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Effect of Suction/Injection on Mixed Convection Flow of an Exothermic Fluid through a Vertical Porous Channel

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ABSTRACT: The study on a steady mixed convection flow of an exothermic fluid with symmetric wall temperature and symmetric wall concentration in the presence of transverse magnetic field, and suction/injection on the fluid flow behavior through a vertical porous channel was investigated. The resulting systems of governing equations were solved semi-analytically using the method of Differential Transformation Method (DTM). The results of the governing equations were obtain and analyzed for different parameters such as, Schmidt number (Sc), Magnetic number (M), Darcy number (Da), mixed convection parameter (λ), constant pressure gradient (γ), Frank-Kamenetskii parameter (K) and sustention parameter (N). The influence of Prandtl number on skin friction, the effect of γ_t on Nusselt number and Skin friction have been analyzed graphically, so also, the maximum and minimum flow velocity, temperature and concentration were recorded at lower and upper plate by showing an increase in symmetric wall temperature and concentration, the effects of Frank-Kamenetskii number and porous medium/Darcy number on velocity were observed.

Keywords: Suction/Injection; Mixed convection; Exothermic fluid; Vertical channel; Porous material.

List of Nomenclatures/Greek letters:

Da : Porous medium

θ : Dimensionless Temperature

u : Dimensionless velocity of the fluid

y : Dimensionless co-ordinate perpendicular to the plate

x : Dimensionless co-ordinate parallel to the plate

N : Sustention parameter

K : Frank-Kamenetskii parameter

λ : Mixed convection parameter

M : Magnetic parameter

γ : Constant pressure gradient

γ_t : Symmetric Wall temperature

γ_c : Symmetric Wall concentration

g: gravitational force

Sc: Schmidt number

ν : Kinematic viscosity

ρ : Density of the fluid

β : Thermal expansion coefficient

σ : Electric conductivity of the fluid

β^* : Solutal expansion coefficient

V_0 : Suction/Injection parameter

I. INTRODUCTION

The problems of Steady free, forced or mixed convection, symmetrically or

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asymmetrically heated/mass fluid flow in a vertical or horizontal porous channels in the presence of chemical reaction of an exothermic fluid have recently received attentions of various researchers from across the globe because of their practical applications in modern electronic equipments, nuclear reactors, geothermal reservoir, thermal insulation, energy storage and conservation, fire control, chemical, food and metallurgical industries, in also petroleum reservoirs. Magnetic field is one of the important factors by which the cooling rate can be controlled and the desired quality of the industrial product can be achieved, many investigations were made to examine mixed convection flow characteristics in porous medium. This type of problem also arises in electronic packages, micro-electronic devices during their operations, as well as in the more convectional fields of the fluid and thermal sciences.

There were many investigations in the past decades on mixed convection flow of an exothermic fluid. Many different configurations and combinations of thermal boundary conditions were considered by various investigators. Some of which are Chamkha et al. (1991) discussed the mixed convection flow in a lid-driven enclosure filled with a fluid-saturated porous medium, Rudraiah et al. (1995) studied the effect of magnetic field on free convection in a rectangular enclosure, Khanafer et al. (1998) discussed the numerical study of laminar natural convection in tilted enclosure with transverse magnetic field. Also in an attempt to study free convection on heat and mass transfer flows, Hamza. (2016) discussed the influence of free convection slip flow of an exothermic fluid in a vertical channel, Chen and Weng. (2005) studied analytically the fully developed natural convection flow in an open-ended vertical parallel plate micro channel with asymmetric wall temperature distribution. This work is further extended by Jha et al. (2015) by taking in to account suction/injection on the micro-channel walls. Jha et al. (2015) reported MHD natural convection flow in a vertical parallel plate micro-channel, Saleh et al. (2013) studied flow of fully developed mixed convection in a vertical channel with chemical reaction, Pop et al. (2010) Investigated numerically by using implicit finite difference scheme, the effect of heat generated by an exothermic reaction on the fully developed mixed convection flow in a vertical channel. The effect of radiation on free convective flow and mass transfer past a vertical isothermal cone surface with chemical reaction in the presence of a transverse magnetic field has been studied by Afify (2004). Hayat et al. (2015) Studied mixed convection flow of casson Nanofluid over a stretching sheet with convectively heated chemical reaction and heat source/sink, also unsteady hydro magnetic Natural convection flow of a heat absorbing fluid within a rotating vertical channel in porous medium with hall effects have been studied by Seth et al .(2015), Srinivas et al .(2010) studied effects of thermal radiation and space porosity on MHD mixed convection flow in a vertical channel using homotophy analysis method. Das and Jana. (2010) analyzed heat and mass transfer effects on unsteady free convection flow near a moving vertical plate in porous medium. Yih. (1997) studied the effect of transpiration in coupled heat and mass transfer on mixed convection over a vertical plate embedded in a saturated porous medium, recently, Ahmad et al. (2017) examine the effect of mass transfer on mixed convection flow of an exothermic fluid in a vertical channel.

