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Applications of S-Type Integral Transform of Some Special Functions

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ABSTRACT: In this paper, have study useful applications of CST of special functions generalized function.

KEYWORD: Canonical transform, Canonical Sine transform, Integral transform, generalized function testing function space.

I. INTRODUCTION:

We have tried to bring completeness to the research work by having the application of "Integral transform as generalized function. The canonical cosine transform of some selected functions are calculated. Now a days, fractional integral transforms play an key role in signal processing, image reconstruction, pattern recognition, accostic signal processing [1],[2]. A new generalized integral transform was obtained by Zayed[9]. Bhosale and Chaudhary [3], S. B. Chavhan [5],[6][7],[8],[9],[10].Had extended fractional Fourier transform to the distribution of compact support. S. B. Chavhan [4], had define the Canonical Sine transform as

$$\{CST\ f(t)\}(s) = -i\sqrt{\frac{1}{2\pi ib}}\, e^{\frac{i}{2}\left(\frac{d}{b}\right)s^2} \int_{-\infty}^{\infty} e^{\frac{i}{2}\left(\frac{a}{b}\right)t^2} \cdot \sin\left(\frac{s}{b}\right) t\, f(t)dt \quad \text{for } b \neq 0$$

Notation and terminology as per Zemanian [10],[11].This paper is organized as section 2 definition of testing function space .Section 3 some useful results of canonical sine transform. Section 4 lastly conclusion is stated.

II. DEFINITION OF TESTING FUNCTION SPACEE:

An infinitely differentiable complex valued function ϕ on R^n belongs to $E(R^n)$, if for each compact set. $I \subset s_a$ where $s_a = \{t :\in R^n,\ |t| \le a, a > 0\}$ and for $k \in R^n$,

$$\gamma_{E,k}\phi(t) = \sup_{t \in I} \left|D^k \phi(t)\right| < \infty \qquad k=0,1,2,3.....$$

Note that space E is complete and a Frechet space,let E' denotes the dual space of E

III. CANONICAL SINE TRANSFORM OF SOME SPECIAL FUNCTIONS:

In this section we have illustrated some examples to demonstrate the transform results.

3.1 CANONICAL SINE TRANSFORM OF $\delta(t-\lambda)$ AND $\delta(t)$:

If $\{CST\ f(t)\}(s)$ is canonical sine transform of $f(t)$

Then $$\{CST\ \delta(t-\lambda)\}(s) = \frac{-i}{\sqrt{2\pi ib}} \cdot e^{\frac{i}{2}\left(\frac{ds^2 + a\lambda^2}{b}\right)} \sin\left(\frac{s}{b}\lambda\right)$$

And $$\{CCT\ \delta(t)\}(s) = 0$$

Solution: By definition of canonical sine transform

$$\{CST\ f(t)\}(s) = -i\sqrt{\frac{1}{2\pi ib}}\, e^{\frac{i}{2}\left(\frac{d}{b}\right)s^2} \int_{-\infty}^{\infty} e^{\frac{i}{2}\left(\frac{a}{b}\right)t^2} \cdot \sin\left(\frac{s}{b}\right) t\, f(t)dt$$

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$$\{CST\ \delta(t-\lambda)\}(s) = -i\sqrt{\frac{1}{2\pi ib}}\ e^{\frac{i}{2}\left(\frac{d}{b}\right)s^2}\int_{-\infty}^{\infty} e^{\frac{i}{2}\left(\frac{a}{b}\right)t^2}\ \sin\left(\frac{s}{b}\right)t\cdot\delta(t-\lambda)dt$$

$$\{CST\ \delta(t-\lambda)\}(s) = -i\sqrt{\frac{1}{2\pi ib}}\cdot e^{\frac{i}{2}\left(\frac{d}{b}\right)s^2}\ e^{\frac{i}{2}\left(\frac{a}{b}\right)\lambda^2}\ \sin\left(\frac{s}{b}\lambda\right)$$

$$\because \int_{-\infty}^{\infty} f(t)\cdot\delta(t-a)dt = f(a)$$

$$\{CST\ \delta(t-\lambda)\}(s) = -i\sqrt{\frac{1}{2\pi ib}}\cdot e^{\frac{i}{2}\left(\frac{d}{b}\right)s^2+\frac{i}{2}\left(\frac{a}{b}\right)\lambda^2}\ \sin\left(\frac{s}{b}\lambda\right)$$

$$\{CST\ \delta(t-\lambda)\}(s) = -i\sqrt{\frac{1}{2\pi ib}}\cdot e^{\frac{i}{2}\left(\frac{ds^2+a\lambda^2}{b}\right)}\ \sin\left(\frac{s}{b}\lambda\right) \qquad (3.1)$$

Putting $\lambda = 0$ in (3.1) we get result

$$\{CST\ \delta(t-0)\}(s) = \sqrt{\frac{1}{2\pi ib}}\cdot e^{\frac{i}{2}\left(\frac{ds^2+0}{b}\right)}\ \sin\left(\frac{s}{b}0\right).$$

$$\{CST\ \delta(t)\}(s) = 0$$

3.2 CANONICAL SINE TRANSFORM OF $\sin t$:

If $\{CST\ f(t)\}(s)$ is canonical sine transform of $f(t)$

Then $$\{CST \sin t\}(s) = \frac{(i)e^{\frac{i}{2}\left(\frac{d}{b}\right)s^2}}{2\sqrt{a}}\left(e^{\frac{-i\left(\frac{s}{b}+1\right)^2 b}{2a}} - e^{\frac{-i\left(\frac{s}{b}-1\right)^2 b}{2a}}\right)$$

