International Journal of Advances in Engineering and Management (IJAEM)
Volume 4, Issue 3 Mar 2022, pp: 1330-1338 www.ijaem.net ISSN: 2395-5252

Combination of stochastic degradation processes based on accelerated degradation tests applied to photovoltaic modules

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Submitted: 20-03-2022 Revised: 28-03-2022 Accepted: 31-03-2022

ABSTRACT: The use of a stochastic process allows us to model and estimate the degradation of a photovoltaic module. Environmental stresses influence the degradation process. Our study is based on classical stochastic processes such as the gamma process, the Wiener process and the inverse Gaussian process. The gamma process is a well-known shock process for modeling damage accumulating in a monotonic way and characterized by independent and positive increments resulting in a wide range of applications such as the calculation of the useful remaining life of systems. , maintenance planning, real-time equipment reliability assessment. And for the Wiener process,
Stochastic mixing consists of adding two stochastic processes to form a single process. In reality the system has several degradation mechanisms. We will combine two stochastic processes by also integrating accelerated degradation models.
KEY WORDS :model combined, stochastic degradation, photovoltaic modul.

I. INTRODUCTION

Stochastic process modeling helps us to model and estimate the degradation of a photovoltaic module. These deteriorations are difficult to assess for technical and financial reasons. Our study is based on combined stochastic processes. It is a question of compiling together the classical stochastic processes such as the gamma process and the Wiener process. The gamma process is a well-known shock process for modeling monotonically accumulating damage and characterized by independent and positive increments resulting in a wide range of applications. We will use the Bayesian and Monte Carlo approach to estimate the different parameters of the model.

II. CLASSICALSTOCHASTIC PROCESS

1. Gamma process

suppose that $\left\{X(t), t \geq 0\right\}$ the degradation random variable and it is defined as a gamma process if $X(t) \square Ga(\alpha\Lambda(t), \beta)$ with α the shape parameter and β scale parameter with $\Lambda(t)$ consists of the monotonically increasing function of time t. The gamma density function translates as follows[3]:

$$f_{gam}(x) = \frac{\beta^{-\alpha\Lambda(t)}}{\Gamma(\alpha\Lambda(t))} x^{\alpha\Lambda(t)-1} \exp\left[-\frac{x}{\beta}\right] \quad (5.63)$$

The variance of the inhomogeneous gamma process $\Gamma(\Lambda(t), \beta)$ for $\forall t \geq 0$.

With $\Gamma(a,z) = \int_z^{\infty} x^{a-1} e^{-x} dx$ is an incomplete function gamma and $\Gamma(a,z)$ the full function. According to the gamma statistical properties, we have the mathematical expectation and the variance respectively:

$$E\left[X(t)\right] = \beta.\rho\Lambda(t) \quad Var\left[X(t)\right] = \beta^2.\rho\Lambda(t)$$

Consider a faulty component when its level of degradation exceeds a critical threshold ρ. That is T_ρ the distribution at which the failure occurs, it is also called the first time of reaching the level of degradation ρ. The distribution of the first passage time is: $T_\rho = \inf\left\{t : t \geq 0 \middle| X(t) \geq \rho\right\}$

The lifetime distribution T_ρ follows the inverse Gaussian distribution. Due to the difficulty on calculating the lifetime distribution, we introduce the Birnbaum-Saunders distribution to approximate the cumulative function as follows [5]:

With $:\Phi(.)$ is a normal distribution with $m=\sqrt{\beta/\rho}$ and $n=\frac{\rho/\beta}{\alpha}$

- **Reliability**:

$$R_{BS}(\Lambda(t)) = 1 - F_{Bs}(t)$$

$$R_T(t) = 1 - \frac{\Gamma\left(\alpha\Lambda(t), \frac{\rho}{\beta}\right)}{\Gamma(\alpha\Lambda(t))} \qquad (5.63)$$