Considering the literature review, no work has been dealt on steady mixed convection flow of an exothermic fluid in a porous medium channel with effect of Suction/Injection in the presence of transverse magnetic field, symmetric wall temperature and concentration. Therefore, the aim of this paper is to study the effect of Suction/Injection on steady mixed convection flow of an exothermic fluid in a vertical porous channel, with symmetric wall temperature and concentration.

II. MATHEMATICAL FORMULATION

By considering steady state heat and mass transfer of mixed convection flow of an exothermic fluid between two vertical porous channel walls, both of the vertical walls are separated by a distance L, while top and bottom of the walls are maintained by a constant and different temperature T_a and T_m respectively, such that $T_a > T_m$ as shown in the figure 1. A coordinate directions were chosen such that the **x**-axis is parallel to the gravitational acceleration vector **g**, but with the opposite direction. The **y**-axis is orthogonal to the channel walls, and the origin of the axes is such that the positions of the channel walls are y = 0 and y = L, respectively. The fluid has a uniform vertical upward stream wise velocity distribution U_0 at the channel entrance, with a magnetic field of strength B_0 is applied in the horizontal direction to the walls. The working fluid were assumed to be Newtonian, incompressible and steady flow by following Pop et al. (2010) and Ahmad et al. (2017), and also, assumed that the heat were supplied to the surrounding fluid by an exothermic surface reaction inside the porous channel. Therefore, based on the aforementioned assumptions, we have the following dimensional forms of Momentum, heat and mass transfer equations;

The Momentum Equation

$$\nu\frac{d^2u'}{dy'^2}+V_0'\frac{du'}{dy'}-\frac{\sigma B_0^2u'}{\rho}-\frac{\nu u'}{k'}+g\beta\left(T'-T_0'\right)+g\beta^*\left(C'-C_0'\right)-\frac{1}{\rho}\frac{dp'}{dx'}=0 \quad 2.1$$

The Heat Transfer Equation

$$\alpha\frac{d^2T'}{dy'^2}+V_0'\frac{dT'}{dy'}+QK_0ae^{-E/RT'}=0 \quad 2.2$$

The Mass Transfer Equation

$$D\frac{d^2C'}{dy'^2}+V_0'\frac{dC'}{dy'}=0 \quad 2.3$$

Where v, g, and ρ are kinematic viscosity, the force of gravity and the density of the fluid, Q is the exothermic factor, β is the fluid thermal expansion coefficient, B_0 is the magnetic induction coefficient, β^* is the fluid solutal expansion coefficient, T_0 is the reference temperature, C_0 is the reference concentration, and assumed that $T_0=(T_a+T_m)/2$ and $C_0=(C_a+C_m)/2$.

The boundary conditions of the problem can be written as;

$$\left.\begin{array}{llll} u'=0, & T=T'_a, & C=C'_a, & y'=0 \\ u'=0, & T'=T_m', & C'=C_m', & y'=L \end{array}\right\} \quad 2.4$$

