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Constructal Design of Heat Exchangers: A Review

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Abstract
A substantial amount of research has been done for improving heat exchangers efficiency by electronic equipment, heating, and air conditions systems, and space vehicles, which were widely employed by domestic applications including thermal power systems, industrial, and transportations. Due to its wide use, its efficiency would therefore be increased and leading to a decrease in the materials, cost, and space remarkably. Constructal design can be applied to different systems of heat transfer such as the electronic package that can be designed optimally with the implementation of the constructal law. As a result, the goal of this work is to conduct a review of recent advancements and their potential for enhancing heat exchanger thermal performance with the aid of constructal design theory. The most recent state-of-the-art developments are thoroughly described, together with their evaluating parameters, and some recommendations for additional research in this field are provided.
Keywords: Forced convection, evolutionary design, Bejan number, multi-scale tubes, Constructal design.

التصميم الإنشائي للمبادلات الحرارية: مراجعة

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الخلاصة:
تم إجراء قدر كبير من الأبحاث لتحسين كفاءة المبادلات الحرارية المستخدمة في المعدات الإلكترونية وأنظمة التدفئة وتكييف الهواء والمركبات الفضائية ، والتي تم استخدامها على نطاق واسع في التطبيقات المحلية بما في ذلك أنظمة الطاقة الحرارية والصناعية ووسائل النقل نظرًا لاستخداماتها الواسعة حيث ستزداد كفاءتها ويؤدي إلى انخفاض في المواد والتكلفة والمساحة بشكل ملحوظ. يمكن تطبيق التصميم الإنشائي على أنظمة مختلفة لنقل الحرارة مثل الحزمة الإلكترونية التي يمكن تصميمها على النحو الأمثل مع تنفيذ القانون الإنشائي. ونتيجة لذلك ، فإن الهدف من هذا العمل هو إجراء مراجعة للتطورات الأخيرة وإمكانياتها في تعزيز الأداء الحراري للمبادل الحراري بمساعدة نظرية التصميم الإنشائي. يتم وصف أحدث التطورات الحديثة بدقة ، جنبًا إلى جنب مع معايير التقييم الخاصة بها ، ويتم تقديم بعض التوصيات للبحث الإضافي في هذا المجال.

I. INTRODUCTION

Heat exchangers are an established technology in general for energy production, cooling, and heat transfer systems. They are also one of the foundations of thermal science. Many kinds of research and articles on heat exchangers appear frequently in a non-constructal design, while the researches or articles utilize the design with constructal theory (DCT) method, depends largely on the constructal theory which focusing mainly on geometric volume optimization, point-to-point and volume flow problems not just in engineering but also in other sciences. The present study will review researches and articles based on two major categories. The first category deals with researches and articles that utilize constructal theory which also reviews both single scale and multiscale researches and articles while the second category review researches and articles that observe non-constructal which will be reviewed in this article.

II. CONSTRUCTAL DESIGN FOR SINGLE SCALE

Constructal design is a method of finding the missing configuration of flow systems based on the evolution of this configuration towards the way which provides easier access of the currents that flow through it. In a heat exchanger with crossflow, the fins that are attached to the tubes increase the heat flow from the heated finned tubes to the cold crossflow. The missing configuration of the finned tubes can be found by the constructal design method. The parallel arrangement of the finned tubes in crossflow makes the design carry out with fixed pressure drop. The literature for constructal design method based on a single-scale system configuration which utilizes constructal theory rather than flow nature, and indorsed convection type shall be reviewed in this is section.

Heat exchanger utilizing tubes required by many industrial engineering which will have designed and sized according to space availability for this heat exchanges. The design of such heat exchangers was studied by R.S.Matos [1] based on the volume occupied by the array of tubes which shall be considered fixed during the design steps. The author mentioned that for the optimal spacing between tube bundles based on the assumption of fixed constraint volume, the maximum overall heat transfer is known as thermal conductance can be estimated. The author adopts two-dimensional circular and elliptic heat exchangers of tube type to evaluate heat transfer analysis. The author referred to previous literature stated that numerical simulation is more accurate through evaluation of a single flow channel and since there is no exchange in fluid and heat transfer between alongside channels, numerical results was carried out with a staggering equilateral triangle element to validate previous experimental results of circular tube heat exchangers. Results were carried out for Reynold number range of ($300 \leq Re_L \leq 800$) knowing that (L) was the swept length of the constraint volume. The author found that the gain of heat transfer raised to (13%) with elliptical configuration compared to that of circular with a (25%) reduction in pressure drop in comparison with previous studies.

R.S.Matos et al. [2] carried out a numerical and experimental study as a continuation of his previous research [1]. The study includes geometrical optimization for a bundle of finned and unfinned heat exchangers for a constant volume and predefined upstream flow where both circular and elliptical tubes were embraced with a staggered arrangement. Equilateral triangle configuration was selected for twelve circular and elliptic tubes used for experimental optimization procedure limited with maximum dimensional spacing to smaller semi-axis of the ellipse (b) of ($S/2b = 1.5$). The numerical solution utilizes only for a two-dimensional domain for unfinned tube arrangement while three-dimensional arrangement for finned tube arrangement knowing that both fluid and solid fin were included within a three-dimensional domain where the entire domain comprises both fluid and solid with imposing heat flux at the interface of solid and fluid having an assumption of zero velocity on the surface of the solid fin. The optimization procedure points out three geometrical degrees of freedom that can attain a maximum rate of heat exchange between fluid and tubes which were the spacing distance between tubes, the eccentricity of tubes, and the spacing between fins. In addition, the author reported that the variance in eccentricity leads to a comparison between elliptic tubes concerning flat plates and circular configuration tubes. The study was carried out for airflow range ($852 \leq Re_L \leq 8520$) were (L) was the swept length of the constraint volume and concluded that heat transfer was enhanced for elliptic optimal arrangement by 20% in comparison to that of circular optimal arrangement. Finally, the author reported that optimization geometrical parameter was found with the spacing distance between tubes and eccentricity. Regarding that optimum values were found with a spacing distance between tubes of ($S/2b = 0.5$) and eccentricity of ($e = 1$). Moreover, R.S.Matos [3] carried out another study based on a three-dimensional domain for both numerical and experimental solution techniques for a fixed constraint volume based on DCT requirement aiming to enhance heat transfer from a finned tubes bundle with a staggered configuration of a circular and elliptic tube geometry.

