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Comparative Analysis of 2 Degree of freedom in Robots

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ABSTRACT-The new non-linear fractional-order PID controller (NLFPID) is designed to control combined and non-linear two-link rigid robots. The structure of the proposed controller consists of a nonlinear hyperbolic instantaneous error function and the present state cascaded to fractions. Order PID (FOPID). The non-linear feature enables adaptive control and the inclusion of the fraction operator increases the flexibility of the designed controller. Learn the comparative advantages of NLF PID controllers.

Keywords-Two Degree of Freedom, PID Controller, MATLAB, Lagrangian, Manipulator.

I. INTRODUCTION

Robot operators are very useful in various real life applications that work in industrial production and automation, medical deposit, safety, harmful conditions and spaceships. Various links and assemblies can process objects and functions of human movements to successfully process objects with accurate positioning and desired locations. Recently, a wide range of robotic hands have created a study in the design and control direction of Eneeffuner in a robot control. Some elements of creating these systems of complex devices include nonlinear during transformation of dynamics, time diseases, and payloads. If a timely is not perfectly removed or oppressed or oppressed, these unwanted amazing elements of the robot system can adversely affect performance. The system may exhibit performance degradation and instability associated with complete trajectory and pick-and-place operations during the execution phase. Because of these factors, it is a difficult task to efficiently, reliably and accurately control a robotic arm with an intelligent controller. Designing and implementing an efficient controller requires expertise, including expertise in the dynamics of a robotic arm due to the torque generated by the actuator. In the past, various control strategies have been widely used in robotic arms to achieve desired performance.

Two degrees of freedom (2DOF) pointing mechanisms are widely used in stable platforms, beacons and other fields. Besides the commonly used serial gimbals, there are two types of parallel pointing mechanisms. H. Equal Diameter Spherical Parallel Manipulator (SPM) and Spherical Pure Rolling (ESPR) Parallel Manipulators are increasingly affected. All of these pointing mechanisms have two rotational DOFs, but exhibit very different motion characteristics. A typical difference that exists between these three pointing mechanisms lies in the properties of proper motion, also called rotational motion by the authors. According to our research, rotational motion is basically a component of the actual rotation of the movable platform. Additionally, using the pointing mechanism as a tracking device, image distortions caused by the 2 rotational motions are identified and distinguished. The conclusions are that it facilitates the design and control of pointing devices and has the potential to improve the measurement accuracy of target pointing and tracking.

1.2 Mathematical Model of Robotic Manipulator

A two-link planar rigid body system of a robotic manipulator with two degrees of freedom is described in this section and shown in Figure 1. A mechanical model of the two-link robotic manipulator under consideration is shown in Figure 1. Located at the end of each link, drive, and encoder. The first link of the system is mounted on a hard floor with friction-free hinges and the second link is mounted on the end of the first link with friction-free ball bearings. The third link is also attached to the second link of the frictionless ball bearing.

1.3 Dynamics of 2 link rigid robots

A double pendulum is an example of a simple physical system that exhibits chaotic behavior. Understanding the two-link manipulator [6] is key to learning the whole robot manipulator. Think of as a two-legged robotic manipulator. This is a classic example of an introductory robotics course. A physical system is shown in Figure 1.

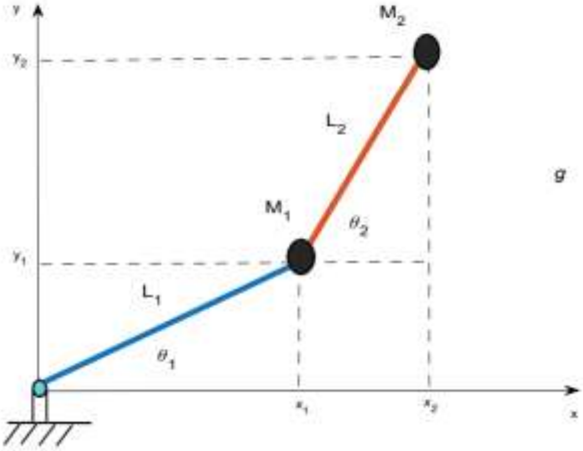

Figure 1

$x_1 = L_1\cos(\theta_1), \quad y_1 = L_1\sin(\theta_1).$ (1.1)

Similarly, the equations for the x-position and the y-position of M_2 are given by

$x_2 = L_1\cos(\theta_1) + L_2\cos(\theta_2), \quad y_2 = L_1\sin(\theta_1) + L_2\sin(\theta_2)$ (1.2)