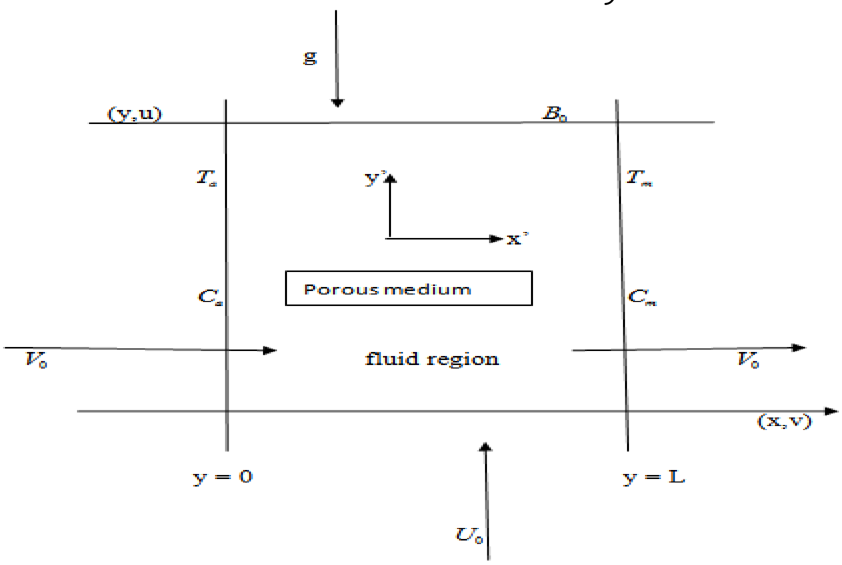

Figure 2.0: Geometric of the Problem.

And then, (2.1) to (2.3) with respect to (2.4) were transformed to dimensionless ordinary differential equations by introducing the following dimensionless variables

$$\left.\begin{array}{lllll} x=\frac{x'}{\operatorname{Re} L}, & y=\frac{y'}{L}, & u=\frac{u'}{u_0}, & p=\frac{p'}{\rho u_0^2}, & \operatorname{Re}=\frac{u_0 L}{v} \\ Gr=\frac{g\beta RT_0^2 L^3}{Ev^2}, & & \theta=\frac{T-T'_0}{RT_0^2/E}, & \phi=\frac{C-C'_0}{RC_0^2/E}, & M^2=\frac{\nabla B_0 L^2}{\rho v} \end{array}\right\}$$

2.5

Substituting (2.5) in to (2.1) to (2.4) we have the following ordinary differential equations of velocity, Temperature and Concentration with respect to their dimensionless boundary conditions;

$$\frac{d^2u}{dy^2}+V_0\frac{du}{dy}-\left(M^2+\frac{1}{Da}\right)u+\lambda\left[\theta+N\phi\right]-\gamma=0 \quad 2.6$$

$$\frac{d^2\theta}{dy^2}+P_rV_0\frac{d\theta}{dy}+Kre^{\theta}=0 \quad 2.7$$

$$\frac{d^2\phi}{dy^2}+ScV_0\frac{d\phi}{dy}=0 \quad 2.8$$

Subject to the boundary conditions

$$\left.\begin{array}{lllll} u=0, & \theta=\gamma_t, & \phi=\gamma_c & at & y=0 \\ u=0, & \theta=-\gamma_t, & \phi=-\gamma_c & at & y=1 \end{array}\right\} \quad 2.9$$

Where λ, N, γ, K, Da, M, γ_c, and γ_t, are mixed convection parameter, sustention parameter, pressure term, Frank-Kamenetskii parameter, porous material, magnetic field, symmetric wall concentration and symmetric wall temperature respectively. Which are defined as:

$$\left.\begin{array}{lll} \lambda=\frac{Gr}{\operatorname{Re}}, & \gamma=\frac{dp}{dx}, & \gamma_t=\frac{T'_a-T'_0}{RT_0^2/E}, \\ \gamma_c=\frac{C'_a-C'_0}{RC_0^2/E}, & & Kr=\frac{EQK_0aL^2}{RT_0^2\alpha}e^{-E/RT_0} \end{array}\right\} \quad 2.10$$

III. METHOD OF SOLUTION

Equations (2.6) to (2.8) subject to (2.9) were solved semi-analytically using the method of Differential Transformation Method (DTM), the expressions of velocity, temperature and concentration are displayed below.