All geometrical configuration and optimal tube spacing distance were adopted similar to the author's previous work [1]. The experimental procedure included a variation in tube spacing distance in addition to other two degrees of freedom such as fin spacing distance and eccentricity. The study was carried out for laminar Reynold number range of ($852 \leq Re_L \leq 1065$) were (L) was the swept length of the fixed constraint volume while the numerical study focused on a three-dimensional optimization also for circular and elliptic tube bundles and validated with the experimental results. Optimum values for the three degrees of freedom reported being ($S/2b \cong 0.5$), ($e \cong 0.5$), and ($\phi \cong 0.06$) for the spacing distance between tubes, the eccentricity of tubes, and spacing distance between fins respectively. The author concluded that an enhancement of 19% was obtained with elliptic tubes which were accompanied by a reduction in heat exchanger material of 32% in comparison to that of circular tube configuration.

A numerical study concerning the optimization of geometrical parameters for parallel stack plates which exhibit as a source of heat generation was carried out by T. Bello-Ochende [4] to include mixed convection region that interposes between natural and forced convection region where the optimal plate spacing occurred. Large cases were adopted for optimization purposes that were conducted systematically for natural, mixed, and forced convection. The author referred that the numerical approach for natural and forced

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convection was used for validation purposes with relevant studies while the numerical approach for mixed convection was aimed to establish the required suitable correlation that describes optimal spacing distance in channels with mixed convection region. For natural convection Bejan number was assumed (Be = 1) with Rayleigh and Prandtl numbers varies in the range (105≤ Ra ≤ 107) and (10-3≤ Pr ≤ 102) respectively. For forced convection Rayleigh number kept on (Ra = 1) and Bejan number varies in the range (105≤ Ra ≤ 107) with different values of spacing distances to find the maximum heat transfer density for Prandtl number ranges of (0.001 ≤ Pr ≤ 1), and (1 ≤ Pr ≤ 100) respectively and also a certain correlation for optimal spacing distance and maximum heat transfer was found. For mixed convection author mentioned that this region was divided into four phases, the first phase started with a strong natural and strong forced convection study with natural Prandtl number equal (Pr = 1) and Bejan number with Rayleigh number ranges of (Ra = B e = 105), while for second phase study started with natural convection having (Ra = 105) and a forced convection limits of (105≤ Be ≤ 107). The third phase fixed forced convection on (Be = 105) while the natural convection in the range (105≤ Ra ≤ 107). The final phase (i.e., fourth one) studies the effect of the Prandtl number in the range (10-3≤ Pr ≤ 102). The author found a global correlation that covers the mixed convection phase of study for optimal spacing distance and maximum heat transfer and reports the important role of (Ra) and (Be) numbers within the outcome correlation.

Marc Joucaviel [5] conducted a study aimed to examine heat transfer density with the existence of constant pressure difference across a bundle of rotating cylinders with a cross-flow configuration. The behavior of the assemble was studied thermally with a numerical model for two types of cylinders rotational configuration, first when the cylinders rotate in the counter direction which means they rotate opposite to each other, and the second when the cylinders rotate in the same direction. Cylinder wall temperatures were assumed constant and distributed uniformly over cylinders surface with a higher value than that of inlet fluid temperature. The author utilized three values for Bejan numbers of (103, 104, and 105) for the optimization of non-dimensional distance that will lead to a maximum value of heat density. Note that the numerical model was validated with another reference works with a non-rotating cylinder configuration which means with a dimensional angular velocity equal to zero. It was concluded that counter cylinder rotation was found to be more efficient than the configuration of cylinder rotation in the same direction.

Elizaldo Domingues dos Santos et al. [6] carried out an investigation for the maximum rate of heat transfer over an arrangement of a square cylinder in crossflow based on constructal theory. The study aimed to observe the number of square cylinders which are considered as the optimum value to ensure the rejection of maximum heat transfer rate for a common constraint. The flow was assumed to be, incompressible, steady, two-dimensional, and laminar flow for solving governing equations of mass, momentum, and energy through using of finite volume method embedded within a commercial software package. Code validation has been done in comparison with previous literature for Reynolds and Prandtl numbers of (60 and 160) and (1, 10, and 20) respectively. The author reported that results agreed with the literature within (4%). Optimization procedure carried out for Reynolds and Prandtl numbers of (60, 100, and 160) and (0.1, 0.72, 1, and 10) respectively. It was concluded that optimal spacing between bluff bodies (square cylinder) decreases with the increasing of Reynolds and Prandtl numbers.

L.G. Page et al. [7] investigated thermal comportment for an in a row cylinder rotating in a counter mode configuration and cooled by convection naturally to optimize the rate of heat transfer density to its maximum value. Flow and temperature fields were solved numerically to examine governing equations through using finite volume code and simple algorithm method. Special attentiveness was achieved and tested for a range of Rayleigh numbers (101 ≤ Ra ≤ 104). It was concluded the increase of Rayleigh number leading to a decrease in the optimum spacing distance that results in a maximum heat transfer density.

R. L. S. Mainardeset al. [8] conducted an experimental investigation endeavored at minimizing the desired pumping power to provide airflow across a finned elliptic tube bundle. The study tested a tube bundle of four rows that possess a plate-fin and are arranged in a staggered configuration. Based on constructal theory, heat exchanger envelope represented by the length, height, and width were considered as the constraint volume which consisted of twelve tubes placed in the four-row arrangement and equilateral triangle array. Dimensionless spacing distance between tubes was optimized through varying its value in the range of (0.1, 0.25, 0.5, and 1.5) in addition to tubes eccentricity which varies in the range of (0.4, 0.5, 0.6, and 1) and the dimensionless fin density which varies in the range of (0.006, 0.0094, and 0.26) so

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as the minimal of both fin spacing, tube eccentricity, and dimensionless fin density can be observed, which accordingly led to minimizing required pumping power for a tube array of an aspect ratio equal to (8.52). Flow conditions reported a turbulent flow for a range of Reynolds numbers of (2650, 5300, 7950, and 10600). It was concluded that utilizing constructal design with elliptic configuration provides a reduction in pumping power of (10%) with optimum spacing distance, eccentricity, dimensionless fin density of (0.5, 0.5, and 0.006) respectively.