$v_1 = \sqrt{\dot{x}_1^2 + \dot{y}_1^2}, v_2 = \sqrt{\dot{x}_2^2 + \dot{y}_2^2}$ (1.3)

where,

$$\dot{x}_1 = -L_1\dot{\theta}_1\sin(\theta_1), \dot{y}_1 = L_1\dot{\theta}_1\cos(\theta_1)$$
$$\dot{x}_2 = -L_1\dot{\theta}_1\sin(\theta_1) - L_2\dot{\theta}_2\sin(\theta_2), \dot{y}_2 = L_1\dot{\theta}_1\cos(\theta_1) + L_2\dot{\theta}_2\cos(\theta_2)$$

Here and below the dot $'$ is a derivative with respect to t, i.e., $\dot{\theta}_k = \frac{d\theta_k}{dt}, \dot{x}_k = \frac{dx_k}{dt}$, for $k = 1,2$. The kinematic energy can be calculated as follows

$KE = \frac{1}{2}M_1 v_1^2 + \frac{1}{2}M_2 v_2^2.$ (1.4)

The equation for the kinetic energy can be written as

$KE = \frac{1}{2}M_1(\dot{x}_1^2 + \dot{y}_1^2) + \frac{1}{2}M_2(\dot{x}_2^2 + \dot{y}_2^2).$ (1.5)

Substituting (1.1) - (1.2) into (1.5), we get

$$KE = \frac{1}{2}M_1\left(\left(-L_1\dot{\theta}_1\sin(\theta_1)\right)^2 + \left(L_1\dot{\theta}_1\cos(\theta_1)\right)^2\right)$$
$$+\frac{1}{2}M_2\begin{pmatrix}\left(-L_1\dot{\theta}_1\sin(\theta_1) - L_2\dot{\theta}_2\sin(\theta_2)\right)^2 + \\ \left(L_1\dot{\theta}_1\cos(\theta_1) + L_2\dot{\theta}_2\cos(\theta_2)\right)^2\end{pmatrix},$$

which can be simplified as

$KE = \frac{1}{2}(M_1 + M_2)L_1^2\dot{\theta}_1^2 + \frac{1}{2}M_2L_2^2\dot{\theta}_2^2 + M_2L_1L_2\dot{\theta}_1\dot{\theta}_2\cos(\theta_1 - \theta_2).$ (1.6)

In order to calculate the Lagrangian, the potential energy PE has to be calculated. By definition the potential energy of the system due to gravity of the i^{th} pendulum is

$PE_i(\theta) = M_i g h_i(\theta), i = 1,2,$ (1.7)

where h_i is the height of the center of mass of the i^{th} pendulum, g is the acceleration due to gravity constant, and M_i is the mass of the i^{th} pendulum. Therefore, the total potential energy for both pendulums can be given by

$$PE = PE(\theta) = \sum_{i=1}^{2} PE_i(\theta) = \sum_{i=1}^{2} M_i g h_i(\theta) = M g L_1\sin(\theta_1) + M_2 g\left(L_1\sin(\theta_1) + L_2\sin(\theta_2)\right) \quad (1.8)$$
$$= (M_1 + M_2)gL_1\sin(\theta_1) + M_2gL_2\sin(\theta_2)$$

Next, by Lagrange Dynamics, we form the

Lagrangian $\mathcal{L}$ which is defined as

$$\mathcal{L} = KE - PE \tag{1.9}$$

Substituting the expressions for the kinetic energy (1.6) and potential energy (1.8) in for KE and PE we get

$$\mathcal{L} = \frac{1}{2}(M_1 + M_2)L_1^2\dot{\theta}_1^2 + \frac{1}{2}M_2L_2^2\dot{\theta}_2^2 + M_2L_1L_2\dot{\theta}_1\dot{\theta}_2\cos(\theta_1 - \theta_2) - (M_1 + M_2) gL_1\sin(\theta_1) - M_2gL_2\sin(\theta_2) \tag{1.10}$$

The Euler-Lagrange equation is given by the equation

$$\frac{d}{dt}\left[\frac{\partial \mathcal{L}}{\partial \dot{\theta}_i}\right] - \frac{\partial \mathcal{L}}{\partial \theta_i} = \tau_i, \quad i = 1,2, \tag{1.11}$$

Where τ_i is the torque applied to the i^{th} link.

The derivations for the Lagrangian equation and Euler-Lagrange equation can be found in [1]. We were more concerned with using the formulas in this problem rather than going into too much in detail for these derivations.