$$u(y)=Zy+\frac{(a3-ZV_0)}{2}y^2+\frac{(a2-V_0(a3-ZV_0)+\alpha Z)}{6}y^3+\frac{(\gamma-\lambda a1-V_0(a2-V_0(a3-ZV_0)+\alpha Z)+\alpha[a3-ZV_0])}{24}y^4 \quad 2.11$$

$$\theta(y)=\gamma_t+Hy-\frac{(HP_rV_0+Kr)}{2}y^2+\frac{(HP_r^2V_0^2+P_rV_0Kr-Kr)}{6}-\frac{(HP_r^3V_0^3+P_r^2V_0^2Kr-P_rV_0Kr+Kr)}{24}y^4 \quad 2.12$$

$$\phi(y)=\gamma_c+Jy-\frac{JS_cV_0}{2}y^2+\frac{JS_c^2V_0^2}{6}y^3-\frac{JS_c^3V_0^3}{24}y^4 \quad 2.13$$

The expression of Skin Friction at the lower plate (y = 0) and upper plate (y = 1) is given below;

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$$\frac{du}{dy}=Z+\left(a3-ZV_0\right)y+\frac{\left(a2-V_0\left(a3-ZV_0\right)+\alpha Z\right)}{2}y^2+\frac{\left(\gamma-\lambda a1-V_0\left(a2-V_0\left(a3-ZV_0\right)+\alpha Z\right)+\alpha\left[a3-ZV_0\right]\right)}{6}y^3 \quad 2.14$$

$$\Rightarrow \frac{du}{dy}|_{y=0}=Z \text{ and } \frac{du}{dy}|_{y=1}=Z+\left(a3-ZV_0\right)+\frac{\left(a2-V_0\left(a3-ZV_0\right)+\alpha Z\right)}{2}+\frac{\left(\gamma-\lambda a1-V_0\left(a2-V_0\left(a3-ZV_0\right)+\alpha Z\right)+\alpha\left[a3-ZV_0\right]\right)}{6} \quad 2.15$$

Similarly, the expression of the rate of Heat Transfer is given below;

$$\frac{d\theta}{dy}=+H-(HP_rV_0+Kr)y+\frac{\left(HP_r^2V_0^2+P_rV_0Kr-Kr\right)}{2}y^2-\frac{\left(HP_r^3V_0^3+P_r^2V_0^2Kr-P_rV_0Kr+Kr\right)}{6}y^3 \quad 2.16$$

$$\Rightarrow \frac{d\theta}{dy}|_{y=0}=H \text{ and }$$

$$\frac{d\theta}{dy}|_{y=1}=+H-HP_rV_0-Kr+\frac{\left(HP_r^2V_0^2+P_rV_0Kr-Kr\right)}{2}-\frac{\left(HP_r^3V_0^3+P_r^2V_0^2Kr-P_rV_0Kr+Kr\right)}{6} \quad 2.17$$

The expression of rate of Mass Transfer at the lower plate (y = 0) and upper plate (y = 1) is given below;

$$\frac{d\phi}{dy}=+J-JS_cV_0y+\frac{JS_c^2V_0^2}{2}y^2-\frac{JS_c^3V_0^3}{6}y^3 \quad 2.18$$

$$\Rightarrow \frac{d\phi}{dy}|_{y=0}=J \text{ and } \frac{d\phi}{dy}|_{y=1}=+J-JS_cV_0+\frac{JS_c^2V_0^2}{2}-\frac{JS_c^3V_0^3}{6} \quad 2.19$$

Where, J, H, Z, and some basic terminologies in Differential Transformation Method (DTM) to be define in the appendix.

IV. RESULT AND DISCUSSION

Steady heat and mass transfer on mixed convection flow of an exothermic fluid through a vertical porous channel has been studied, with effect of suction/injection considering some fluid flow parameters like Magnetic number, Schmidt number, Frank-Kamenetskii number. The Differential Transformation Method, which is a semi-analytical method were used and obtained the solution of the governing equations. Mat Lab programming package has been used to compute, analyzed and validate the result graphically.