Gongnan Xie [9] investigate the topology of pin fins that are used in systems of thermal energy storage via optimizing fins shape, diameter, and length through employing entropy generation minimization and constructal law as a mutually coupled technique. Configuration for the structure of the pin fin heat exchanger adopted an optimization procedure based on the three-dimensional degree of freedom which represents the diameter, horizontal spacing, and vertical spacing with two-dimensional, incompressible, laminar, and steady-state flow conditions. Optimization started first with the EGM technique to acquire pin fin optimal length, then the constructal theory was used to acquire pin fin optimal shape and diameter. Mass, momentum, and energy conservation equation were simulated numerically with CFD analysis, and the results compared with previous literature and found in a good agreement within a percentage of (1.4%). It was concluded that the adopted optimization technique for the utilized pin fin exchanger increases thermal energy stored by (10.2%).

G. Lorenzini et al. [10] conduct a numerical study for the lower surface of an intruded rectangular fin having a lid-driven square cavity with a steady, laminar, and mixed convection flow. The study aimed to enhance the rate of heat transfer with aid of internal mixed convection flow and the surface of the rectangular fin. Fluent software was used to solve the governing equation for a different arrangement of fin geometry through adopting of velocity pressure coupling solver with SIMPLEC algorithm. System configuration proposed a ratio of the fin to cavity area equal to (0.05) with one degree of freedom represented by fin aspect ratio (Fin height / Fin width) which altered in the range of (0.1 - 10). The range of Rayleigh number and Reynolds numbers equal (103, 104, 105, and 106) and (10, 102, 3 × 102, 5 × 102, 7 × 102, and 103) respectively. It was deduced that the fin aspect ratio largely affects the fin Nusselt number with a considerable change for different values of Rayleigh numbers.

Lingen Chen [11] examine the heat transfer density of a pin fin heat sink with a cylindrical shape and simulated with finite volume method for a system having a constraint volume and fin material with a constant pressure drop. Analysis of the study model postulates a three-dimensional steady turbulent incompressible flow with no penetration and no-slip on the wall surface. Uniform air velocity with constant thermal and physical properties for solid and air was adopted in addition to neglect viscous dissipation and the rate of heat transfer by radiation for a given air temperature and fin base temperature. A (κ-ε) turbulence model was used and solved according to boundary conditions mentioned before and SIMPLEC algorithm solution that made use of ANSYS Fluent software. Domain configuration consists of a length (7.5), a width of (1.5), heat sink top distance of (0.4), a dimensionless velocity of (1), and a Prandtl number of (0.712). It was concluded that for a given heat sink shape, fin optimum diameter based on constructal design maximizes heat density with the decreasing of dimensional pressure difference and the pressure drop was found increasing with increases in fluid velocity.

G.M. Barros et. al [12] adopted the constructal design method to investigate a triangular configuration of circular cylinders geometrically which exposed two-dimensional, laminar, incompressible transient flow with the assistance of mixed convection. The proposed problem focused on two major objectives, the first was to enhance the value of Nusselt numbers to their maximum values, the second was to reduce the drag coefficient. Since the study dealt with mixed convection flow, Grashof numbers to observe the role of buoyancy forces lead to various Richardson numbers in the range of (0.1, 0.5, 1, 5, and 10) with a fixed Reynolds number of (100) and Prandtl number of (0.71). The design employed two constraints represented by (cylinders cross-sectional area and the occupied area of the proposed arrangement) and two degrees of freedom represented by (longitudinal distance and transversal distance between cylinders). The enhancement method utilized the numerical solution of governing equations with aid of the finite volume method embedded within Fluent software. It was concluded that the maximum Nusselt number observed with a transversal distance of (5) and (2.5) for Richardson numbers of (0.1 and 10) respectively.

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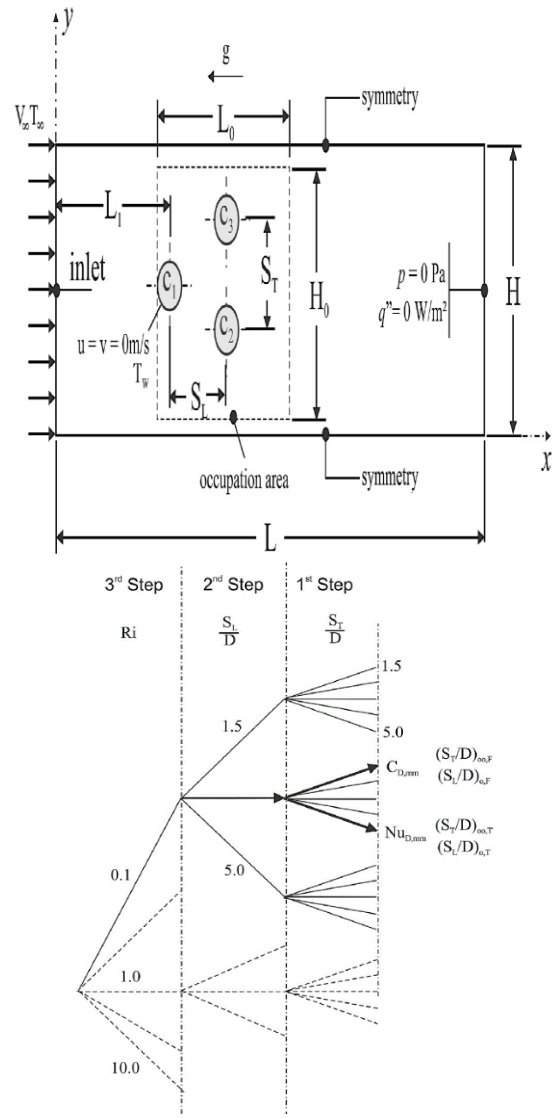

Figure (1): Computational domain for cylinders and optimization procedure.