From (1.10), we have

$$\frac{\partial \mathcal{L}}{\partial \dot{\theta}_i} = (M_1 + M_2)L_1^2\dot{\theta}_1 + M_2L_1L_2\dot{\theta}_2\cos(\theta_1 - \theta_2),$$

$$\frac{\partial \mathcal{L}}{\partial \theta_1} = -M_2L_1L_2\dot{\theta}_1\dot{\theta}_2\sin(\theta_1 - \theta_2) - (M_1 + M_2)gL_1\cos(\theta_1),$$

and

$$\frac{d}{dt}\left[\frac{\partial \mathcal{L}}{\partial \dot{\theta}_i}\right] = (M_1 + M_2)L_1^2\ddot{\theta}_1 + M_2L_1L_2\ddot{\theta}_2\cos(\theta_1 - \theta_2) - M_2L_1L_2\dot{\theta}_2(\dot{\theta}_1 - \dot{\theta}_2)\sin(\theta_1 - \theta_2).$$

Similarly, we compute

$$\frac{\partial \mathcal{L}}{\partial \dot{\theta}_2} = M_2L_2^2\dot{\theta}_2 + M_2L_1L_2\dot{\theta}_1\cos(\theta_1 - \theta_2)$$

$$\frac{\partial \mathcal{L}}{\partial \theta_2} = M_2L_1L_2\dot{\theta}_1\dot{\theta}_2\sin(\theta_1 - \theta_2) - M_2gL_2\cos(\theta_2)$$

and

$$\frac{d}{dt}\left[\frac{\partial \mathcal{L}}{\partial \dot{\theta}_2}\right] = M_2L_2^2\ddot{\theta}_2 + M_2L_1L_2\ddot{\theta}_1\cos(\theta_1 - \theta_2) - M_2L_1L_2\dot{\theta}_1(\dot{\theta}_1 - \dot{\theta}_2)\sin(\theta_1 - \theta_2).$$

Therefore, (1.11) gives the following two nonlinear equations of motion which are second-order system of ordinary differential equations

$$(M_1 + M_2)L_2^2\ddot{\theta}_1 + M_2L_1L_2\ddot{\theta}_2\cos(\theta_1 - \theta_2) + M_2L_1L_2\dot{\theta}_2^2\sin(\theta_1 - \theta_2) + (M_1 + M_2)gL_1\cos(\theta_1) = \tau_1, \tag{1.12}$$

$$M_2L_2^2\ddot{\theta}_2 + M_2L_1L_2\ddot{\theta}_2\cos(\theta_1 - \theta_2) - M_2L_1L_2\dot{\theta}_1^2\sin(\theta_1 - \theta_2) + M_2gL_2\cos(\theta_2) = \tau_2,$$

or equivalently,

$$L_1\ddot{\theta}_1 + \delta L_1L_2\ddot{\theta}_2\cos(\theta_1 - \theta_2) = \frac{\delta\tau_1}{M_2L_1} - \delta L_2\dot{\theta}_2^2\sin(\theta_1 - \theta_2) - g\cos(\theta_1),$$

$$L_2\ddot{\theta}_2 + L_1\ddot{\theta}_1\cos(\theta_1 - \theta_2) = \frac{\tau_2}{M_2L_2} + L_1\dot{\theta}_1^2\sin(\theta_1 - \theta_2) - g\cos(\theta_2),$$

where, $\delta = \frac{M_2}{M_1 + M_2}$

Solving for $\ddot{\theta}_1$ and $\ddot{\theta}_2$, we get the normal form of the dynamics equations

$$\ddot{\theta}_1 = g_1(t, \theta_1, \theta_2, \dot{\theta}_1, \dot{\theta}_2), \ddot{\theta}_2 = g_2(t, \theta_1, \theta_2, \dot{\theta}_1, \dot{\theta}_2),$$

$$g_1 = \frac{\frac{\delta\tau_1}{M_2L_1} - \delta L_2\dot{\theta}_2^2\sin(\theta_1-\theta_2) - g\cos(\theta_1) - \delta\cos(\theta_1-\theta_2)\left(\frac{\tau_2}{M_2L_2} + L_1\dot{\theta}_1^2\sin(\theta_1-\theta_2) - g\cos(\theta_2)\right)}{L_1\left(1-\delta\cos^2(\theta_1-\theta_2)\right)},$$

$$g_2 = \frac{\frac{\tau_2}{M_2L_2} + L_1\dot{\theta}_1^2\sin(\theta_1-\theta_2) - g\cos(\theta_2) - \cos(\theta_1-\theta_2)\left(\frac{\delta\tau_1}{M_2L_1} + \delta L_2\dot{\theta}_2^2\sin(\theta_1-\theta_2) - g\cos(\theta_1)\right)}{L_2\left(1-\delta\cos^2(\theta_1-\theta_2)\right)}.$$

In order to solve for the angles θ_1 and θ_2, we need to solve the above second-order system of ordinary differential equations. To do this, we first reduce the system into an equivalent system of first-order ordinary differential equations.
Let us introduce four new variables

$$u_1 = \theta_1, u_2 = \theta_2, u_3 = \dot{\theta}_1, u_4 = \dot{\theta}_2.$$

After differentiating, we have

$$\dot{u}_1 = \dot{\theta}_1 = u_3, \dot{u}_2 = \dot{\theta}_2 = u_4, \dot{u}_3 = \ddot{\theta}_1 = g_1(t, u_1, u_2, u_3, u_4), \dot{u}_4 = \ddot{\theta}_2 = g_2(t, u_1, u_2, u_3, u_4)$$

