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Dynamic Modeling and Performance Analysis of an Autonomous Quadrotor Using Linear and Nonlinear Control Techniques

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ABSTRACT: This paper is on modeling, design and control of a multirotor Unmanned aerial Vehicle (UAV). Specifically, the quadrotor UAV with emphasis on the autonomous operation of the vehicle in linear and nonlinear operating points. The mathematical model of the quadrotor is presented for simulation and control using Newton-Euler formulation. Based on the mathematical model, linear and nonlinear control algorithms are designed, implemented and used to simulate the quadrotor.

Linear and Nonlinear control algorithms were developed to control the altitude, attitude and heading of the quadrotor in space.

The parameters of the controllers were tuned using MATLAB control toolbox to improve the performance of the system dynamic responses. Computer based experiments and simulations were carried out to evaluate the performance of the quadrotor for autonomous operation under the control of the three control techniques developed. A detailed comparison of the system's dynamic responses, system's stability and the effect of disturbances is presented.

Keywords: Modeling, Model, Quadrotor, UAV, Controller, High Gain Robust.

I. INTRODUCTION

Flying vehicles are known to be fascinating to man. This has prompted all kinds of research and development. The challenge in the design and control of Unmanned Aerial Vehicles (UAV) to operate in a clustered environment and the lack of existing solutions has increased the drive for finding solutions. Also, the application of UAV research and application in the Military and Civilian markets and spaces has brought much needed interest and funding to the development and research of UAV related projects.

The advancement in small sensors systems, in microcontrollers and microcomputer technologies and the development in aerodynamics has impacted positivity in advancement of small Unmanned Aerial Vehicles related researches. Due to theirsmall sizes, low cost and maneuverability, UAVs have become the bedrock for larger class of application.

The Vertical Takeoff and Landing (VTOL) capability of a quadrotor is one of its most essential features and advantages over other Unmanned Aerial Vehicles (UAVs). This feature is what makes it ideal for use in small spaces[19]. Their hovering ability makes it ideal for used in surveillance and monitoring applications. The presence of four rotors in a Quadrotors provide higher payload over conventional helicopters, and also have better maneuverability, [19]. Quadrotors are used extensively in areas ranging from civil, commercial and military applications including search and rescue, earth science, surveillance and monitoring, law enforcement and border control [1].

However, challenges such as noise associated with small sensors systems, susceptibility to environmental disturbances due to its miniature structure and size are some of the inherent difficulties with operation of UAVs.

Onboard stabilization and trajectory capabilities is the core of any autonomous operation of aerial vehicles. Significant effort has to be put in place in other to achieved a stable flight. These problems are compounded when operating smaller size vehicles which are have a wingspan of less than one meter.

The nonlinearity, multivariable and under-actuated system of a Quadrotor has prompted a number of researches into finding efficient and effective methods for controlling and stabilization of quadrotor's six Degrees of Freedom (DOF).

Due to the increased in the complex nature of application for systems, the control algorithms must also evolve in order to provide better and efficient performance and versatility. In the past, linear control techniques were seen to be adequate and were employed for the purpose of ease of computation and to achieved linear flight operating point such as hover flight. However, with the advent of better modelling techniques, micro computational abilities and the use of small sensor system, it is possible to deploy nonlinear control techniques to be run on real time in order to attain a specific objective. With nonlinear control techniques, improved performance of the system and robustness is guaranteed.

A. Objectives

The objectives of this paper is to compare the performance of the linear and nonlinear control algorithms on autonomous operation of a quadrotor's altitude, attitude and heading and its ability to stabilize the vehicle, track flight trajectories and resilience to environmental disturbances.A cascading linear proportional controller is developed to control the quadrotor's altitude, attitude and heading. Also, two nonlinear control techniques: High Gain Robust and Backstepping controllers are formulated and their performances in their ability to control and stabilize the UAV is analyzed.

II. LITERATURE REVIEW

B.Dynamic Model of a Quadrotor

The dynamic model of a quadrotor describes the altitude, attitude and position of the system using sets of differential equations to describe the forces and moment acting on the quadrotor at any given time. Depending on the control inputs, two dynamic model types are used to describe and simulate the response of the quadrotor and the control systems. The first dynamic model use motor's speed without necessary considering the dynamic system model of the motors while the second model take into account the dynamic model of the motor while using the motors' voltages as the control inputs.

Generally, quadrotor system modeling is divided into body modeling and propulsion system modeling.

B1.Body Modeling

Different methods have been adapted in modeling the body dynamics of a quadrotor. [9] adopted the Lagrangian approach. [26] used the Newton–Euler approach and [14] used a quaternion formation of equations to model the body dynamics. Superposition method and the system identification were used in dynamic modeling of quadrotors' body by [3], [24] and [20] respectively. In effort to model an unknown dynamic system of a quadrotor, [8] used the neural network. [17] presented a dynamic of the quadrotor at the presence of a sliding mass. [11] on the other hand used a method called Differential Algebric Splitter (DAS) to simulate the nonlinearity of the dynamics of a quadrotor. However, all of the methods highlighted in the literature based the model of the quadrotor on having the exact model knowledge of the system. It was immediate apparent that there is need to develop a quadrotor model that can cater for unknown parameters of the vehicle model. This type of model will not just be more accurate but robust to disturbances.

B2.Motor Modelling

[5] developed and analyzed a nonlinear airframe of a quadrotor comprising of the body and propulsion system. [2] on the other hand, developed a complete model of motor including mechanical parts (gear and various frictions) and electrical parts using speed reduction gear ratios to control the speed of the motor. [21] employed the identification method to predict the model of a motor. In all the researches highlighted, a brushless DC motor was used as the propulsion system while, [27] modelled a brush DC motor for a quadrotor.