Ahmed Waheed Mustafa et al. [13] mentioned that the constructal law is used to design a set of vertical flat tubes cooled by natural convection and put in a finite size space. The size of the space where the tubes are positioned is considered as the constraint in this design. The constructal law is used to estimate the optimum spacing between the tubes. (Ra=103, 104, and 105) are the Rayleigh numbers utilized by author. A values of (0.2) to (1) is observed for the dimensionless tube diameter (tube diameter/tube height. The tubes are all heated to the same temperature on the wall surface and a Prandtl number (Pr = 0.72) of the air used to cool the tubes. The finite volume approach is used to solve the equations of mass, momentum, and energy for steady, two-dimensional, and incompressible flow. Author concluded that for all tube diameters, the optimum spacing distance at a given Rayleigh number remains constant. The results also showed that for the same Rayleigh number and space size, the number of small diameter tubes must be greater than the number of big diameter tubes to make heat flow from the tubes to the coolant better.

A.L. Razera [14] conducted an optimization process based on geometrical variation with the aid of constructal design. Mixed convection was utilized to cool a semi-elliptical fin that can alter its dimensions and is located within a lid-driven cavity of square shape. Governing equations of continuity, momentum, and energy were solved numerically with aid of the finite volume method for bouncy forces. Heat rejected by fins to its surrounding was enhanced for various values of Reynolds and Rayleigh numbers of (10, 102, and 103) and (103, 104, 105, and 106) respectively with a constant Prandtl number of (0.71). Design with constructal theory allocated two constraints represented by the area of cavity and fin in addition to one degree of freedom represented by the aspect ratio of fin height to its length. It was deduced that both Reynolds and Rayleigh numbers were highly affected the optimum shape that can be obtained. This optimum shape was found to have a thermal performance gain of (40%) in comparison with other proposed geometries.

Ahmed Waheed [15] investigate the optimal distance that can be observed with a diamond shape tube array employed within a constraint volume through using constructal theory. Numerical study of a free convection cooling for isothermal tubes aligned in row shape with comprising of finite volume to solve governing equations with SIMPLEC algorithm and collocated mesh grid. The study was conducted for a range of Rayleigh number (103 ≤ Ra ≤ 105) accompanied with diamond axis ratio range of (0 ≤ e ≤ 0.5) and air fluid of Prandtl number (0.71). Exploring grid independence test showing that heat density converge via constant values with (50×50) control volume and a percentage of difference equal to (1.5%). Upstream and downstream extension distances were selected to be (0.5) and (2) respectively. Numerical results was validated with a previous literatures and found acceptable with a percentage difference of (0.77 %). Author concluded that with the increasing of Rayleigh number, the optimum distance decreases in accordance. Moreover, maximum heat density appeared with a value of axis ratio equal to zero as it behave as a flat plate while the minimum value appeared with a value of axis ratio equal to (0.5) as it behave as rhombic tube.

Ahmed Waheed [16] conducted a study on an array of rhombic tubes to evaluate the rate of heat density that was cooled with forced convection utilizing constructal design method. Rhombic tubes were placed in constraint volume with a row placed in a parallel arrangement. The author referred that the tubes' horizontal axis was kept constant while the tubes' spacing distance and vertical axis were altered to enhance heat transfer from tubes to fluid flow. Constant pressure drop was observed based on the concept of constructal theory with an isothermal condition on the tube surface for steady, incompressible, two-dimensional laminar forced convection flow. Mass, momentum, and energy equations were solved numerically with the aid of the finite volume method and a SIMPLEC algorithm. Dimensionless pressure drop number (Bejan number) implemented in the work in the range of (103 to 105) and a Prandtl number of (0.71) with the vertical axis value altered in the range of ($0.2 \leq B \leq 2$). The author deduced that with the decreasing of optimum spacing, heat density increases to a maximum value as the Bejan number raised for all tubes vertical axes. Moreover, the Bejan number was also found to possess a considerable effect on vortex formation and separation on all sides of tubes and with the value of optimum spacing distance.

Ahmed W. Mustafa [17] perform a study for a flat-tube array aligned vertically in crossflow to estimate the maximum heat density that can be obtained with constant pressure drop with aid of constructal theory. The volume occupied by the flat tubes was adopted as the system design constraint. The study considered two degrees of freedom which were tube spacing distance and flat tube length with isothermal temperature conditions exist on the tube surface. Investigation observed Bejan number as the driving number which varied in the range of ($103 \leq Be \leq 105$) with tube flat length of ($0 \leq F \leq 0.8$) and a Prandtl number of (0.72). It was reported that the decreasing of tube flatness with a constant Bejan number leads to the decreasing of the optimum heat density. Also, heat density increases with increasing of Bejan number at the constant value of tube flatness. Finally, at constant Bejan number, it was concluded that spacing distance was found constant as it was independent of tube flatness.

A.L. Razera et. al,[18] carried out an investigation work with the aid of constructal design to inspect the enhancement of heat transfer density rejected from elliptic cylindrical bodies subjected to forced convection externally. Optimization also included the ratio of horizontal to vertical cross-section area. A two-dimensional model was adopted with laminar, incompressible, steady flow. The pressure difference is considered as the driving force represented by the Bejan number with thermal and fluid properties represented by a Prandtl number of (0.72). The finite volume method was used to solve all conservation equations symbolized by mass, momentum, and energy equations. The study objective was to acquire the optimum spatial arrangement that can lead to maximizing the amount of heat transfer density rejected or removed. Proposed values for Bejan numbers were considered (10, and 5 ×10). It was concluded that optimization with a constructal design enhanced heat density in the range of (50% to 97%) as compared with cases that utilize fewer degrees of freedom.