Thus, we obtain a system of first-order nonlinear differential equations of the form

$$\frac{d\mathbf{U}}{dt} = \mathbf{S}(t, \mathbf{U}), \mathbf{U}(0) = \mathbf{U}_0, \quad (1.16a)$$

Where $\mathbf{U} = [u_1, u_2, u_3, u_4]^t$ and $\mathbf{S} = [s_1, s_2, s_3, s_4]^t$ with

$$s_1 = u_3, s_2 = u_4, s_3 = g_1(t, u_1, u_2, u_3, u_4), s_4 = g_2(t, u_1, u_2, u_3, u_4)$$

The initial conditions are given by

$$\mathbf{U}_0 = [u_1(0), u_2(0), u_3(0), u_4(0),]^t, \quad (1.16b)$$

Where,

$$u_1(0) = \theta_1(0), u_2(0) = \theta_2(0), u_3(0) = \dot{\theta}_1(0), u_4(0) = \dot{\theta}_2(0).$$

The system (1.16a) subject to the initial condition (1.16b) can be solved for the unknown vector**U**, using, for example, the ode45 command defined in MATLAB to solve the system of ordinary differential equations numerically. This command is based on the fourth-order Runge-Kutta method. Table 1: The PID terms and their effect on a control system.

Term	Math Function	Effect on Control System
P Proportional	$K_p\, V_{error}$	Typically the main drive in a control loop, K_p reduces a large part of the overall error.
I Integral	$K_I \int V_{error}\, dt$	Reduces the final error in a system. Summing even a small error over time produces a drive signal large enough to move the system toward a smaller error.
D Derivative	$k_D \frac{dV_{error}}{dt}$	Counteracts the K_p and K_I terms when the output changes quickly. This helps reduce overshoot and ringing. It has no effect on the final error.

Table 1

PID Controller Design

Robotic manipulators are generally difficult to control. In particular, it is a challenging task to stabilize a robot manipulator at a fixed, accurate position. In this section, we focus mainly on control of the robot manipulator to get the desired position using computed torque control method [4]. After deriving the equation of motion, control simulation is represented using MATLAB.

We define control as the ability to hold the system of two links in a particular position on the xy-plane. Having control gives us the ability to hold each link at a particular angle θ_i with respect to the positive x-axis. The proportional-integral-derivative (PID) controller is a common control algorithm [5]. The "P" in PID stands for Proportional control, the "I" stands for Integral control, and the "D" stands for Derivative control. This algorithm works by defining an error variable $V_{error} = V_{set} - V_{sensor}$ that takes the position we want to go (V_{set}) minus the position we are actually at (V_{sensor}). We get the proportional part of the PID control by taking a constant defined as K_p and multiplying it by the error. The I comes from taking a constant K_1 and multiplying it by the integral of the error with respect to time. Derivative control is defined as a constant K_D multiplied by the derivative of the error with respect to time. Many industrial processes are controlled using PID controllers. Below is a table describing PID control Table 1.

The equations of motion (1.12) can be written compactly as

$$M(\boldsymbol{\theta})\ddot{\boldsymbol{\theta}} + \mathbf{c}(\boldsymbol{\theta},\dot{\boldsymbol{\theta}}) + \mathbf{G}(\boldsymbol{\theta}) = \mathbf{F}$$

where,

$$\boldsymbol{\theta} = \begin{bmatrix}\theta_1\\ \theta_2\end{bmatrix}, M(\boldsymbol{\theta}) = \begin{bmatrix}(M_1+M_2)L_1^2 & M_2L_1L_2\cos(\theta_1-\theta_2)\\ M_2L_1L_2\cos(\theta_1-\theta_2) & M_2L_2^2\end{bmatrix}, \mathbf{c}(\boldsymbol{\theta},\dot{\boldsymbol{\theta}}) = \begin{bmatrix}M_2L_1L_2\dot{\theta}_2^2\sin(\theta_1-\theta_2)\\ -M_2L_1L_2\dot{\theta}_1^2\sin(\theta_1-\theta_2)\end{bmatrix},$$

$$\mathbf{G}(\boldsymbol{\theta}) = \begin{bmatrix}(M_1+M_2)gL_1\cos(\theta_1)\\ M_2gL_2\cos(\theta_2)\end{bmatrix}, = \mathbf{F}\begin{bmatrix}\tau_1\\ \tau_2\end{bmatrix},$$

