.10, 0.30, 0), C_{45} = (0.10, 0.20, 0.30, 0.40, 0),

Substituting into Eqn.(10) , we obtain the values of the actual mixes as follows:

Control 1 C_1

$$\begin{Bmatrix} Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \\ Z_5 \end{Bmatrix} = \begin{pmatrix} 0.67 & 0.56 & 0.5 & 0.7 & 0.75 \\ 1 & 1 & 1 & 1 & 1 \\ 1.7 & 1.6 & 1.2 & 1 & 1.3 \\ 2 & 1.8 & 1.7 & 1.8 & 1.2 \\ 0.5 & 0.8 & 1 & 1.2 & 1.5 \end{pmatrix} \begin{pmatrix} 0.25 \\ 0.25 \\ 0.25 \\ 0.25 \\ 0 \end{pmatrix} = \begin{Bmatrix} 0.61 \\ 1 \\ 1.38 \\ 1.83 \\ 0.5 \end{Bmatrix} \quad \textbf{(14)}$$

The restof the results are represented in Table 2

Table 2: Actual (Z_i) And Pseudo (X_i) Component OfScheffe's (5, 2) Simplex Lattice Control Point For SFRC

Points	Pseudo					Actual				
	Water	Cement	Fine Aggregate	Coarse Aggregate	Steel Fibre	Water	Cement	Fine Aggregate	Coarse Aggregate	SteelFibre
C_1	0.25	0.25	0.25	0.25	0	0.61	1	1.38	1.83	0.5
C_2	0.25	0.25	0.25	0	0.25	0.62	1	1.45	1.68	0.8
C_3	0.25	0.25	0	0.25	0.25	0.67	1	1.4	1.7	1
C_4	0.25	0	0.25	0.25	0.25	0.66	1	1.3	1.68	1.2
C_5	0	0.25	0.25	0.25	0.25	0.63	1	1.28	1.63	1.5
C_{12}	0.2	0.2	0.2	0.2	0.2	0.64	1	1.36	1.7	0.65
C_{13}	0.3	0.3	0.3	0.1	0	0.59	1	1.45	1.83	0.75
C_{14}	0.3	0.3	0.3	0	0.1	0.59	1	1.48	1.77	0.85
C_{15}	0.3	0.3	0	0.3	0.1	0.65	1	1.42	1.8	1
C_{23}	0.3	0	0.3	0.3	0.1	0.64	1	1.3	1.77	0.9
C_{24}	0	0.3	0.3	0.3	0.1	0.6	1	1.27	1.71	1
C_{25}	0.1	0.3	0.3	0.3	0	0.6	1	1.31	1.79	1.15
C_{34}	0.3	0.1	0.3	0.3	0	0.62	1	1.33	1.83	1.1
C_{35}	0.3	0.3	0.1	0.3	0	0.63	1	1.41	1.85	1.25
C_{45}	0.1	0.2	0.3	0.4	0	0.61	1	1.25	1.79	1.35

III. MATERIALS AND METHODS

3.1 MATERIALS

The materials investigated are the mixture of cement, water, fine and coarse aggregate and steel fibre. The cement is a brand of Ordinary Portland Cement, conforming to British Standard Institution BS 12 (1978). The fine aggregate, whose size ranges from 0.05 - 4.5mm was procured from the local river. Crushed granite of 20mm size downgraded to 4.75mm obtained from a local stone market was used in the experimental investigation., as it is important to note that when mixing fibre-reinforced concrete, the maximum size of the coarse aggregates should not be more than 10 mm to avoid reducing the strength of the concrete. Steel Fibre used is of 60mm in length and 0.75 mm in diameter as shown in Figure 1. Also, potable water drawn from the clean water source was used in the experimental investigation.

3.2. METHOD

3.2.1. SPECIMEN PREPARATION / BATCHING/ CURING

The specimens for the compressive strength were concrete cubes. They were cast in steel mould measuring 150mm*150mm*150mm. The mould and its base were damped together during concrete casting to prevent leakage of mortar. Thin engine oil was applied to the inner surface of the moulds to make for easy removal of the cubes. Batching of all the constituent material was done by weight using a weighing balance of 50kg capacity based on the adapted mix ratios and water cement ratios. A total number of 30 mix ratios were to be used to produce 60 prototype concrete cubes. Fifteen (15) out of the 30 mix ratios were as control mix ratios to produce 30 cubes for the conformation of the adequacy of the mixture design given by the Eqn. (8).. Curing commenced 24hours after moulding. The specimens were removed from the moulds and were placed in clean water for curing. After 28days of curing the specimens were taken out of the curing tank.