III. CONSTRUCTAL DESIGN FOR MULTI-SCALE CONFIGURATION

A. Bejan [19] managed to exhibit a new conception for utilizing a flow system of finite size based on a multi-scale framework that can achieve a maximum rate of heat density within a fixed known volume or what was named by constraint. The proposed target was achieved by employing of forced convection flow of laminar nature accompanied with isothermal parallel blades embedded within the constraint volume. The constructal theory is used to optimize spacing distances between blades to gain over maximum available heat density. Optimization procedure with a multi-scale concept focused on inserting small blades (i.e., shorter) in the unused fluid volume region that surrounds the boundary layer formed on the longer blades and with the existence of a constraint pressure across the fixed volume. The author referred that the technique of inserting small blades aimed to add more regions of heat transfer at the zone of unused flow where the boundary layers are thinner and more small blades can be inserted in these zone with a representative number of multi-scale configuration (m). It was deduced that the unused flow region can be filled with more small blades to produce a more efficient system.

T. Bello-Ochende [20] conduct a numerical study for a geometry consist of parallel plates that can generate heat subjected to laminar forced convection with optimal spacing enhanced through inserted smaller plates at the entrance region to the constraint volume and possess the proper length to fit in the region of unused flow. Numerical simulation performs a range of Bejan number of ($105 \leq Be \leq 108$) with Prandtl number of (1). Optimization procedure includes different degrees of freedom with two optimization stages, first one with flow between parallel plates in single scale criteria with a fixed constraint volume and seeking for optimum spacing while second criteria with an inserted plate between the single scale plates to form a multi-scale configuration where optimization process includes the varying of inserted plate length and the spacing formed with large scale plates (single scale). The author concluded that performance enhanced as system complexity increases and as the number of inserted plates increases to three. Also, the author mentioned that the optimal length of plates increases with increasing of the Bejan number.

T. Bello-Ochende [21] managed to implement the concept of constructal theory on a row of circular cylinders having a multi-scale configuration in crossflow with a degree of freedom of up to four degrees. The structure of the flow was subjected to laminar forced convection for a given Bejan number. Multi-scale cylinders of various sizes distributed throughout constraint volume non-uniformly. Cylinders of smaller sizes were inserted at the entrance to cylinders of the vertical row where the unused flow is available as that area utilized three diameter sizes of small cylinder and degrees of freedom up to four counts. The proposed study utilizes a numerical solution that adopted the finite element method. The study first starts with a large scale (single scale) of one diameter size where the constraint is represented by the combination of the total bundle height, diameter, and cylinder width for a given pressure difference. The second round of optimization procedure starts with a more complex configuration toward implementing of multi-scale concept. Smaller diameter cylinders were inserted at the middle distance located between large-scale cylinders which are considered as the constraint for the proposed problem. Two degrees of freedom were observed which can be exemplified by the dimensionless diameter of the small cylinder and the dimensionless spacing between large-scale cylinders. The third round added another small cylinder diameters which are smaller than the one inserted in the second round leading to four degrees of freedom represented. All four rounds utilize a range of Bejan number of (103 ≤ Be ≤ 106) and a Prandtl number of (0.72) and aimed to obtain the maximum heat density out of the constraint volume. It was concluded that the flow structure gets less permeable as the number of plates grows, and the flow rate falls. At the same time, the solid structure's overall heat transfer rate density rises. Also, the increase in heat transfer density was discovered to follow the nonuniform distribution of length scales through the accessible space.

Alexandre K. da Silva [22] conducted a new design conception to produce a configuration of multi-scale criteria subjected to natural convection aiming to maximize heat transfer density for a given constraint that was equipped with equidistant blades that were heated and placed in a vertical position. Spacing between heated blades was varied seeking maximum heat density removal. The author proposed implementing constructal theory to enhance heat transfer density by inserting more plates with shorter lengths in the regions of unheated fluid to maximize the thermal conductance inside the constraint volume. The numerical study utilizes Rayleigh numbers in the range of (107 ≤ Ra ≤ 108). The author deduced that once the complexity regarding flow structure optimized then the rate of heat transfer density was found to be maximized.

T. Bello-Ochende et. al [23] investigated how to raise and maximize the convective heat transfer density progressively with the existence of natural convection subjected to cylindrical assemblies. The constructal design was used for that purpose by inserting small cylinders of diverse diameters at the constraint entrance plane and more precisely in the region of unheated fluid (i.e., unused flow). The optimization procedure involves one and two degrees of freedom through using one and two-cylinder sizes for multi-scale optimization criteria. Finite element code was used for numerical solution. A numerical study performs a Rayleigh number in the range of (103 ≤ Ra ≤ 105). The first stage of optimization works on the varying spacing between large scale cylinders with one degree of freedom to find optimum spacing that leads to maximizing the heat transfer density, with the second stage author intended to increase the complexity of configuration by adding a small cylinder at the entrance of constraint volume and between the large scale cylinders to utilizes as much as possible from unused coolant fluid. It was deduced that the spacing between large scale cylinder increases with the increase of complexity by inserting the small cylinders in the unheated region and when the flow grows faster, then spacing became smaller while the diameters of cylinders reveal a minor change.

T. Bello-Ochende et. al [24] conducted a three-dimensional study on a parallel plate with the merit of generating heat. These plates were arranged in a stack configuration leading to the formation of a fluid channel that was driven by the differential pressure across the plates. Numerical simulation was conducted numerous trials with different values for spacing between plates to found optimal spacing leading to maximum heat transfer density and that was considered as the first round of the optimization process. The second round of optimization started with inserting shorter plates between the original large-scale plates to occupy the unheated fluid and increase the overall thermal conductance. A third round of optimization process aimed at utilizing the available coolant outcome from the second round with on firm target toward maximizing the rate of heat transfer density for a given constraint volume. Governing equations were solved for a three-dimensional, steady-state, incompressible, laminar flow that had a constant Newtonian fluid property. Plate width was taken to be unity and thickness in the range of (0.01 ≤ t ≤ 0.05) with a driving pressure difference represented by the Bejan number in the range of (105 ≤ Be ≤ 107). The author deduced that results were in fair agreement with foretelling analytical outcomes.