We can solve for some theoretical values of forces given certain initial inputs. Solving for $\ddot{\boldsymbol{\theta}}$ we get

$$\ddot{\theta} = -M^{-1}(\theta)[\mathbf{c}(\boldsymbol{\theta},\dot{\theta}) + \mathbf{G}(\boldsymbol{\theta})] + \hat{\mathbf{F}}$$

Where

$$\hat{\mathbf{F}} = M^{-1}(\boldsymbol{\theta})\mathbf{F}.$$

Thus, we decoupled the system to have the new input

$$\hat{\mathbf{F}} = \begin{bmatrix}f_1\\ f_2\end{bmatrix}$$

However, the physical torque inputs to the system arè

$$\mathbf{F} = \begin{bmatrix}\tau_1\\ \tau_2\end{bmatrix} = M(\boldsymbol{\theta})\begin{bmatrix}f_1\\ f_2\end{bmatrix}$$

Let us denote the error signals by

$$e(\theta_1) = \theta_{1f} - \theta_1, e(\theta_2) = \theta_{2f} - \theta_2,$$

Where the target positions of M_1 and M_2 are given by the angles θ_{1f} and θ_{2f}, respectively.
We assume that the system has initial positions

$$\theta_0 = \begin{bmatrix}\theta_1(0)\\ \theta_2(0)\end{bmatrix}.$$

A common technique for controlling a system with input is to use the following general structure of PID controller

$$f = K_P e + K_D\dot{e} + K_1\int edt$$

In our situation, the technique for controlling the double pendulum system with inputs f_1 and f_2 is to employ two independent controllers, one for each link, as follows

$$f_1 = K_{P_1}e_1(\theta_1) + K_{D_1}\dot{e}_1(\theta_1) + K_{I_1}\int e(\theta_1)dt = K_{P_1}(\theta_{1f}-\theta_1) - K_{D_1}\dot{\theta}_1 + K_{I_1}\int(\theta_{1f}-\theta_1)dt,$$

$$f_2 = K_{P_2}e_2(\theta_2) + K_{D_2}\dot{e}_2(\theta_2) + K_{I_2}\int e(\theta_{21})dt = K_{P_2}(\theta_{2f}-\theta_2) - K_{D_2}\dot{\theta}_2 + K_{I_2}\int(\theta_{2f}-\theta_2)dt,$$

Where θ_{1f} and θ_{2f} are given constants.
The complete system of equations with control is then

$$\ddot{\theta} = -M^{-1}(\boldsymbol{\theta})[\mathbf{c}(\boldsymbol{\theta},\dot{\theta}) + \mathbf{G}(\boldsymbol{\theta})] + \hat{\mathbf{F}},$$

Where

$$\hat{\mathbf{F}} = \begin{bmatrix}f_1\\ f_2\end{bmatrix} = \begin{bmatrix}K_{P_1}(\theta_{1f}-\theta_1) - K_{D_1}\dot{\theta}_1 + K_{I_1}\int(\theta_{1f}-\theta_1)dt\\ K_{P_2}(\theta_{2f}-\theta_2) - K_{D_2}\dot{\theta}_2 + K_{I_2}\int(\theta_{2f}-\theta_2)dt\end{bmatrix}.$$

We would like to emphasize that the actual physical torques are

$$\begin{bmatrix}\tau_1\\ \tau_2\end{bmatrix} = M(\boldsymbol{\theta})\begin{bmatrix}f_1\\ f_2\end{bmatrix}$$

To implement the PID controller, we introduce the following new states

$$x_1 = \int e(\theta_1)dt, x_2 = \int e(\theta_2)dt.$$

Differentiating with respect to t gives

$$\dot{x}_1 = e(\theta_1) = \theta_{1f} - \theta_1, \dot{x}_2 = e(\theta_2) = \theta_{2f} - \theta_2.$$

The complete equations are

$$\dot{x}_1 = \theta_{1f} - \theta_1,$$
$$\dot{x}_2 = \theta_{2f} - \theta_2,$$
$$\begin{bmatrix}\ddot{\theta}_1\\ \ddot{\theta}_2\end{bmatrix} = -M^{-1}(\boldsymbol{\theta})\left[\mathbf{c}(\boldsymbol{\theta},\dot{\theta}) + \mathbf{G}(\boldsymbol{\theta})\right] + \begin{bmatrix}K_{P_1}(\theta_{1f}-\theta_1) - K_{D_1}\dot{\theta}_1 + K_{I_1}x_1\\ K_{P_2}(\theta_{2f}-\theta_2) - K_{D_2}\dot{\theta}_2 + K_{I_2}x_2\end{bmatrix}$$
$$\begin{bmatrix}\tau_1\\ \tau_2\end{bmatrix} = M(\boldsymbol{\theta})\begin{bmatrix}K_{P_1}(\theta_{1f}-\theta_1) - K_{D_1}\dot{\theta}_1 + K_{I_1}x_1\\ K_{P_2}(\theta_{2f}-\theta_2) - K_{D_2}\dot{\theta}_2 + K_{I_2}x_2\end{bmatrix}$$