B3.Propeller Modeling

This type of modeling describes the aerodynamic equations of thrust, drag forces and moments as a result of the rotors' speed of rotation and the aerodynamic coefficients.[6], [13] and [23] described the identification of aerodynamic coefficients and the driving aerodynamic equations of blade. It was observed that a ducted fan propeller is more efficient when compared with an ordinary propeller. A detailed comparison between ordinary propeller and a ducted fan propeller is presented by [25].

C. Control Theory

Various control approaches have been proposed for quadrotor's control, for positioning, regulatory and trajectory tracking problems. Previous researchers including but not limited to [7] ,[5]&[22] have shown the possibility and successful control of quadrotor using linear control techniques howbeit, by linearizing the nonlinear dynamics of the system around an operating point, usually chosen to be the hovering point.

There are several control techniques that can be used to control a quadrotor varying between the classical linear Proportional-Integral-Derivative (PID), Proportional-Derivative (PD) controller to complex nonlinear approaches such as backstepping and sliding-mode control algorithms. The fight control systems can be classified into four main categories which are: linear flight control systems, non-linear flight control systems, hybrid and learning-based flight control systems [22]. It was found that the

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most common control technique used is the PID or PD controller. Although it is a linear controller used for the nonlinear multivariable quadrotor system.

C1.Linear Flight Control Systems

PID and LQ: [5] proposed the usage of PID and LQ control techniques and was found out that these two types of controllers performed comparably and were able to stabilize the quadrotor's attitude around its linear operating point when it undergoes little disturbances.[16] controlled and stabilized the position and orientation of the quadrotor using PID controller under the influence of low-speed windy environment.

C2.Nonlinear Flight Control Systems

Due to the fact that the dynamics of the quadrotor is of a nonlinear nature, developing nonlinear control algorithms to be used as flight controllers was necessary. There is a variety of nonlinear control algorithms applied to quadrotors including: feedback linearization, model predictive control, backstepping and sliding-mode.

Backstepping and Sliding-mode: Backstepping is a recursive control technique that can be applied to both linear and nonlinear systems [18].[4] proposed the use of backstepping and sliding-mode nonlinear control approaches to control the quadrotor with better performances in the presence of disturbances. [26] developed controllers that stabilized the quadrotor in an outdoors environment. They compared the performance of an integral sliding-mode controller with a reinforcement learning controller. [10] merged backstepping controller with an adaptive controller to overcome the problems of model uncertainties and external disturbances. [15] also controlled the position and attitude of the quadrotor using a backstepping controller. The controller was then tested in a noisy environment and produces a satisfactory performance. [12] proposed using a chattering free sliding mode controller to control the altitude of a quadrotor. The proposed controller performed well in both simulations and on a real system in the presence of disturbances. However, none of the reviews mentioned here developed or used a High Gain Robust controller which will be one of the nonlinear controllers proposed in this paper.

III. QUADROTORDYNAMIC MODEL

In the derivation of quadrotors' kinematics and dynamics models using Newton-Euler method, the following assumptions are made:

a) The quadrotor's structure is rigid
b) The quadrotor's structure is symmetrical
c) The center of gravity of the quadrotor's body is the same with the center of origin of the body fixed frame
d) The structure of the propellers is rigid
e) Lift and drag forces are proportional to the square of the propeller's speed

D. Kinematic Model of a Quadrotor

The kinetic model of a quadrotor is derived both in the earth inertia and body reference frames.
The inertia frame, or earth fixed frame, is used to describe the absolute position in the space. Figure 1.1 shows the Earth reference frame designated as NED axes and the body frame defined as x,y and z axes. The Earth frame is an inertia frame fixed on a specific place at ground level as its name implies, it uses the NED notation to describe the North, East and Downwards axes respectively. On the other hand, the body frame unites with the barycenter of the quadrotor body, with its x,y and z axes pointing towards propeller 1, 2 and upward respectively.

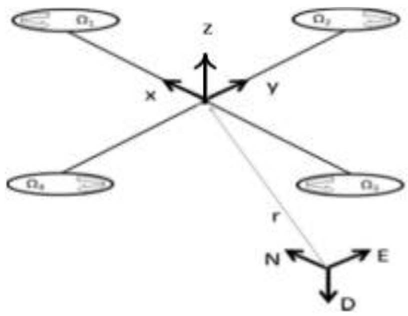

Fig.1.1: Quadrotor Reference Frames

The coordinate vector r describe the distance between the Inertia frame and the body frame from the absolute position of the center of mass is given as $r = [xyz]^T$. The rotation matrix R is used to rotate from the body frame to the inertial frame, where $R \in SO3$ describes the orientation of the quadrotor. The orientation of the quadrotor is described using yaw, pitch and roll angles ($\phi, \theta$and ψ) describing rotations about the Z, Y and X-axes respectively. Taking the order of rotation to be yaw (ϕ), pitch (θ) and roll (ψ), then the rotation matrix R which is derived based on the sequence of ZYX rotations is:

$$R = \begin{bmatrix} c\phi c\theta & -c\theta s\phi & s\theta \\ c\psi s\phi + c\phi s\psi s\theta & c\phi c\psi - s\phi s\psi s\theta & -c\theta s\psi \\ s\phi s\psi - c\phi c\psi s\theta & c\phi s\psi + c\psi s\phi s\theta & c\psi c\theta \end{bmatrix} \#(1)$$

where c and srepresentscos and sin angles respectively.