T. Bello-Ochende [25] proceed in utilizing constructal theory to observe the optimal configuration of pin fins aligned in two rows to maximize heat transfer density. The upstream flow was considered uniform and with constant temperature, while the surface temperature of fins was considered also constant (i.e., isothermal) with forced convection heat transfer of laminar type. Pin fin materials were presumed fixed during the optimization procedure. Governing equations were solved with a code of computational fluid dynamics used the finite volume method while the pressure velocity coupling implemented SIMPLIC algorithm. It was concluded that the optimal configuration for a pin fin obtained with Reynolds number equal to (50) with a spacing varied between (0.05) and (0.2) for short plates while a ratio of pin fins diameters varied in the range of (1) and (1.2), and a ratio of the large fin to small fin height range from (0.9) and (1.2).

Y. Kim [26] conducted a numerical investigation for a configuration of the heat exchanger of cross-flow having two vertical tubes attached into two horizontal plena. Natural convection drives the cold-side fluid in vertical round tubes joined by two plenums. By forced convection, the hot-side fluid flows perpendicular to the vertical tubes and heats them. On both sides, the flow is laminar. From a two-tube design to three and four-tube configurations, the hunt for the flow configuration continues. This criterion of heating phenomenal results in a difference of the fluid density allowing for bouncy to lead circulations of upward flow inside the leading tube and downward flow through trailing tube in addition to the plenums horizontal flow which accompanied circulation loops in vertical tubes. Rejected heat by convection was removed with the aid of cold fluid moving slowly to form a stream that flows through the upper plenum from the left to the right. In general, flow conditions within heat exchangers included natural convection for internal flow and forced convection for external flow. Grashof number of order equal to (104) and Reynolds of the cold streamflow in the upper plenum was taken to be of the order (500). Heat exchanger configuration was subdivided into four cases, two vertical tubes of equal diameters, a case of three vertical tubes of equal diameters, three vertical tubes on of them was small with variable position, and four vertical tubes having two equal tube diameters while the other two tubes different in diameters and their position (i.e., tube spacing). The author mentioned that the outcomes of the study revealed that for a two-tube heat exchanger, the best performance appeared with two vertical tubes having almost the same diameters while for the three vertical tubes it was better to include the spacing distance between tubes for better performance while the case of four tubes leads to the privilege of the case that possesses two vertical tubes of almost same diameters.

T. Bello-Ochende et. al [27] carried out a numerical study aimed to enhance the rate of heat transfer density to its maximum value which was rejected from a cross-flow heat exchanger subjected to a fixed pressure drop applied for a constraint volume involve rotating multi-scale tubes applied to laminar forced convection of a steady-state nature. The study referred to the existence of two configurations, the first one involves two large scale cylinders in addition to another two small cylinders inserted in the middle spacing distance of the large one where all cylinders aligned on the same centerline while the second configuration include the same large scale and small scale cylinders with same spacing criteria but they were not aligned on the same centerline rather than having the leading edge of the four cylinders aligned on the same direction. The author adopts counter spinning and co-spinning with one target for stationary and rotational cases which were to maximize the rate of heat transfer density through optimization of cylinder diameters and the spacing between them. In addition, the role of the location of rotational centers, Bejan number (pressure drop dimensionless number), spacing, and the optimum cylinder diameters were investigated. The numerical solution adopted a range of smaller cylinder diameters between (0.1) and (1) while the large scale diameters stayed constant with a range of dimensionless pressure numbers equal (10 ≤ Be ≤ 104). The author deduced that enhancements were found influential with the case of rotating cylinders that aligned on the same axis of rotation rather than

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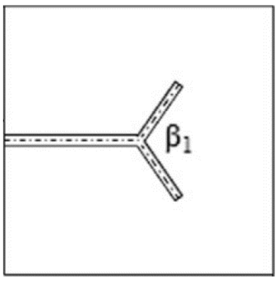
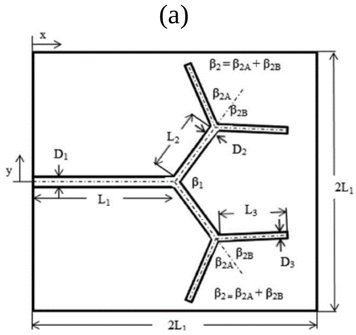
the case of cylinders aligned on the plane of the leading edge.

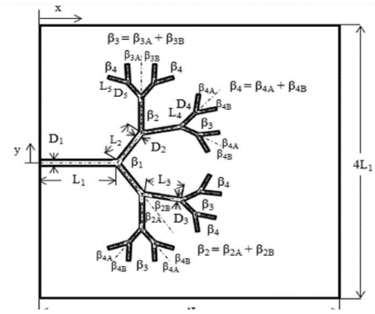
H. Kobayashi [28] investigate the optimum structure that can expedite heat transfer between the solid heated body and the proposed multi scale tree shape of the coolant stream. Tree shape has a vascular configuration with the privilege of affording greater heat transfer density with evolving of tree structure. Solid heated body was analyzed for two branch sizes, first for a cube of small size with tree shape that increases in size to the second scale of divergence while the second for a cube of larger size with tree shape that increases in size to the fourth scale of divergence. The implanted tree shape configuration was made of pipes with isothermal surface temperature which was also embedded within the solid cube at different temperature. Cube volume was considered as the constraint for the implementation of constructal design and set as a function of the length of one scale degree of freedom (trunk) which was considered as constant. Initially, the tree shape tubes start with a temperatures lower than the temperature of the cube and with time the zone of cooled region start to grow with time. Evolution of multi scale tree shape was generated in the form of (Y) shape divergence and all other divergence followed were sized as a function of each other's. Author deduced that asymmetry in the morphing and evolution of the tree shape configuration leaded to better optimum design.



(a)



(b)



(c)

Figure (2): Morphing of tree shape evolution (a) First divergence. (b) Second divergence. (c) Fourth divergence.