- **Mean time to failure**:

$$MTTF_{BS} = \int_0^\infty \Lambda(t).f_{BS}(\Lambda(t))d\Lambda(t) \qquad (5.63)$$

$$\approx \left[\frac{\rho/\beta}{\alpha} + \frac{1}{2\alpha}\right]^c$$

1. Wiener process

consider $\{X_1(t), t \geq 0\}$ a Wiener process and the stochastic degradation process can be described if the distribution follows the following relationship:

$$X(t) = \mu\Lambda(t) + \sigma_B B(\Lambda(t)) \qquad (5.61)$$

average degradation $\mu\Lambda(t)$ and standard deviation $\sigma\sqrt{\Lambda(t)}$. Hence obtaining the relation $\Delta X(t) \sim N(\mu_1\Delta\Lambda(t), \sigma^2\Delta\Lambda(t))$ and $\Lambda(t)$ denotes the non-negative increasing function by describing the approximation as a function of time. Itsprobabilitydensityis written as follows

$$f_{gau}(\Delta x(t); \mu, \sigma_B^2) = \frac{1}{\sqrt{2\pi\sigma_B{}^2\Delta\Lambda(t)}} \qquad (5.62)$$

$$\exp\left[-\frac{(\Delta x - \mu\Delta\Lambda(t))^2}{2\sigma_B{}^2\Delta\Lambda(t)}\right]$$

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- **Reliability:**

$$R_{T_\rho}(t) = \Phi\left(\frac{\mu\Lambda(t)-\rho}{\sqrt{\sigma_B{}^2\Lambda(t)}}\right) \tag{5.62}$$

$$-\exp\left[-\frac{2\mu\rho}{2\sigma_B{}^2}\right]\times\Phi\left(\frac{\mu\Lambda(t)+\rho}{2\sigma_B{}^2\Lambda(t)}\right)$$

- **Mean time to failure**: (5.62)

$$MTTF_T = E(T) = \int_0^\infty t.Ff_T(t)\,dt \approx \frac{\rho}{w}\hat{\mu}$$

1. Maximum likelihood method

The likelihood estimation method is the method proposed to estimate unknown parameters based on accelerated degradation tests. Consider z_{ijk} the characteristic degradation of stress by noting j^{th} the measure of sampling at the level i^{th} on the sampling value and k the number of measures. We can then write t_{ijk} measuring time with Notons $\Delta z_{ijk} = z_{ijk} - z_{i(j-1)k}$ the increment of degradation and $\Delta t_{ijk} = t_{ijk} - t_{i(j-1)k}$ the time increment function after transformation ($\Lambda(t) = t^q$ the function is nonlinear and if $q=1$ the function will be linear).

We have the log-likelihood functions of the two stochastic processes:

$$\ln L(a(S_k),\beta) = \sum_{k=1}^{N_1}\sum_{i=1}^{N_2}\sum_{j=1}^{N_3}\left[\begin{array}{l}(\alpha(S_k)\Delta t_{ijk}-1)\ln\Delta z_{ijk} - \frac{\Delta z_{ijk}}{\beta}\\ -\ln\Gamma(\alpha(S_k)\Delta t_{ijk}) - \alpha(S_k)\Delta t_{ijk} - \ln\beta\frac{\Delta z_{ijk}}{\beta}\end{array}\right]$$

$$\ln L(\Phi|Z) = -\frac{mNL}{2}\left(\ln(2\pi)+\ln\left(\sigma_B^2\right)\right) - \frac{mN}{2}$$

$$\times\sum_{k=1}^{L}\ln\left(\Lambda(t_k)-\Lambda(t_{k-1})\right) - \frac{1}{2\sigma_B^2}$$

$$\times\sum_{i=1}^{m}\sum_{j=n_{n-1}+1}^{n_i}\sum_{k=1}^{L}\frac{z_{ijk} - N\left[ae^{-\frac{b}{S_i}}\left(\Lambda(t_k)-\Lambda(t_{k-1})\right)\right]}{\left(\Lambda(t_k)-\Lambda(t_{k-1})\right)}$$

I. Combined stochastic process

In this approach, we describe the combined degradation $(D_t)_{t\geq 0}$ of a system by the combination of a gamma process $(Y_t)_{t\geq 0}$ and a Brownian motion $(B_t)_{t\geq 0}$ of which it is independent of $(Y_t)_{t\geq 0}$, and multiplied by a constant $\tau \in \square$. The degradation model is therefore the combined degradation model $DC = (D_t)_{t\geq 0}$ which combines the two classical stochastic processes: gamma process and Brownian motion, which is represented by the following relationship:

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$$\forall t \geq 0, D_t = Y_t + \tau B_t \quad (1.1)$$