The rotation matrix R in equation (1) is used to formulate the dynamic model of the quadrotor. The significance of matrix R is due to the fact that some of the vehicle state are measured in the body frame such as Lift and drag forces produced by the propellers while forces such as the gravitational pull and the position of the quadrotor are measured in the earth frame. It then became imperative to have a type of transformation from one frame of reference to another as the need arises. This conversion is achieve using the rotation matrix.

The relationship between Euler rates $\dot{\eta} = [\dot{\psi}\dot{\theta}\dot{\phi}]^T$ measured in the inertia frame and angular body rates $\omega = [p\ q\ r]^T$, is described by the transformation matrix W as follows:

$$\omega = W\dot{\eta} \#(2)$$

Where

$$W = \begin{bmatrix} \sin\theta & 0 & 1 \\ -\cos\theta\sin\psi & \cos\psi & 0 \\ \cos\psi\cos\theta & \sin\psi & 0 \end{bmatrix} \#(3)$$

W is known as the Wronskian Matrix. Around the linear operating point, small angle assumption is made where $\cos\phi \equiv 1$, $\cos\theta \equiv 1$ and $\sin\phi = \sin\theta = 0$. Therefore, W becomes an identity matrix I.

E. Dynamic Model

The dynamic model of a quadrotor is divided into two subsystems: translational and rotational subsystems.

The translational equations of motion of the quadrotor is derived in the earth inertia frame using Newton's second law of motion and is given as

$$m\ddot{r} = \begin{bmatrix} 0 \\ 0 \\ -mg \end{bmatrix} + RF_{vb} \#(4)$$

Where$r = [x\ y\ z]^T$ Quadrotor's distance from the inertia frame, m is the mass of the quadrotor, g is the gravitational pull$= 9.81m/s^2$ and F_{vb} are the lift forces acting on the body of the quadrotor

The first term in equation (4) is the force due to gravity acting in the downward direction of the earth inertia frame. The negative sign implies that the direction of the force of gravity is in opposite direction to the thrust forces generated by the propeller shown as the positive z-axis pointing upward in figure 1.1.

When a quadrotor is on a horizontal orientation, the only forces acting on it is the lift force produced by the rotors which is directly proportional the square of the speed of rotation of the propellers given as $F_{vb} = K_f\Omega_b^2$. Thus, the lift force acting on the quadrotor, F_B, can be expressed as,

$$F_B = \begin{bmatrix} 0 \\ 0 \\ K_f(\Omega_1^2 + \Omega_2^2 + \Omega_3^2 + \Omega_4^2 \end{bmatrix} \#(5)$$

By multiplying F_B with the rotation matrix R, it is then possible to transform the thrust forces generated by the rotors from the body frame to earth inertia frame in order to make it possible to apply the force equation to any orientation of the quadrotor.

The rotational equation of motion is described by the Newton-Euler method in the body frame as follows:

$$I\dot{\omega} + \omega\ x\ I\omega = M_B \#(6)$$

Where$I$is Principal of Inertia, $\omega$is the angular body velocity and M_B is the moments acting on body of the quadrotor.

The first two terms in equation (6), $I\dot{\omega}$ and $\omega \times I\omega$, represent the rate of change of angular momentum in the body frame.

The main reason why rotational equation of motion of a quadrotor is derived in the body frame as opposed to earth frame is to be able to express the diagonal inertia matrix as time independent.

Due to the assumption that the structure of the quadrotor is symmetrical, therefore, the principal Matrix of Inertia for the quadrotor become a diagonal matrix making the off-diagonal (product of inerta) elements zero as follows:

$$P = \begin{bmatrix} I_{xx} & 0 & 0 \\ 0 & I_{yy} & 0 \\ 0 & 0 & I_{zz} \end{bmatrix} \in \mathcal{R}^{3x3} \#(7)$$

Where $I_{xx}, I_{yy}, and\ I_{zz}$ are the moments about the x, y and z principal axes in the body frame.

The last term of equation (6) is needed to define two physical parameters; the aerodynamic forces and moments produced by a rotor. When a body rotates, a force is generated and is called the aerodynamic force or the lift force and the moment is referred to as the aerodynamic moment. The aerodynamic forces and moments are given in the following equation as follows:

$$F_{vb} = K_f\Omega_i^2 \#(8)$$
$$M_{vb} = K_M\Omega_i^2 \#(9)$$

Where K_f and K_M are the constants of aerodynamic force and moment respectively and Ω_i is the angular speed of rotor i.

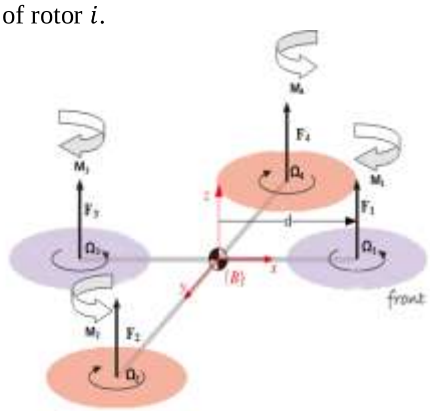

Fig. 1.2: Forces and Moment on a Quadrotor

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Fig. 1.2 shows the forces and moments on the quadrotor. Each rotor generates an upwards lift force F_{vb} and moment M_{vb} in the direction opposing the direction of rotation of the corresponding rotor i. Pairwise differences in speed of the rotors induced moments causing the vehicle to rotate.