The influence of γ_c on the concentration field is displayed in figure 4.1, and it can be deduce that the concentration of the fluid increases with increase in the value of γ_c at the lower plate $\left(y=0\right)$, until the Steady state is attained, while different behavior were observed at the upper plate $\left(y=1\right)$. Figure 4.2 shows the effect of Schmidt number $\left(Sc\right)$ on concentration field, where it is observed that the increase in value of $\left(Sc\right)$ accelerate the concentration of the fluid. Figure 4.3 is the representation of the temperature profile showing the influence of Frank-Kamenetskii number $\left(K\right)$ on the temperature field, it is observed from the figure that the increase in the value of $\left(K\right)$ leads to the significant increase in the temperature of the fluid at the lower and upper plate until the steady state is attained. It can also be deduce that the velocity of the fluid increases with the increase in the value of Darcy number $\left(Da\right)$ as illustrated in figure 4.4. The influence of γ_t on velocity is displayed in figure 4.5, from the figure it can be seen that, whenever the value of γ_t is increase it will accelerate the velocity field. Figure 4.6 displayed the effect of

Magnetic number on the velocity of the fluid, and it can be deduce from the figure that increasing the value of magnetic number M can decreases the velocity field and vice-versa.

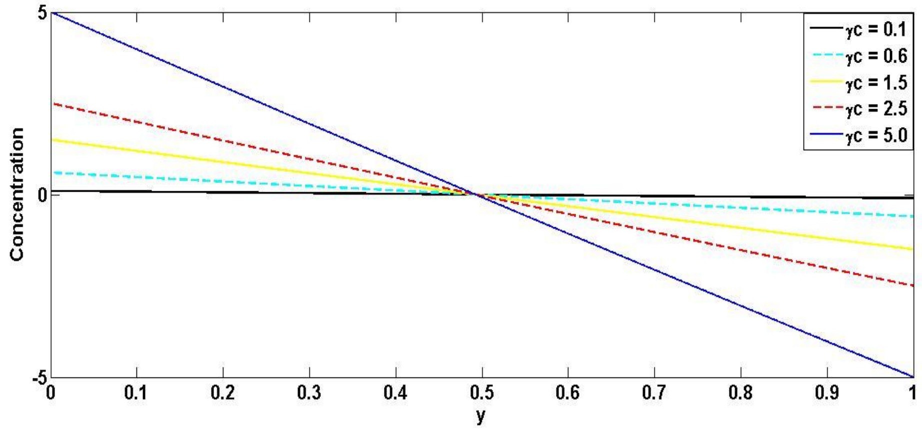

Figure 4.1: Concentration profile for $Da = 0.1, \lambda = 100, Kr = 1.5, \Pr = 0.71, Sc = 0.62$ with different values of γ_c

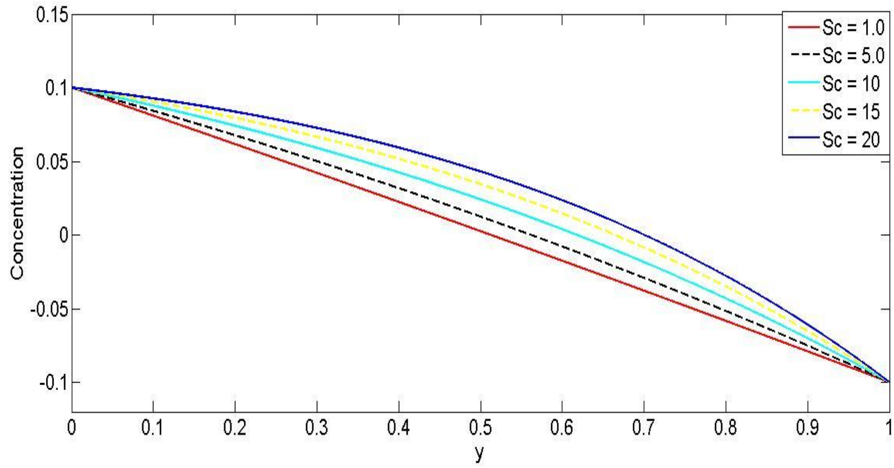

Figure 4.2: Effect of Schmidt number (Sc) on Concentration profile.