H. Matsushima et. al [29] carried out a review on the techniques of conducting wall design evolution targeting the enhancement of thermal surface contact with the cooling fluid in order to maximize the rate of heat transfer. Morphing freely was the major criteria adopted by author through reviewing of former literatures that utilizes constructal theory. Wall conducting volume shall be reviewed for variation of wall thickness, spread of fins inserted on wall with no constraint to alter freely. Systematic evolution endeavor three criteria, first morphing of base thickness, equidistance spread of identical fins and the second both fin thickness and spacing distance was allowed to morph freely with an identical fin shapes while the third criteria permit for a free morph of both fin size and fin base thickness with fixed spacing distances. Author reported the optimal cases that lead to maximize the rate of heat transfer for all criteria's adopted which lead to connect optimum heat transfer with the spacing distance between fins in constructal design.

L.G. Page et. al [30] investigated a configuration of multi-scale counter-rotating cylinders arranged in a staggered bundle and exposed to cooling with natural convection to maximize heat transfer density. A numerical model was utilized to deal with governing equations regarding temperature and flow in addition to the establishment of a mathematical algorithm for the optimization procedure. Two parameters adopted in the study are the Rayleigh number and the cylindrical rotational speed that later will be optimized. Large-scale cylinders were positioned on their centerline while the smaller cylinder of multi-scale order was inserted in the unused fluid region at the entrance of the constraint volume. Both large and small cylinders were kept at an equal constant surface temperature which was higher than that of the upstream cooling fluid. The study aimed to found the optimum spacing between large cylinders and small cylinders that led to maximum heat transfer density removal. Numerical study performs a Rayleigh number in the range of (101 ≤ Ra ≤ 104). The author conducted that the maximum heat transfer density was not influenced by the rotation of cylinders in comparison with a stationary case at large values of Rayleigh numbers and the optimum spacing was found to be decreased with the increasing of cylinders rotational speed. Finally, the author deduced that for multi-scale configuration and no cylinder rotation, the maximum rate of heat transfer density was found larger than the single-scale configuration with rotating cylinders.

Ahmed Waheed Mustafa [31] utilized constructal design methodology to maximize the rate of heat transfer density from pin-fins of diamond multi-scale shape subjected to mixed convection. The author observed new criteria for perceiving the onset of mixed convection instead of Richardson number which was the ratio of Rayleigh to Bejan numbers to include pressure drop implementation according to the assumption of the constructal theory. A multi-scale diamond pin fins shape were inserted in a constraint volume and aligned in the vertical arrangement of a crossflow. Small size pin fins were implanted in the entry zone for the unheated fluid at the middle spacing distance between large scale pin fins so as all pin fins aligned on the line of their leading edge. Numerical solution was used for the solution of governing equations through using of finite volume method with an assumption of incompressible, laminar, two-dimensional steady flow. In addition, the buoyancy pressure equation was solved with the isothermal pin fin surface temperature assumption. Boundary conditions for computational domain presumed a constant Rayleigh number of (105) while Bejan number varied in the range of (104 ≤ Be ≤ 106) with a range of Rayleigh to Bejan numbers of (0.1 ≤ Ra/Be ≤ 10) and a fluid Prandtl number of (0.71). The angle of the leading edge was altered for optimization in the range of (30o to 60o). The author deduced that utilizing multi-scale arrangement enhanced heat transfer density twice that of the single-scale arrangement. Also, it was concluded that heat transfer density enhanced for a leading-edge angle of (30o) for all values of (Ra/Be).

Ahmed Waheed et. al [32] investigated the best arrangement of two scale elliptic tubes in crossflow, which was observed based on constructal design. Larger tubes are fitted within a fixed length and height region. Smaller tubes are put in the entry region of the same domain between the bigger tubes at the mid leading edge distance of the bigger tubes. To determine the best configuration, the spacing between the bigger tubes, the semi-minor axis of the bigger tubes, the major axis of the smaller tubes, and the semi-minor axis of the smaller tubes are all freely adjusted inside the domain. Author mentioned that there are two optimal configurations: one without the smaller tubes and the other with the presence of the smaller tubes. Both the larger and the smaller tubes are heated at a constant surface temperature. The flow is induced by a fixed pressure difference. The equations for steady, laminar, two dimensional, and incompressible flow are solved by finite volume method and the range of Bejan number (dimensionless pressure drop) is selected to be in the range (103 ≤ Be ≤ 105) in the absence of the smaller tubes and Be =105 in the presence of the smaller tubes. The semi minor axis range of the dimensionless bigger tubes is varied between (0.1 and 0.4). The row of tubes with a Prandtl number of (0.7) is cooled with air. Author concluded that when the smaller tubes are inserted between the bigger tubes, the heat transfer rate is increased for different semi minor axes of the larger tubes.

IV. NON-CONSTRUCTAL DESIGN:

S.C. Haldar et. al [33] carried out a study for a cylinder lying in a horizontal position with longitudinal fins attached to the external surface of the cylinder and possess a finite fin thickness. A conjugate solution with a numerical base for laminar free convection was conducted. The physical domain consisted of a longitudinal solid fin attached to the cylinder. The fin geometry was considered rectangular since the tip and root of the fin were assumed straight flat. The author reported that the assumptions adopted by previous research such as isothermal fin base and neglecting the effect of fin thickness were not considered in the proposed study which leads results to be dependent on fin thickness and the thermal conductivity of fin. The study focused on investigating the influence of various parameters that might affect the rate of heat rejected from longitudinal fin for a range of equally spaced fin number (0≤ N ≤18), range of fin dimensionless length (0.1 ≤ L ≤ 0.6), range of fin dimensionless thickness (0.01 ≤ t ≤0.05), and a Grashof number equal to 105 with a Prandtl number of (0.7) value for air surrounding the finned cylinder. The author reported that maximum heat transfer can be obtained with thin fins of (0.01 m), optimum fin number of (6), and optimum fin length of (0.2 m).