The moment about the $x-axis$ occur on the body of the airframe due to pairwise difference between the rotor 2 and 4 as shown the fig. 1.3 and expressed in the following equation:
The torque about the vehicle's x-axis, the rolling torque, is generated by the moments,

$$M_x = dF_4 - dF_2$$
$$= d\left(K_f \Omega_4^2\right) - d\left(K_f \Omega_2^2\right)$$
$$= dK_f\left(\Omega_4^2 - \Omega_2^2\right)\#(10)$$

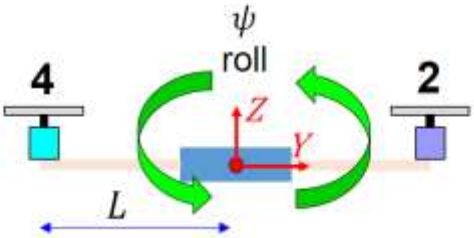

Fig. 1.3 Rolling Moment

Moment generated about the body frame's y-axis is due to the pairwise difference between the speed of rotation of propeller 1 and 3 as shown in Figure 1.4.

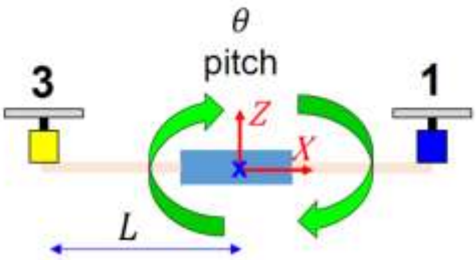

Figure 1.4: Pitching Moment

The pitching moment can be expressed as:

$$M_y = dF_1 - dF_3$$
$$= d\left(K_f \Omega_1^2\right) - d\left(K_f \Omega_3^2\right)$$
$$= dK_f\left(\Omega_1^2 - \Omega_3^2\right)\#(11)$$

A yaw torque is generated by appropriately controlling all four rotors speeds as shown in the figure 1.5.

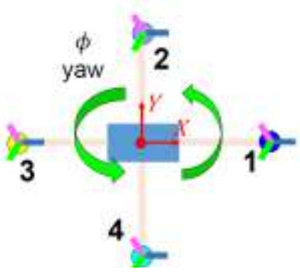

Figure 1.5: Yaw Moment

By using the right-hand-rule, the moment about the body frame's z-axis can be expressed as,

$$M_z = M_1 - M_2 + M_3 - M_4$$
$$= K_M\left(\Omega_1^2 - \Omega_2^2 + \Omega_3^2 - \Omega_4^2\right)\#(12)$$

Combining equation (10), (11) and (12) in a vector form, we have:

$$M_B = \begin{bmatrix} dK_f\left(\Omega_4^2 - \Omega_2^2\right) \\ dK_f\left(\Omega_1^2 - \Omega_3^2\right) \\ K_M\left(\Omega_1^2 - \Omega_2^2 + \Omega_3^2 - \Omega_4^2\right) \end{bmatrix}\#(13)$$

where d is the arm length, that is, the distance between the center of mass of the quadrotor to the axis of rotation of each of the rotor.
Combining the translational and rotational subsystem, the complete dynamic model of the system is expressed as follows:

$$\begin{cases} \ddot{x} = \dfrac{U_f}{m}(sin\phi sin\psi - cos\phi cos\psi sin\theta) \\ \ddot{y} = \dfrac{U_f}{m}(cos\phi sin\psi + cos\psi sin\phi sin\theta) \\ \ddot{z} = \dfrac{U_f}{m}(cos\psi cos\theta) - g \\ \ddot{\psi} = \dfrac{dU_x}{I_{xx}} + \dfrac{\dot{\theta}\dot{\phi}\left(I_{yy} - I_{zz}\right)}{I_{xx}} \\ \ddot{\theta} = \dfrac{dU_y}{I_{yy}} + \dfrac{\dot{\psi}\dot{\phi}\left(I_{zz} - I_{xx}\right)}{I_{yy}} \\ \ddot{\phi} = \dfrac{U_z}{I_{zz}} + \dfrac{\dot{\theta}\dot{\psi}\left(I_{xx} - I_{yy}\right)}{I_{zz}} \end{cases} \#(14)$$

To simplify further, the following are defined as:

$$a_1 = \frac{I_{yy} - I_{zz}}{I_{xx}} \text{ and } b_1 = \frac{d}{I_{xx}}$$
$$a_2 = \frac{I_{zz} - I_{xx}}{I_{yy}} \text{ and } b_2 = \frac{d}{I_{yy}}$$
$$a_3 = \frac{I_{xx} - I_{yy}}{I_{zz}} \text{ and } b_3 = \frac{1}{I_{zz}}$$

F.Controller Designs

F.1Linear Proportional Controllers

In other to make it possible to design multiple Proportional controllers for the dynamic model of the system derived in equation (14), one has to linearized the nonlinear equation around a particular operating point (in this case, the hovering point) by removing the nonlinear cross couplings in equation (14).

A cascading proportional controller is developed to control the states of the quadrotor through the control inputs U_f, U_x, U_y and U_z and is expressed as:

$$\begin{cases} U_f = k_{p1}(z - z_d) + k_{p2}(\dot{z} - \dot{z}_d) \\ U_x = k_{p1}(\psi - \psi_d) + k_{p2}(\dot{\psi} - \dot{\psi}_d) \\ U_y = k_{p1}(\theta - \theta_d) + k_{p2}(\dot{\theta} - \dot{\theta}_d) \\ U_z = k_{p1}(\phi - \phi_d) + k_{p2}(\dot{\phi} - \dot{\phi}_d) \end{cases} \#(15)$$

F.2 High Gain Robust Nonlinear Controller

Robust controller uses high gain to stabilize a system and achieves great feedback performance in stability and accuracy. Due to the nature of its high gain, it is able to provide robustness to system uncertainties, parameter variations and environmental disturbances. High Gain Robust controller reduces the effects of external disturbances and parameter variations. The High Gain Robust controller is derived based on the approach by Khalil (2002).