Or $(Y_t)_{t\geq 0}$ is a homogeneous gamma process of parameters $\beta \in \mathbb{R}_+^*$ and $\eta_t = \alpha t$ with $\alpha \succ 0$ and $(B_t)_{t\geq 0}$ is a Brownian motion.

$$\forall x \in \mathbb{R}, f_{\theta,\delta}(x) = \frac{\xi^{\frac{\alpha T}{N}}}{\Gamma\left(\frac{\alpha T}{N}\right)} \exp\left\{\frac{\tau^2 \xi^2 T}{2N} - x\xi\right\}$$

$$\int_0^{+\infty} \left(2\pi\tau^2 T/N\right)^{-\frac{1}{2}} \exp\left(-\frac{1}{2}\left[u - \left(x - \tau^2 \xi T/N\right)\right]^2 \Big/ \left(\frac{\tau^2 T}{N}\right)\right) u^{\frac{\alpha T}{N}-1} du$$

That is (D_t) the degradation model and which admits the following properties:

- Let's take $\Delta D_t = D_{t+\delta} - D_t = \Delta Y_t + \tau \Delta B_t$, we can write that the density of ΔD_t is defined by

$$f_{\Delta D_t}(x) = \frac{\xi^{\Delta\eta_t}\left(\tau^2\delta\right)^{\Delta\eta_t/2} e^{\delta\tau 2 \xi^{2}/4}}{\tau\sqrt{2\pi\delta}} \quad (1.2)$$

$$\exp\left(-\frac{1}{4}\frac{x^2}{\delta\tau^2} - \frac{x\delta}{2}\right) D_{-\Delta\eta_t}\left(\left(\xi - \frac{x}{\delta\tau^2}\right)\tau\sqrt{\delta}\right)$$

Or $D_p(z) = \frac{e^{-\frac{z^2}{4}}}{\Gamma(-p)} \int_0^{+\infty} \exp\left(-zx - \frac{x^2}{2}\right) x^{-p-1} dx$, if the real part $\operatorname{Re}(p) \prec 0$, so D_p is called parabolic cylindrical function

- By referring to the density of the truncated normal law, we obtain the density of increments as follows:

$$\forall x \in \mathbb{R}, f_{\Delta D_t}(x) = \frac{\xi^{\Delta\eta_t}}{\Gamma(\Delta\eta_t)} \quad (1.3)$$

$$\exp\left(\frac{\delta\tau^2\xi^2}{2} - x\xi\right) P\left[Q \succ 0\right] h\left(\Delta\eta_t - 1\right),$$

We can deduce the density which is written as follows:

$$f\Delta D_t(x) = \frac{\xi^{\Delta\eta_t}}{\Gamma(\Delta\eta_t)} \exp\left(\frac{\delta\tau^2\xi^2}{2} - x\xi\right) \quad (1.5)$$

$$\int_0^{+\infty} \left(2\pi\delta\tau^2\right)^{-\frac{1}{2}} e^{e^{-1/2\left[u-\left(x-\delta\tau^2\xi\right)\right]^2/\delta\tau^2}} u^{\Delta\eta_t - 1} du$$

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1. Maximum likelihood method

In this maximum likelihood method we can make an estimate of the different parameters of the models.

With $\theta = (\xi, \alpha, \tau^2)$ and $\delta = T/N$. And note that $\underline{x} = \left(x_j^{(i)}; 1 \le i \le n \text{ et } 1 \le j \le N\right)$.

The likelihood function of increments is denoted by: $L(\underline{x}|\xi, \alpha, \tau^2)$. Hence the following relationship:

$$L\left(\underline{x}|\xi,\alpha,\tau^2\right) = \prod_{i=1}^{n}\prod_{j=1}^{N}\left(\begin{array}{c} \dfrac{\xi^{\frac{\alpha T}{N}}}{\Gamma\left(\frac{\alpha T}{N}\right)}\exp\left\{\dfrac{\tau^2\xi^2 T}{2N} - x_j^{(i)}\xi\right\} \\ \int_0^{+\infty} \begin{array}{c}\left(2\pi\tau^2 T/N\right)^{-\frac{1}{2}} \\ \exp\left(-\frac{1}{2}\left[u - \left(x_j^{(i)} - \tau^2\xi T/N\right)\right]^2 \Big/ \frac{\tau^2 T}{N}\right)\end{array} u^{\frac{\alpha T}{N}-1} du \end{array}\right)$$

2. Combined process with covariate

We introduce the accelerate model on the performance degradation that is obtained by the traditional accelerated life test model. The Arrhenius model is renowned for describing the evolution of degradation by the effect of temperature.