3.2.2. COMPRESSIVE STRENGTH TEST

Compressive strength testing was done in accordance with BS 1881 – part 116 (1983) - Method of determination of compressive strength of concrete cube and ACI (1989) guideline .Two samples were crushed for each mix ratio.The compressive strength was then calculated using Eqn. (15)

Compressive Strength = Average failure Load, **P**(N)**(15)**

Cross- sectional Area**, A** (mm^2)

IV. RESULTS AND DISCUSSION

4.1 COMPRESSIVE STRENGTH TEST RESULTS FOR SFRC BASED ON SCHEFFE'S (5,2) SIMPLEX LATTICE

4.1.1 EXPERIMENTAL TEST RESULTS

The results of compressive strength test basedon Eqn. (15) are shown in Table 3

Table 3:28th DayCompressive Strength Test Results for SFRC

Points	Experiment No.	Response Y_i, N/mm^2	Response Symbol	$\sum Y_i$	Average Response Y, N/mm^2
1	1A 1B	23.75 22.54	Y_1	46.29	23.15
2	2A 2B	19.78 18.68	Y_2	38.46	19.23
3	3A 3B	19.34 19.56	Y_3	38.90	19.45
4	4A 4B	27.86 27.75	Y_4	55.61	27.81
5	5A 5B	20.34 20.68	Y_5	41.02	20.51
12	6A 6B	21.53 21.64	Y_{12}	43.17	21.59
13	7A 7B	20.99 20.65	Y_{13}	41.64	20.82
14	8A 8B	19.58 19.56	Y_{14}	39.14	19.57
15	9A 9B	19.88 19.87	Y_{15}	39.75	19.88
23	10A 10B	22.78 22.76	Y_{23}	45.54	22.77
24	11A 11B	21.94 21.86	Y_{24}	43.80	21.90

25	12A 12B	19.64 19.76	Y_{25}	39.40	19.70
34	13A 13B	20.82 20.78	Y_{34}	41.60	20.80
35	14A 14B	24.54 24.62	Y_{35}	49.16	24.58
45	15A 15B	19.86 20.22	Y_{45}	40.04	20.04

4.1.2SCHEFFE'S (5,2) MATHEMATICAL MODEL EQUATION FOR OPTIMIZATION OF COMPRESSIVE STRENGTH OF SFRC.

By substituting the values of Y_1, Y_2, ….Y_{45} from Table 3 into Eqn. (8) yields:

$$Y = 23.15X_1(2X_1 - 1) + 19.23X_2(2X_2 - 1) + 19.45X_3(2X_3 - 1) + 27.81X_4(2X_4 - 1) + 20.51X_5(2X_5 - 1) + 4(21.59)X_1X_2 + 4(20.82)X_1X_3 + 4(19.57)X_1X_4 + 4(19.88)X_1X_5 + 4(22.77)X_2X_3 + 4(21.90)X_2X_4 + 4(19.70)X_2X_5 + 4(20.80)X_3X_4 + 4(24.58)X_3X_5 + 4(20.04)X_4X_5 \quad \textbf{(16)}$$

Equation (16) is the Scheffe's (5,2) Second Degree Mathematical Model Equation .

4.1.3. EXPERIMENTAL (CONTROL) TEST RESULTS

The response (compressive strength) of control points from experimental tests is shown in Table 4

Table 4: Response Of Control Points From Experimental Tests For SFRC

Points	Experiment No.	Response(Compressive Strength) N/mm^2	Z_1	Z_2	Z_3	Z_4	Z_5	Average Response		
C_1	1A 1B	24.34 24.22	0.61	1	1.38	1.83	0.5	24.28	10.42	
C_2	2A 2B	19.66 19.56	0.62	1	1.45	1.68	0.8	19.61	9.04	
C_3	3A 3B	18.78 18.86	0.67	1	1.4	1.7	1	18.82	7.33	
C_4	4A 4B	26.98 27.16	0.66	1	1.3	1.68	1.2	27.07	7.89	
C_5	5A 5B	19.66 19.88	0.63	1	1.28	1.63	1.5	19.77	12.81	
C_{12}	6A 6B	20.68 21.34	0.64	1	1.36	1.7	0.65	21.01	10.77	
C_{13}	7A 7B	19.98 20.12	0.59	1	1.45	1.83	0.75	20.05	7.6	
C_{14}	8A 8B	19.84 19.96	0.59	1	1.48	1.77	0.85	19.90	8.1	
C_{15}	9A 9B	19.75 19.48	0.65	1	1.42	1.8	1	19.62	7.05	
C_{23}	10A	21.98							7.25	

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	10B	21.86	0.64	1	1.3	1.77	0.9	21.92		
C_{24}	11A 11B	20.96 21.32	0.6	1	1.27	1.71	1	21.14	8.04	
C_{25}	12A 12B	19.22 19.54	0.6	1	1.31	1.79	1.15	19.38	7.96	
C_{34}	13A 13B	21.22 20.96	0.62	1	1.33	1.83	1.1	21.09	8.14	
C_{35}	14A 14B	23.56 23.98	0.63	1	1.41	1.85	1.25	23.77	10.54	
C_{45}	15A 15B	19.88 20.12	0.61	1	1.25	1.79	1.35	20.00	11.02	

4.2SCHEFFE'S (5,2) SIMPLEX MODEL RESULTS FOR SFRC

4.2.1. RESPONSE OF EXPERIMENTAL POINTS FROM SCHEFFE'S (5, 2) SIMPLEX MODEL RESULTS

Substituting the pseudo mix ratio points of the initial experiment A_1, A_2, A_3, A_4, A_5, A_{12}, A_{13}, A_{14}, A_{15}, A_{23}, A_{24}, A_{25}, A_{34}, A_{35}, and A_{45} of Table 1 into Eqn. (16), we obtain theScheffe's second degree model response as shown in Table 5. Note that the results are the same as those obtained in Table 3.