The high gain robust control is expressed as

$$U = -kx + V_R \#(16)$$

WhereK_x is the feedback term which ensures stability and $V_R = K_2x$ is the virtual gain of the controller which deals with noise and disturbances.

The roll controller is design by first considering the second derivative of the roll angle in terms of the Euler rates: $\ddot{\psi} = b_1U_x + a_1\dot{\theta}\dot{\phi}$.

For the second step, a Lyapunov function $V(r) = \frac{1}{2}r^2$ $(Positive\ Definite)$ is considered and r is given as: $r = \dot{e} + \alpha e$.

Where$r$ is the filter tracking error, e is the error signal $(\psi_d - \psi)$, $\dot{e} = (\dot{\psi}_d - \dot{\psi})$ and $\ddot{e} = (\ddot{\psi}_d - \ddot{\psi})$and α is a positive constant.

Taking the derivative of the Lyapunov function,

$$\dot{V}(r) = r\dot{r}$$

$$\dot{V} = r(\ddot{e} + \alpha\dot{e})$$

$$\dot{V}(r) = r(\ddot{\psi}_d - \ddot{\psi} + \alpha\dot{e})\#(17)$$

Since the control input is a function of the second derivative of the roll angle, the controller is designed as follows:

$$\dot{V}(r) = r(\ddot{\psi}_d - (b_1U_x + a_1\dot{\theta}\dot{\phi}) + \alpha\dot{e})\#(18)$$

From equation (18), the controller U_x can be designed as follows:

$$U_x = \frac{1}{b_1}(\ddot{\psi}_d - a_1\dot{\theta}\dot{\phi} + \alpha\dot{e} + kr)\#(19)$$

Substituting equation (19) into equation (18), the following is obtained:

$$\dot{V}(r) = r(-kr) = -kr^2\#(20)$$

From equation (20), the derivative of the Lyapunov function is Negative Definite which proves that the controller designed in equation (19) achieves global asymptotic stability of the system.

This same approach is adopted to design $U_y, U_z, U_f, \theta_d\, and\, \psi_d$ for Pitch, Yaw, Altitude and position (x and y) controllers respectively in the equation (21) in terms of system state variables.

$$\begin{cases} U_y = \frac{1}{b_2}(\ddot{\theta}_d - a_2\dot{\psi}\dot{\phi} + \alpha\dot{e} + kr) \\ U_z = \frac{1}{b_3}(\ddot{\phi}_d - a_3\dot{\psi}\dot{\theta} + \alpha\dot{e} + kr) \\ U_f = \frac{m}{\cos\theta\cos\phi} + \ddot{z}_d - g + \alpha\dot{e} + kr \\ \theta_d = \frac{m}{U_f}\left(\frac{1}{\cos\phi} + \psi_d\sin\phi\right) - \alpha\dot{e} - kr - \ddot{x}_d \\ \psi_d = \frac{m}{U_f}\left(\frac{1}{\cos\phi} - \theta_d\sin\phi\right) - \alpha\dot{e} - kr - \ddot{y}_d \end{cases} \#(21)$$

IV. RESULTS AND DISCUSSION

G.Proportional Controller

The parameters and gain of the proportional controllers were tuned using MATLAB control toolbox PID tuner. The Single Input Single Output (SISO) Proportional controllers of equation (15) are implemented as cascading controllers. They are various advantages of using cascading controllers such as improved system response over single loop control and can limits disturbances or effect of gain variation from the outer loop on the inner loop.

The control structure is given in fig 1.6 below.

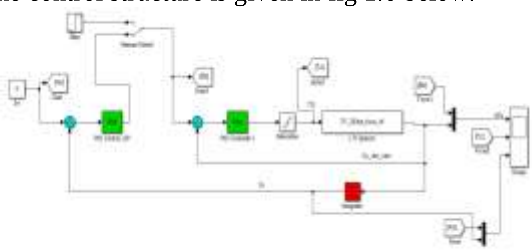

Figure 1.6: SISO Cascading Altitude Controller

G1.Proportional Controller Simulation (Linearized Model of the Quadrotor)

The control gains produced by the MATLAB PID tuner for outer and inner loops of the altitude controller were $K_p = 3.3$ and $K_p = 9.3$ respectively. The designed requirement used to tune the gains of the controllers were rise time and gain margin.

Table 1.1 shows a summary of the control gains and performance of the system in terms of its settling time and overshoot. The response of the system is shown in figure 1.7 - 1.10 with the respective control inputs.

Due to the symmetry of the quadrotor, the controller for the pitch rotation is equivalent to that of the roll rotation.