To discretize the above system of differential equations in time, we transform them into a system of first-order ordinary differential equations. To do this, we define six new variables as follows

$$u_1 = x_1, u_2 = x_2, u_3 = \theta_1, u_4 = \theta_2, u_5 = \dot{\theta}_1, u_6 = \dot{\theta}_2,$$

After differentiating, we have

$$\dot{u}_1 = \dot{x}_1 = \theta_{1f} - u_3, \dot{u}_2 = \dot{x}_2 = \theta_{2f} - u_4,$$
$$\dot{u}_3 = \dot{\theta}_1 = u_5, \dot{u}_4 = \dot{\theta}_2 = u_6,$$
$$\ddot{u}_5 = \ddot{\theta}_1 = \phi(t,u_1,u_2,u_3,u_4,u_5,u_6), \ddot{u}_6 = \ddot{\theta}_2 = \psi(t,u_1,u_2,u_3,u_4,u_5,u_6),$$

where ϕ and ψ are expressed in terms of $u_{k^k}k = 1-6$, as

$$\begin{bmatrix}\phi\\ \psi\end{bmatrix} = -M^{-1}(\boldsymbol{\theta})[\mathbf{c}(\boldsymbol{\theta},\dot{\boldsymbol{\theta}}) + \mathbf{G}(\boldsymbol{\theta})] + \begin{bmatrix}K_{P_1}(\theta_{1f}-u_3) - K_{D_1}u_5 + K_{I_1}u_1\\ K_{P_2}(\theta_{2f}-u_4) - K_{D_2}u_6 + K_{I_2}u_2\end{bmatrix}$$

and $\boldsymbol{\theta} = \begin{bmatrix}u_3\\ u_4\end{bmatrix}$.

A simple calculation shows that

$$\phi = \frac{-M_2L_2u_6^2\sin(u_3-u_4) - (M_1+M_2)g\cos(u_3) - M_2\cos(u_3-u_4)\left(L_1u_5^2\sin(u_3-u_4) - g\cos(u_4)\right)}{L_1\left(M_1+M_2-M_2\cos^2(u_3-u_4)\right)}$$
$$+K_{P_1}(\theta_{1f}-u_3) - K_{D_1}u_5 + K_{I_1}u_1,$$
$$\psi = \frac{\cos(u_3-u_4)\left(M_2L_2u_6^2\sin(u_3-u_4) + (M_1+M_2)g\cos(u_3)\right) + (M_1+M_2)\left(L_1u_5^2\sin(u_3-u_4) - g\cos(u_4)\right)}{L_2\left(M_1+M_2-M_2\cos^2(u_3-u_4)\right)}$$
$$+K_{P_2}(\theta_{2f}-u_4) - K_{D_2}u_6 + K_{I_2}u_2$$

Thus, we obtain a system of first-order nonlinear differential equations of the form

$$\frac{d\boldsymbol{U}}{dt} = \boldsymbol{H}(t,\boldsymbol{U}), \boldsymbol{U}(0) = \boldsymbol{U}_0,$$

Where $\boldsymbol{U} = [u_1,u_2,u_3,u_4,u_5,u_6]^t$ and $\boldsymbol{H} = [h_1,h_2,h_3,h_4,h_5,h_6]^t$ with
$h_1 = \theta_{1f} - u_3, h_2 = \theta_{2f} - u_4, h_3 = u_5, h_4 = u_6,$
$h_5 = \phi(t,u_1,u_2,u_3,u_4,u_5,u_6), h_6 = \psi(t,u_1,u_2,u_3,u_4,u_5,u_6).$
The initial conditions are given by

$$\boldsymbol{U}_0 = [u_1(0),u_2(0),u_3(0),u_4(0),u_5(0),u_6(0)]^t,$$

Where,

$$u_1(0) = x_1(0), x_2(0) = \theta_2(0), u_3(0) = \theta_1(0), u_4(0) = \theta_2(0), u_5(0) = \dot{\theta}_1(0), u_6(0) = \dot{\theta}_2(0).$$

The system [3] can be solved for the unknown vector $\boldsymbol{U}$, using, for example, the ode45 command defined in MATLAB to solve the system of ordinary differential equations numerically. This command is based on the fourth-order Runge-Kutta method. For more details consult[2].