The Arrhenius model is defined by [3]:

$$A(T) = a\exp\left(-\frac{E_a}{KT}\right)$$

With : $A(T)$ represents the rate of the reaction, T absolute temperature, and E_a activation energy, K represents the Boltzmann constant.

On the accelerated degradation analysis, the function $A(.)$ and the rate of degradation are indicators of parameters associated with the degradation of product performance

III. ESTIMATION BY BAYESIAN APPROACH AND THE MARKOV AND MONTE CARLO CHAIN ALGORITHM

The usual method is no longer enough, we need to add another approach like Bayesian inference. The latter is able to estimate parameters based on Bayes' theorem. It consists in observing the prior distribution with the probability of the data observed by the likelihood function in order to obtain the posterior distribution.

This method focuses upstream by the distribution parameter the prior distribution and the maximum likelihood. And downstream by the distribution of the a posteriori law.

The prior distribution contains different information about the useful data and we can classify the parameters as conjugate prior, informative prior, non-informative. If it is noninformative about the parameters, there is no a priori conjugate. According to the Bayesian theorem, we have the posterior distribution as follows:

$$\pi(\Phi|z) = \frac{L(z|\Phi)\times p(\Phi)}{\int L(z|\Phi)\times p(\Phi)\,d\Phi} \propto L(z|\Phi)\times p(\Phi)$$

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With Φ is an unknown vector of the parameter model; $\pi(\Phi|z)$ a posteriori distribution; $p(\Phi)$ prior distribution; $L(z|\Phi)$ likelihood function; z observational data.

Due to the complexity on the resolution of the numerical calculation concerning the parameter estimation, we combine with a method of Markov chain and Monte Carlo (MCMC). It is a question of generating a vector algorithm with the Markov chain with the Open BUGS software in order to find the posterior distribution.

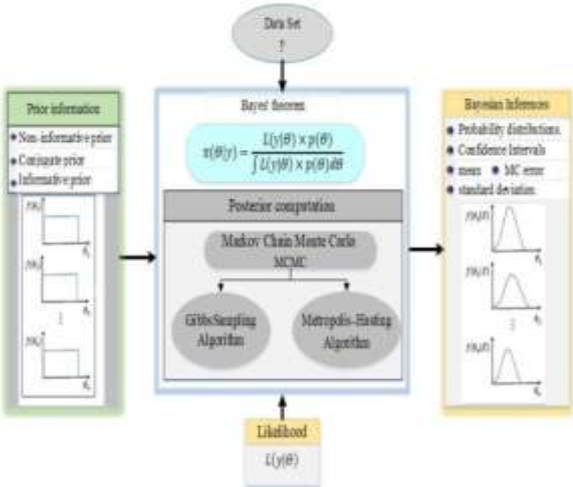

Figure :Bayesian Markov Chain Monte Carlo method

2. Model selection criteria

To better compare on the proposed models, we will make a selection based on the deviance information criterion. (DIC).

IV. APPLICATION

1. Simulation data

Our field of application relates to the degradation data of the photovoltaic modules, it is about degradation of the power supplied by the module. The observation begins at the start of the start-up. The goal is to assess degradation in order to predict maintenance.

Table 1: parameters and lifetime estimation for a photovoltaic module [2]

Table 1.1: Photovoltaic module lifetime estimation parameter

Module	Equation	at	b	ct	k	Initial power
S70L 45	1	-	-	-	-	100
	2	100	-	4299	- 9.54. 10-4	98.4
	3	43	55.5	2875. 4	- 0.001 9	98.6

We illustrate in the following table, the different data, parameter and estimates of the photovoltaic modules that we use during our studies

The model will describe the relationship between performance degradation and usage time whether it is a linear or non-linear regression. The model is quite simple, it is obtained by estimating the remaining life of the module.

The following relationship describes the degradation [2]:

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$$P(t) = P_0 - D(t)$$

With :

- P: output power at time t, P_0 : rated output power; $D(t)$ the degradation random variable.