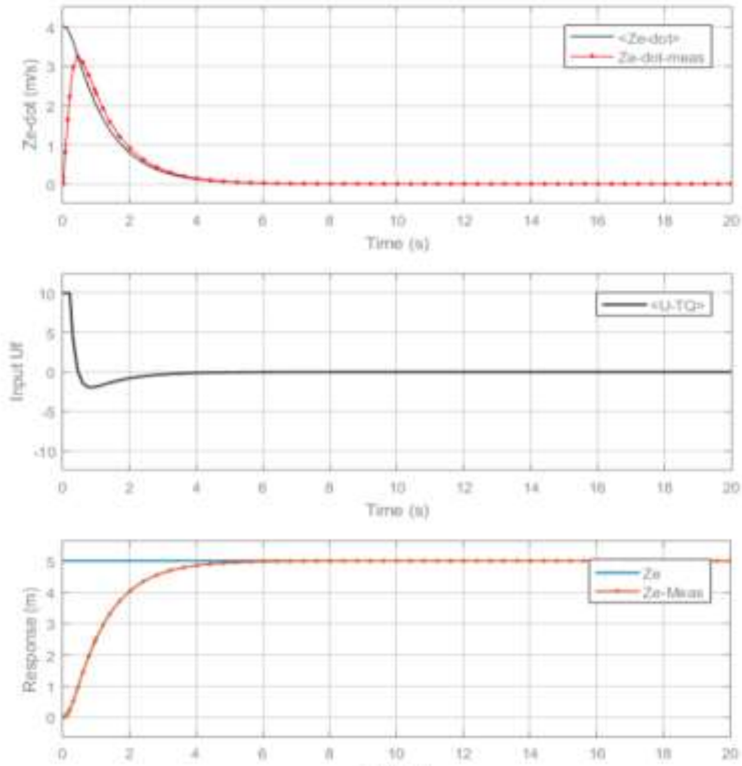

Figure 1.7: Proportional Control Inputs and Simulation Response for Altitude Control

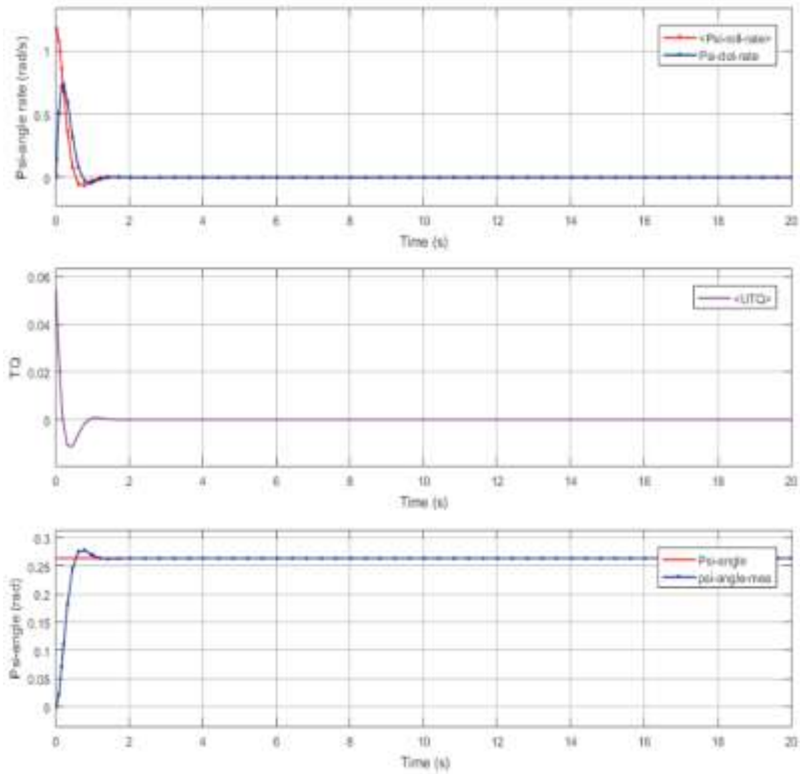

Figure 1.8: Proportional Control Inputs and Simulation Response for Psi-Roll Angle

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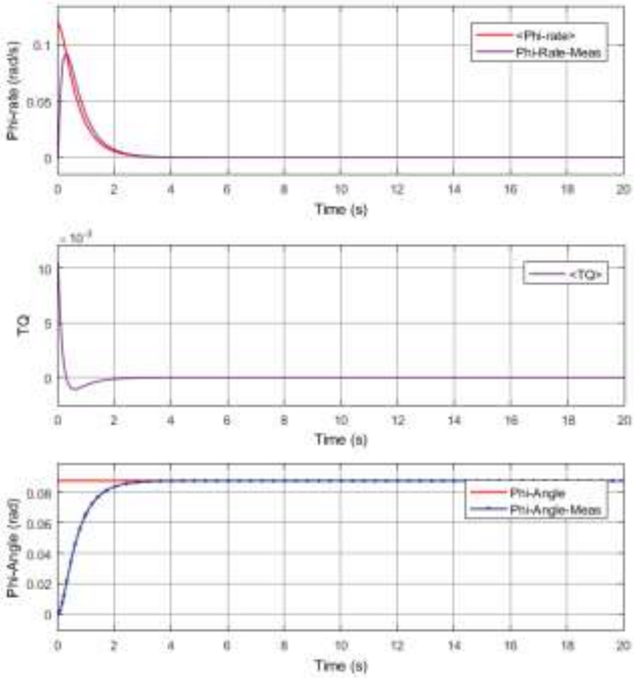

Figure 1.9: Proportional Control Inputs and Simulation Response for Phi-Yaw Heading Angle

Table 1.1: Proportional Controller Results

	Reference Value	**K_p (Outer loop)**	**K_p (Inner Loop)**	**Rise Tiime**	**Settling Time**	**Overshoot**
Altitude (Ze)	5 m	0.805	6.181	2.5	4.9s	0.5%
Attitude (ψ and θ)	15^o (0.26 rad)	4.47	0.05	0.5	1.2s	5.85%
Heading (ϕ)	5^o (0.087 rad)	1.35	0.089	1.5	2.6s	0.5%

G2.Proportional Controller Simulation on Nonlinear Model of the Quadrotor

Although the Proportional controllers were developed and applied on the linearized version of the quadrotor model, however, in reality, quadrotor is a nonlinear vehicle; the proportional controllers were then tested on the nonlinear model and the following results were obtained.