Once we solve for, we obtain to torques using $\boldsymbol{U} = (u_1,u_2,u_3,u_4,u_5,u_6)^t$

$$\begin{bmatrix}\tau_1\\ \tau_2\end{bmatrix} = M(\boldsymbol{\theta})\begin{bmatrix}K_{P_1}(\theta_{1f}-u_3) - K_{D_1}u_5 + K_{I_1}u_1\\ K_{P_2}(\theta_{2f}-u_4) - K_{D_2}u_6 + K_{I_2}u_2\end{bmatrix},$$

or equivalently,

1.3
$$\tau_1 = (M_1+M_2)L_1^2\left(K_{P_1}(\theta_{1f}-u_3) - K_{D_1}u_5 + K_{I_1}u_1\right) + M_2L_1L_2\cos(u_3-u_4)\left(K_{P_2}(\theta_{2f}-u_4) - K_{D_2}u_6 + K_{I_2}u_2\right),$$
$$\tau_2 = M_2L_1L_2\cos(u_3-u_4)\left(K_{P_1}(\theta_{1f}-u_3) - K_{D_1}u_5 + K_{I_1}u_1\right) + M_2L_2^2\left(K_{P_2}(\theta_{2f}-u_4) - K_{D_2}u_6 + K_{I_2}zu_2\right).$$

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II. CONCLUSION

THE PID CONTROLLER HELPS GET THE OUTPUTS, WHICH ARE THE FINAL POSITIONS OF M_1 AND M_2 DETERMINED BY THE ANGLES θ_1 AND θ_2, WHERE WE WANT IT, IN A SHORT TIME, WITH MINIMAL OVERSHOOT, AND WITH LITTLE ERROR. THIS WILL BE DEMONSTRATED USING THE FOLLOWING EXAMPLES.

Example 1. We consider the simplified model of a two-link manipulator shown in Figure 1. In this experiment, we take the following parameters

$$M_1 = M_2 = 1, L_2 = 2, L_2 = 1.$$

$M_1 = M_2 = 1, L_2 = 2, L_2 = 1$

The target positions (final positions) are $\begin{bmatrix}\theta_{1f}\\ \theta_{2f}\end{bmatrix} = \begin{bmatrix}\pi/2\\ 0\end{bmatrix}$

The initial positions, initial angle velocities, and initial states are, respectively, taken as

$$\begin{bmatrix}\theta_1(0)\\ \theta_2(0)\end{bmatrix} = \begin{bmatrix}\pi/2\\ \pi/2\end{bmatrix}, \begin{bmatrix}\dot{\theta}_1(0)\\ \dot{\theta}_2(0)\end{bmatrix} = \begin{bmatrix}0\\ 0\end{bmatrix}, \begin{bmatrix}x_1(0)\\ x_2(0)\end{bmatrix} = \begin{bmatrix}0\\ 0\end{bmatrix}.$$

The PID parameters for θ_1 and θ_2, are taken as

$$KP_1 = 30, \quad KD_1 = 15, \quad KI_1 = 20,$$
$$KP_2 = 30, \quad KD_2 = 10, \quad KI_2 = 20.$$

In Figure 2, we show the initial and target positions of the two-link manipulator. Next, we plot the positions θ_1 and θ_2 of M_1 and M_2 versus time over the interval [0,30] in Figure 3.

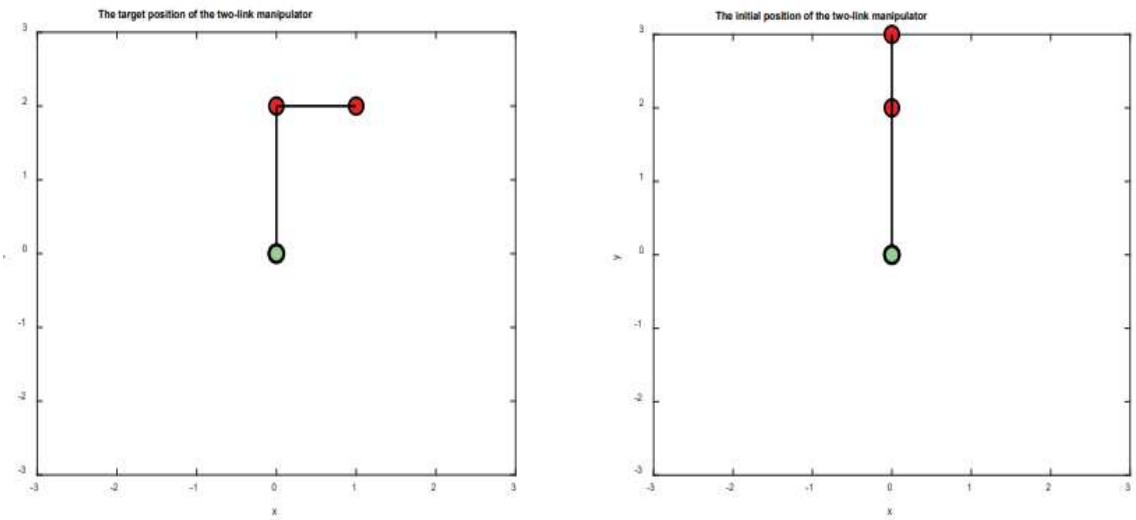

Figure 2: The initial positions (left) and the final position (right) of M1 and M2 for example 1.