We assume that the shape of the degradation modeling curve is similar to the sigmoid function:

$$P(t) = \frac{a}{\left(1+\exp\left(-k*t\left(t-t_c\right)\right)\right)}$$

With : a , k and t_c are regression coefficients. The rated power which is exposed outdoors to $t=0$ which is not greater than 100. So we take $P_0 = 100$ and $a = 100$

We add another regression coefficient b in order to have a lifetime between 6 to 10 years with a power degradation no more than 80% of its initial value.

$$P(t) = b + \frac{a}{\left(1+\exp\left(-k*t\left(t-t_c\right)\right)\right)}$$

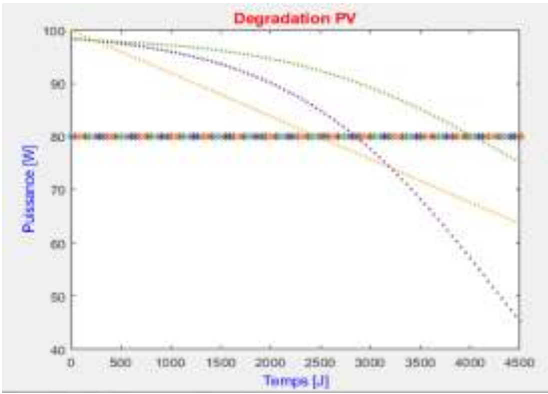


Figure 2: S70L45 Photovoltaic Module Lifetime

Table 1.01 :Estimation Model and Degradation

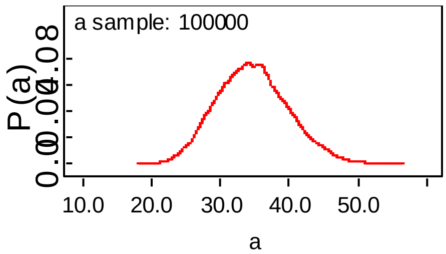
The photovoltaic module power degradation data is obtained by the three equations of the deterministic model.

2. Bayesian estimation and Monte Carlo method

We can write the following relation:

$$\pi_G\left(a,b,\beta,\alpha|\Delta z\right) \propto L_G\left(\Delta z|a,b,\beta,\alpha\right) \cdot p(a)\cdot p(b)\cdot p(\beta)\cdot p(\alpha)$$

By using the Monte Carlo method and the Gibbs algorithm, and the open software BUGS, the unknown parameters are obtained from 100000 iterations. The estimate is composed of the posterior distribution mean, standard error, standard mounted Carlo.



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Volume 4, Issue 3 Mar 2022, pp: 1330-1338 www.ijaem.net ISSN: 2395-5252

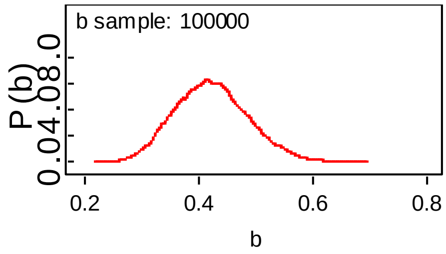
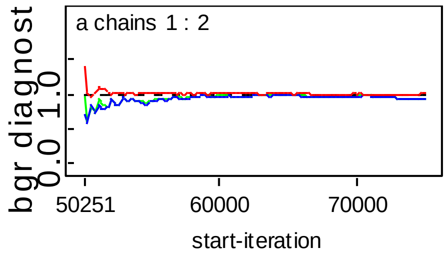


Figure 3: a and b probability density of the combined model



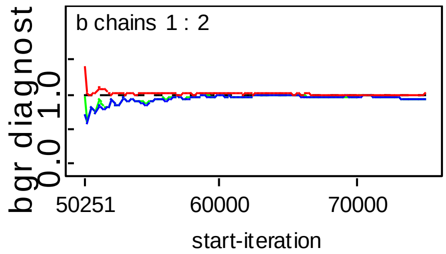


Figure 3: diagnosis for parameters a and b

Table 1.2: comparisonbetweenstochastic processes

Stochastic process	Mean time to failure (hour)	Durability (years)	Deviance Information Criterion
Wiener process	1.9466e+03	5.33	787.0
Gamma process	2.3937e+03	6.6	768.7
Combined process	2.8375e+03	7.77	769.9

We find that the Wiener process has the highest deviance information criterion unlike the gamma process which is lower. Regarding sustainability, the processcombined with the greatest age of all, this proves that this combined process models the degradation process well, the second rank is the gamma process. We find that the combination of two processes increases the viability of the system before being put into maintenance or replacement.

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