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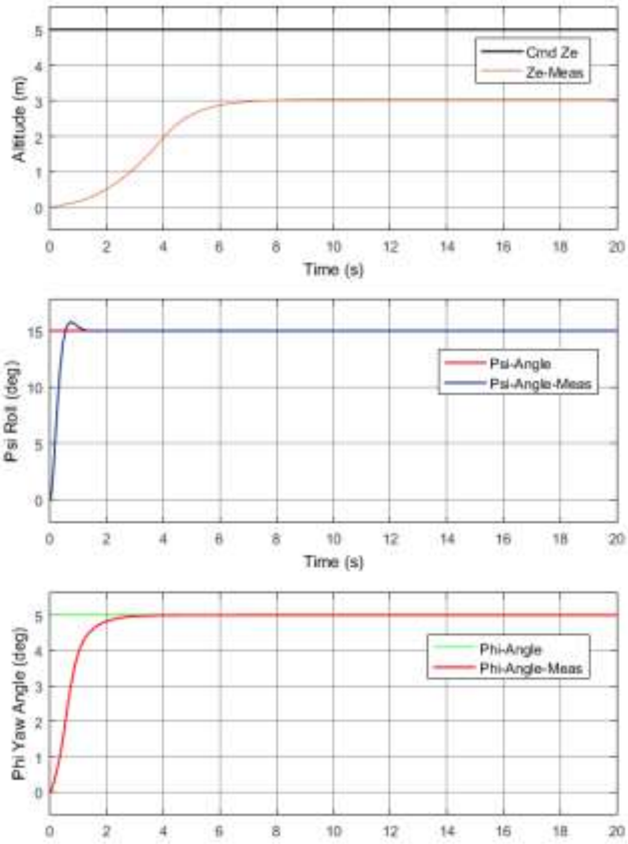

Figure 1.10: Proportional Control of Nonlinear Quadrotor Dynamics (Hovering Point)

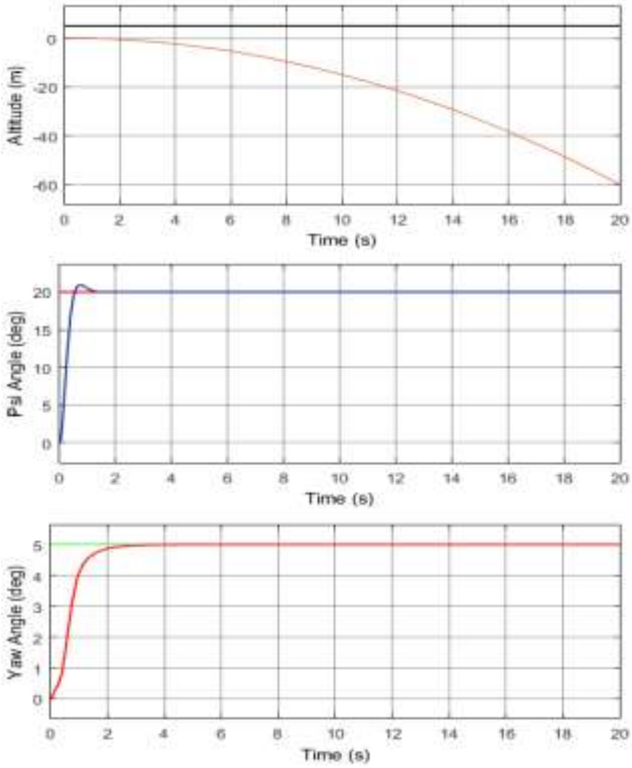

Figure 1.11: Proportional Control of Nonlinear Quadrotor Dynamics (Outside Linear Region)

Table 1.2: Proportional Controller Results on Nonlinear Quadrotor Model

	Reference Value	K_p Outer loop	K_p Inner Loop	Settling Time	Overshoot	Steady State Error
Altitude (Ze)	5 m	0.805	6.181	-	0%	(- 40%)
Attitude (ψ and θ)	15^o (0.26 rad)	4.47	0.05	1.2s	5.85%	0%
Heading (ϕ)	5^o (0.087 rad)	1.35	0.089	2.6s	0.5%	0%

From table 1.2, 40% steady state error was observed on the altitude plot whereas there was no steady state error observed on the attitude and heading plots as shown in figure 1.10. This was possible because of the small angle assumption (less than 20^o). However, when the attitude and heading angle was above 20^o the controller was unable to stabilized the system outside the linear region as shown in figure 1.11.

H. Control using High Gain Robust Technique

The results of the simulation on the application of High Gain Robust Controllers on the Quadrotor Nonlinear Airframe is presented in Table 1.3. Figure 1.12 shows the result when running the simulation using constants specified in Table 1.3.