Table 5: Response Of Experimental Points FromScheffe's (5, 2)Model inEqn. (16) For SFRC

points	**X_1**	**X_2**	**X_3**	**X_4**	**X_5**	**Response N/mm^2**
1	1	0	0	0	0	23.15
2	0	1	0	0	0	19.23
3	0	0	1	0	0	19.45
4	0	0	0	1	0	27.81
5	0	0	0	0	1	20.51
12	0.5	0.5	0	0	0	21.59
13	0.5	0	0.5	0	0	20.82
14	0.5	0	0	0.5	0	19.57
15	0.5	0	0	0	0.5	19.88
23	0	0.5	0.5	0	0	22.77
24	0	0.5	0	0.5	0	21.90
25	0	0.5	0	0	0.5	19.70
34	0	0	0.5	0.5	0	20.80

35	0	0	0.5	0	0.5	24.58
45	0	0	0	0.5	0.5	20.04

4.2.2. RESPONSE OF CONTROL POINTS FROM SCHEFFE'S (5,2) SIMPLEX MODEL RESULTS

By substituting the pseudo mix ratio into points c_1, c_2, c_3, c_4, c_5, c_{12}, c_{13}, c_{14}, c_{15}, c_{23}, c_{24}, c_{25}, c_{34}, c_{35}, and c_{45} of Table 2 into Eqn.(16) , we obtain the second degree model response as shown in Table 6

Table 6: SFRC Responses of Control PointsFromScheffe's (5, 2) Model in Eqn. (16)

Points	X_1	X_2	X_3	X_4	X_5	Response, N/mm^2
C_1	0.25	0.25	0.25	0.25	0	20.67
C_2	0.25	0.25	0.25	0	0.25	22.07
C_3	0.25	0.25	0	0.25	0.25	19.37
C_4	0.25	0	0.25	0.25	0.25	20.07
C_5	0	0.25	0.25	0.25	0.25	21.59
C_{12}	0.2	0.2	0.2	0.2	0.2	21.43
C_{13}	0.3	0.3	0.3	0.1	0	21.31
C_{14}	0.3	0.3	0.3	0	0.1	22.71
C_{15}	0.3	0.3	0	0.3	0.1	20.38
C_{23}	0.3	0	0.3	0.3	0.1	19.69
C_{24}	0	0.3	0.3	0.3	0.1	22.26
C_{25}	0.1	0.3	0.3	0.3	0	21.18
C_{34}	0.3	0.1	0.3	0.3	0	20.00
C_{35}	0.3	0.3	0.1	0.3	0	20.04
C_{45}	0.1	0.2	0.3	0.4	0	21.10

4.3: TEST OF THE ADEQUACY OF THE MODEL USING STUDENT'S – T - TEST

Here, the Student's – T - test is adopted to check if there is any significant difference between the lab responses (compressive strength results) given in Table 4 and model responses given in Table 5. The procedures for using the Student's – T - test have been explained by Nwachukwu and others (2022 b). The outcome of the test shows that there is no significant difference between the experimental results and model results.Thus, the modelisveryadequate for predicting the compressive strength of SFRC.

4.4. DISCUSSION OF RESULTS

The highest compressive strength of 27.81 Nmm^{-2} corresponding to mix ratio of 0.70:1:1.00:1.80:1.2 for water, cement, fine and coarse aggregate and steelfibre respectively was obtained through the Scheffe's Model. The lowest strength was found to be 19.23Nmm^{-2} corresponding to mix ratio of 0.56:1:1.60:1.80:0.8.The maximum strength value from themodel wasgreater than the minimum value specified by the American Concrete Institute for the compressive strength of good concrete.Using the model, compressive strength of SFRC of all points in the simplex can be determined.

V. CONCLUSION

Scheffe's Second Degree (5,2) Mathematical Model was used to predict/ formulate the mix ratios/propotions, as well as a model for predicting the compressive strength of SFRC cubes. Using Scheffe's (5,2) simplex model the values of the compressive strength were obtained for SFRC. One of the advantages of the model is that it can be used predict the compressive strength of the SFRC concrete cubes if the mix ratios are known and vice versa.As confirmed through student's t-test, the strengths predicted by the models are in good agreement with the corresponding experimentally observed results.The maximum attainable compressive strength of SFRC predicted by the Scheffe's (5,2) model at the 28th day was 27.81N/mm$^{2.}$. This valuemeets the minimum standard requirement stipulated by American Concrete Institute (ACI) of 20N/mm^2 for the compressive strength.of good concrete. With the model, any desired strength of SteelFibre Reinforced Concrete, given any mix proportions can be easily predicted and evaluated.Thus the problem of having to gothrough vigorous mix-design procedures to obtain a desiringstrength of SFRC has been reduced by the utilization of this optimization model.

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