E. Ibrahim [34] achieved an experimental study to observe heat transfer from an elliptic tube as well as entropy generation accompanied with longitudinal fins. Experimental investigation for geometrical configuration utilized for study, observe the existence of longitudinal fin at the tube front leading edge, tube back edge, front and back. Moreover, different rates of heat generated from the elliptic tubes from the internal embedded electrical heater which was inspected inside a square section wind tunnel. The proposed configuration was tested with airflow having Reynolds number in the range of (4.75 × 103) to (3.96 × 104) and elliptic tubes surface temperatures were monitored and registered for the different adopted cases. The elliptic tube was made of copper material with an axis ratio of (2:1) with a major axis of (30 mm) and a thickness of (1.5 mm). Fin width equal to (20 mm) and welded to tube surface while heat source obtained with employing of resistance electrical heater of flat shape inserted inside ceramic rod then heaters placed at the center of axis and the gaps between heaters and tubes surface was filled with sand to acquire regular allocation of input power to the inner surfaces of elliptic tubes. The author mentioned that irreversibility ratio, friction factor, and the coefficient of heat transfer affected by the position of fin concerning elliptic tube surface.

S. C. HALDAR [35] conducted a numerical solution for a cylinder aligned in the horizontal direction with longitudinal fins attached externally and subjected to laminar free convection. Both cylinder and fins surfaces' temperatures were assumed uniform. The fluid pulled over a wide domain angularly through a vertical narrow strip that subjected to thickness reduction once the buoyancy increase. The fin thickness was negligible in comparison to tube diameters with the assumption that both fin faces reject heat and the range of Grashof number was (102 ≤ Gr ≤ 106) with fin number in the range of (0) to (18). The author concluded that as the Grashof number increases which is accompanied by an increase in both fins number and length then heat transfer rate also increases.

J. M. WU et. al [36] carried out a numerical study to simulate natural laminar convection heat transfer over a finned horizontal tube. Governing equations were discretized through using of finite volume method and the coupling of velocity and pressure which was performed with a SIMPLE algorithm. Flow condition was presumed Rayleigh number of (7120). The author deduced that for a range of fins numbers (4 to 10), there will be a decrease in fin efficiency with the increasing of fins number. Numerical results for eight fins configuration reveal that there should be no fins aligned in the vertical plane.

Chidanand K. Mangrulkar et al. [37] performed an experimental and three-dimensional numerical simulation of a cross-flow tube bank in staggered configurations with and without the splitter plate connection. The tests were carried out in an open channel wind tunnel with flow control, pressure differential, and velocity and temperature measurement equipment. The test rig is made up of a rectangular air duct with a length of (600 mm) and a cross-section of (150 mm). Thirteen test tubes and four half-dummy tubes are placed in five rows with staggered orientation in the test portion. In the case of a circular cylinder with splitter plate arrangements, a similar test section is supplied. With Reynolds numbers ranging from 5500 to 14,500, and a splitter plate length to tube diameter ratio of (1), the study is carried out. The pitch ratios of the longitudinal and transverse tubes were kept constant at (1.75) and (2.0), respectively.

The author discovered that adding a splitter plate to a fluid flow raises the Nusselt number, which boosts heat transfer while lowering pressure drop within the tube bank when compared to a bare cylinder. Furthermore, the splitter plate attachment's total thermal performance is far greater than that of the bare cylinder, indicating better heat transmission with a lower friction factor. At a Reynolds number of 5500, the usage of the splitter plate over the bare cylinder resulted in a maximum (60-80 percent) gain in total thermal performance.

Chidanand K. Mangrulkar et. al [38] reported a review study for recent evolution in fluid flow and heat transfer characteristics in cross-flow heat exchangers The cross-flow heat exchanger is a special type of heat exchanger involving the exchange of heat in the case of two fluids flowing in an orthogonal direction. The heat exchanger often comprises several cylinders carrying hot fluid, along with the colder fluid flowing across/over the tubes or cylinders. The system also comprises additional fins across the tube cross-section. Thereby, the objective of the author's work is to review the recent developments and their potential candidature for improving the thermal performance of heat exchangers. One of the reviewed papers was illustrated by Sparrow et al [39] who states that providing the single fin at the front, back, and the combination of front and back in case of the circular tubes. The experimental work consists of an unfinned tube bank where the tubes were positioned on equilateral triangular centers. Each row consisted of six tubes with a total of 15 rows in the tube bank. The Reynolds number for the cross fluid is up to 8000. The author refers that the heat transfer enhancements obtained by finning were compared with those attainable by the use of increased diameter unfinned tubes. For the same pressure drop, finning yields significantly greater enhancements. It was concluded also that at fixed mass flow, greater enhancements are attained with increased diameter unfinned tubes, but at a high penalty in pressure drop and pumping power. For the same pumping power, the two methods of adding surface area yield comparable enhancements, with that for finning being slightly greater.

V. CONCLUSION

Based on the former surveyed evolutionary constructal designs, the increase of the system complexity provides easier access of the heat flow and since system complexity increases with increasing of system scales. It can be noticed from constructal and non-constructal under review literature that, it is clear that first, there is no constructal design for longitudinally finned tubes based on constructal design of longitudinally finned tubes in cross-flow which is driven by fixed pressure drop with different fin locations. Also it was noticed that there is no constructal design articles for single scale longitudinally finned tubes with two degrees of freedom. The first degree of freedom is the tube-to-tube spacing, and the second is the fin length.

Moreover, no articles was found that utilizes system complexity increases based on a constructal design concept with observing of an evolutionary design having three degrees of freedom for a row of tubes with planted fins in crossflow and implemented theoretically and experimentally. Such type of evolution proposed that the first evolution consists of a row of isothermal circular tubes placed in a two-dimensional domain of fixed length and height. In the second evolution, isothermal fins are planted in three different locations of tube surface (front, back, and the front-back) in the same domain of the first evolution. The length of the fin is selected from the second evolution while in the last stage of evolution, isothermal diameters are inserted between the finned tubes and varied seeking for optimal diameter leading to maximum heat density rejected from the entire domain.

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