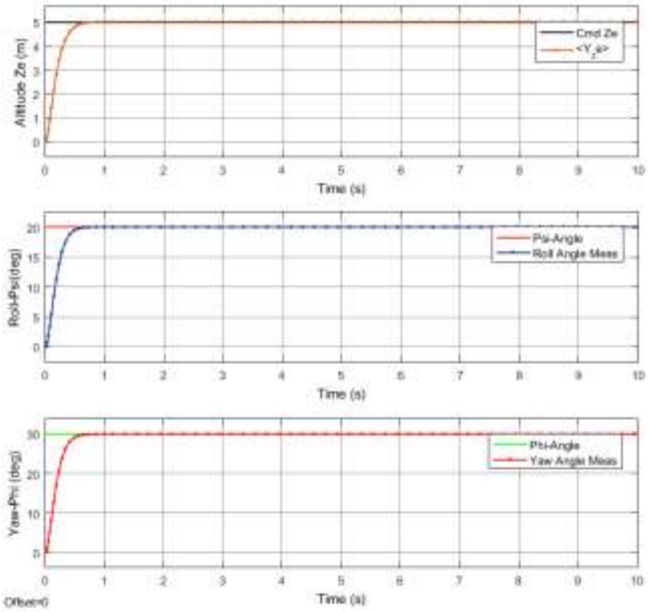

Figure 1.12: High Gain Control of Nonlinear Quadrotor Model

Table 1.3: High Gain Robust Controllers Results

	Reference Value	k	α	Rise Time (s)	Settling Time (s)	Overshoot	Steady State Error
Altitude (Ze)	5 m	10	8	0.34	0.6	0.03%	+0.03%
Attitude (ψ and θ)	20^o (0.35 rad)	10	8	0.3	0.6	0.5%	0%
Heading (ϕ)	30^o (0.52 rad)	10	8	0.31	0.6	0.05%	0%

H1.Performance with Disturbance

4.3.2 Performance with Disturbance

Disturbance was added to the quadrotor model in the form of additional force to give the effect of operating the quadrotor with disturbances. The force was modeled as Gaussian noise with zero mean and with a maximum value of 2N. The system was commanded to follow a varying path. The

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performance of the system under the effect of disturbances is shown in figure 1.13.

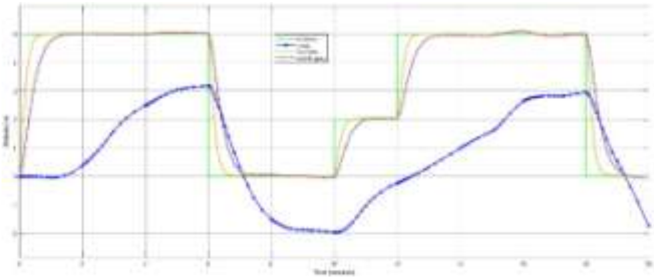

Figure 4.14: Altitude Response with Disturbance

The green trace represents the reference path while the blue and yellow traces represent the linear and High Gain Robust controller responses respectively.

I. Summary of Results

I1. Linear Operation

Two controllers developed to control the quadrotor model performed considerably well in terms of rise time, settling time and overshoot when they were deployed around the linear region of the quadrotor.

Tables 1.4 and 1.5 gives a quantitative analysis between the performance of the Linear and High Gain in terms of the settling time and overshoot of the system's response respectively.

Table 1.4: System Settling Time Response Under Two Controllers in the Linear Region

Controller	**Altitude (5 m)**	**Attitude (15^o)**	**Heading (5^o)**
Linear	4.9s	1.2s	2.6s
High Gain	0.6s	0.3s	0.2s

Table 1.5: System Overshoot Response Under Different Controllers in the Linear Region

Controller	**Altitude (5 m)**	**Attitude (15^o)**	**Heading (5^o)**
Linear	0	5.85%	0.5%
High Gain	0.03%	0.5%	0.05%

I2.Nonlinear Operation

When the controllers were applied to control the quadrotor outside the linear operating region, the Proportional controllers failed to stabilize the system due to the fact that Proportional controller is a type of linear controller. On the other hand, the High Gain was able to stabilize the system with a good dynamic performance as presented in Tables 1.6.

Table 1.6: System Responses Under High Gain Controller in the Nonlinear Region

System Parameters	**Altitude (5 m)**	**Attitude (20^o)**	**Heading (30^o)**
Settling Time (s)	0.6s	0.6s	0.6s
System Overshoot (%)	0.03%	0.5%	0.05%
Steady State Error (%)	0.03%	0%	0%

I3.Control Effort Comparison:

Comparing the two developed controllers in terms of their gains and generated control signals, the proportional controller comes out to be the most energy efficient controller.

V. CONCLUSION

In this paper, a detailed and accurate mathematical model for a quadrotor Unmanned Aerial Vehicle (UAV) was derived, a cascading linear proportional and High Gain Robust nonlinear control techniques were also developed to stabilize and the control the states variables of the quadrotor including its altitude, attitude and heading in space and a verifiable performance statistics of the developed controllers via MATLAB simulation and results were presented.

The system performance characteristics were evaluated, analyzed given different inputs.

Tuning the parameters and gains of the three implemented controllers was done using MATLAB control toolbox. The design requirement for the tuning of the controllers was the dynamic responses of the system in terms of rise time, settling time, overshoot and steady state errors.

The two controllers performed comparatively well in the linear operating region of the quadrotor in the range of 0 to 20° of attitude and heading. However, when the quadrotor was commanded to follow a varying trajectory, the Linear Proportional controller was unable to track the reference path due to the nonlinearity of the quadrotor dynamics. The High Gain controller gave better performance and was able to track the varying trajectory outside the linear operating region and was resilient to environmental disturbances which was modeled as external force on the quadrotor body frame due to its nonlinear specification.

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