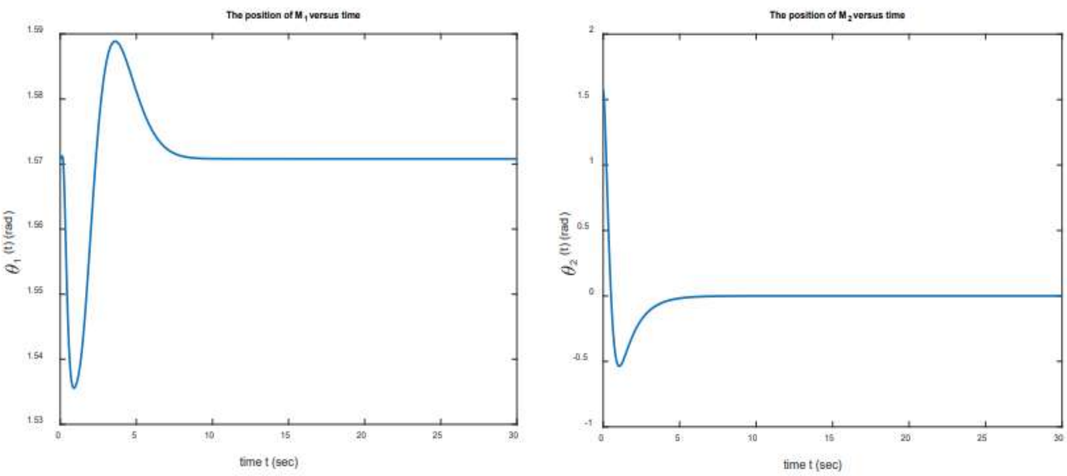

Figure 3: The positions of M1 (left) and M2 (right) versus time for example 1.

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In Figure 4 we present the difference between where we want to go and where we are actually at i.e.,

$$e(\theta_1) = \theta_{1f} - \theta_1, e(\theta_2) = \theta_{2f} - \theta_2,$$

where θ_{1f} and θ_{2f} are the target positions and $\theta_1(t)$ and $\theta_2(t)$ are the numerical approximations obtained by solving the system of ODEs. These results indicate that the PID controller gets the final positions.

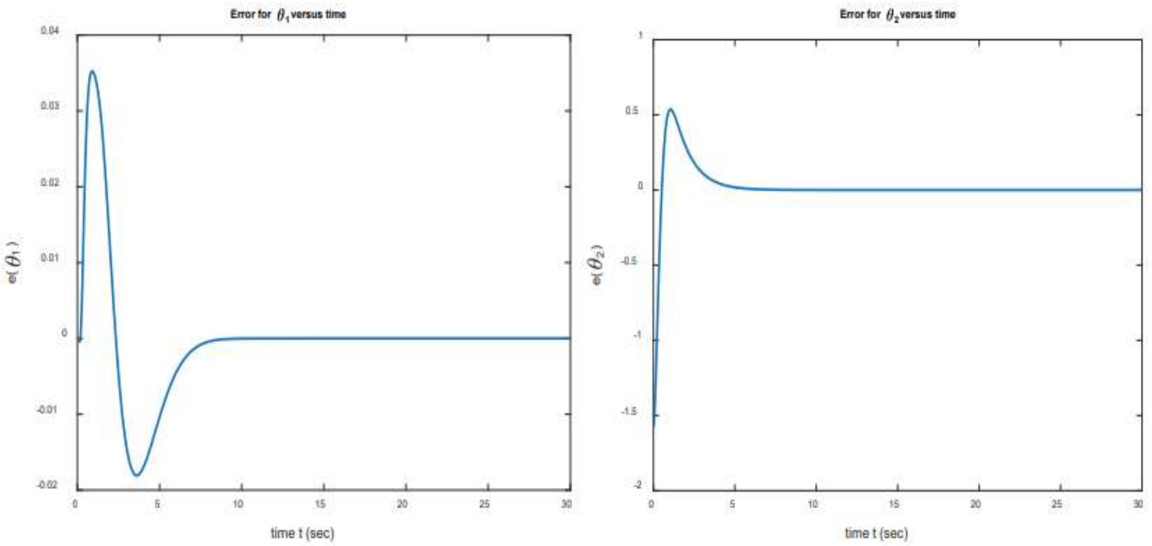

Figure 4: The errors $e(\theta_1) = \theta_1(t) - \theta_{1f}$ and $e(\theta_2)=\theta_2(t) - \theta_{2f}$ versus time for example 1.

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