Solution: By definition of canonical sine transform

$$\{CST\ f(t)\}(s) = -i\sqrt{\frac{1}{2\pi ib}}\ e^{\frac{i}{2}\left(\frac{d}{b}\right)s^2}\int_{-\infty}^{\infty} e^{\frac{i}{2}\left(\frac{a}{b}\right)t^2}\cdot\sin\left(\frac{s}{b}\right)t\ f(t)dt$$

$$\{CST\ (\sin t)\}(s) = -\sqrt{\frac{1}{2\pi ib}}\cdot e^{\frac{i}{2}\left(\frac{d}{b}\right)s^2}\int_{-\infty}^{\infty} e^{\frac{i}{2}\left(\frac{a}{b}\right)t^2}\cdot i\sin\left(\frac{s}{b}\right)t\ \sin t\cdot dt$$

$$= (i)\sqrt{\frac{1}{2\pi ib}}\cdot e^{\frac{i}{2}\left(\frac{d}{b}\right)s^2}\frac{1}{2}\int_{-\infty}^{\infty} e^{\frac{i}{2}\left(\frac{a}{b}\right)t^2}\left[\cos\left(\frac{s}{b}+1\right)t - \cos\left(\frac{s}{b}-1\right)t\right]dt$$

$$= i\sqrt{\frac{1}{2\pi ib}}\ e^{\frac{i}{2}\left(\frac{d}{b}\right)s^2}\left[\frac{1}{2}\int_{-\infty}^{\infty} e^{\frac{i}{2}\left(\frac{a}{b}\right)t^2}\ \cos\left(\frac{s}{b}+1\right)t\ dt - \frac{1}{2}\int_{-\infty}^{\infty} e^{\frac{i}{2}\left(\frac{a}{b}\right)t^2}\ \cos\left(\frac{s}{b}-1\right)t\right]dt$$

$$= (i)\sqrt{\frac{1}{2\pi ib}}\cdot e^{\frac{i}{2}\left(\frac{d}{b}\right)s^2}\frac{1}{4}\left(\frac{\sqrt{\pi i}\cdot e^{\frac{-i\left(\frac{s}{b}+1\right)^2}{4\frac{a}{2b}}}}{\sqrt{\frac{a}{2b}}} - \frac{\sqrt{\pi i}\cdot e^{\frac{-i\left(\frac{s}{b}-1\right)^2}{4\frac{a}{2b}}}}{\sqrt{\frac{a}{2b}}}\right)$$

$$= (i)\sqrt{\frac{1}{2\pi ib}}\cdot e^{\frac{i}{2}\left(\frac{d}{b}\right)s^2}\frac{1}{2}\left(\frac{\sqrt{2b}\sqrt{\pi i}\cdot e^{\frac{-i\left(\frac{s}{b}+1\right)^2}{\frac{2a}{b}}}}{\sqrt{a}}-\frac{\sqrt{\pi i}\sqrt{2b}\cdot e^{\frac{-i\left(\frac{s}{b}-1\right)^2}{4\frac{a}{2b}}}}{\sqrt{a}}\right)$$

$$= (i)\sqrt{\frac{1}{2\pi ib}}\cdot e^{\frac{i}{2}\left(\frac{d}{b}\right)s^2}\frac{1}{2}\frac{\sqrt{2b}\sqrt{\pi i}}{\sqrt{a}}\left(e^{\frac{-i\left(\frac{s}{b}+1\right)^2 b}{2a}}-e^{\frac{-i\left(\frac{s}{b}-1\right)^2 b}{2a}}\right)$$

$$\{CST\sin t\}(s) = \frac{(i)e^{\frac{i}{2}\left(\frac{d}{b}\right)s^2}}{2\sqrt{a}}\left(e^{\frac{-i\left(\frac{s}{b}+1\right)^2 b}{2a}}-e^{\frac{-i\left(\frac{s}{b}-1\right)^2 b}{2a}}\right)$$

3.3 CANONICAL SINE TRANSFORM OF 1:

If $\{CST\ f(t)\}(s)$ is canonical sine transform of $f(t)$ then $\{CST1\}(s)=0$

Solution: By definition of canonical sine transform

$$\{CST\ f(t)\}(s) = -i\sqrt{\frac{1}{2\pi ib}}\ e^{\frac{i}{2}\left(\frac{d}{b}\right)s^2}\int_{-\infty}^{\infty} e^{\frac{i}{2}\left(\frac{a}{b}\right)t^2}\sin\left(\frac{s}{b}\right)t\ f(t)dt$$

$$\{CST1\}(s) = (-i)\sqrt{\frac{1}{2\pi ib}}\cdot e^{\frac{i}{2}\left(\frac{d}{b}\right)s^2}\int_{-\infty}^{\infty} e^{\frac{i}{2}\left(\frac{a}{b}\right)t^2}\cdot\sin\left(\frac{s}{b}\right)t\ dt$$

$$= (-i)\sqrt{\frac{1}{2\pi ib}}\cdot e^{\frac{i}{2}\left(\frac{d}{b}\right)s^2}\cdot(0)$$

$$\because \int_{-\infty}^{\infty} e^{iax^2}\cdot\sin(bx) = 0$$

$$\{CST1\}(s) = 0$$

IV. CONCLUSION:

In this paper canonical sine transforms is generalized in the form the distributional sense, we have obtained application of CST of some Special Functions.

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