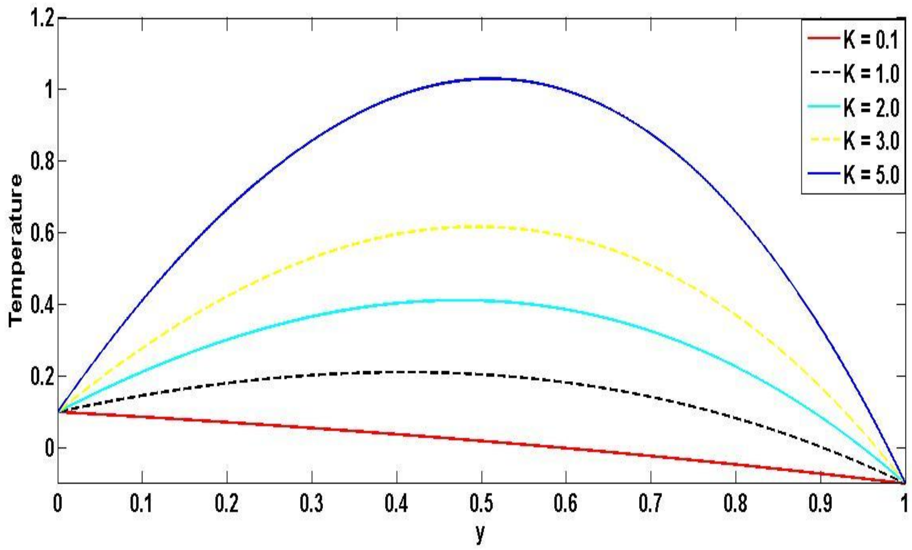

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Figure 4.3: Showing the temperature profile for $Da = 0.1, \lambda = 100, V = 0.1, \Pr = 0.71, Sc = 0.62$ with different values of Frank-Kamenetskii number K.

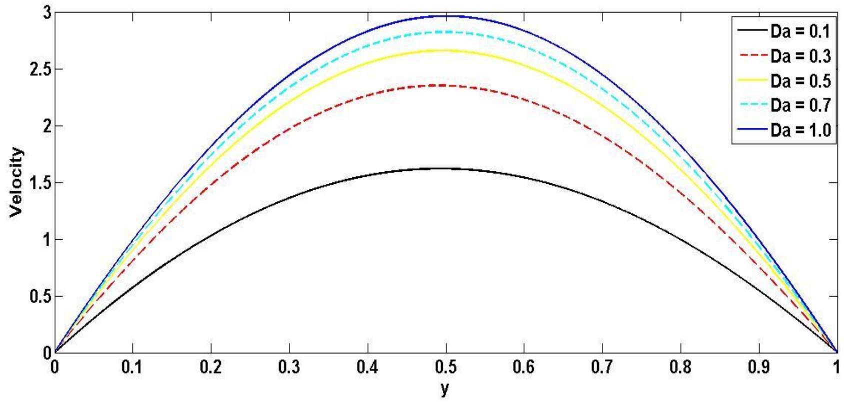

Figure 4.4: Velocity profile for $\lambda = 100, V = 0.1, \Pr = 0.71, Sc = 0.62, N = 0.01, Kr = 1.5$ with different values of Da

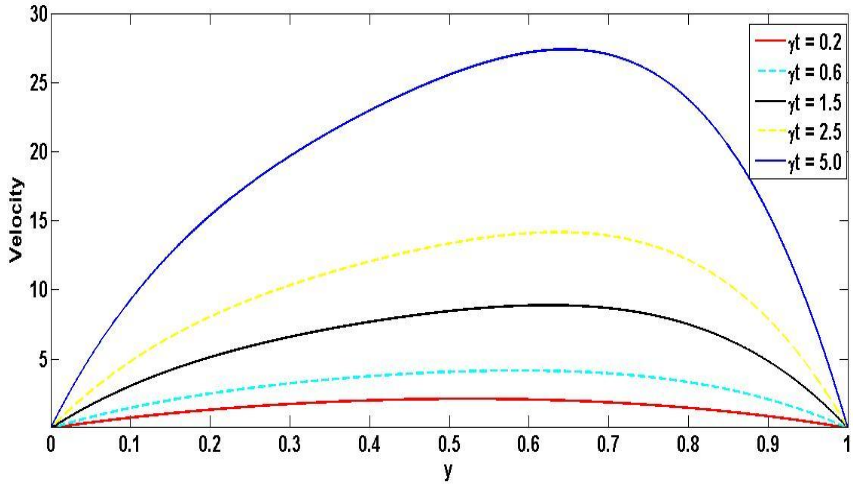

Figure 4.5: Velocity profile for different values of γ_t

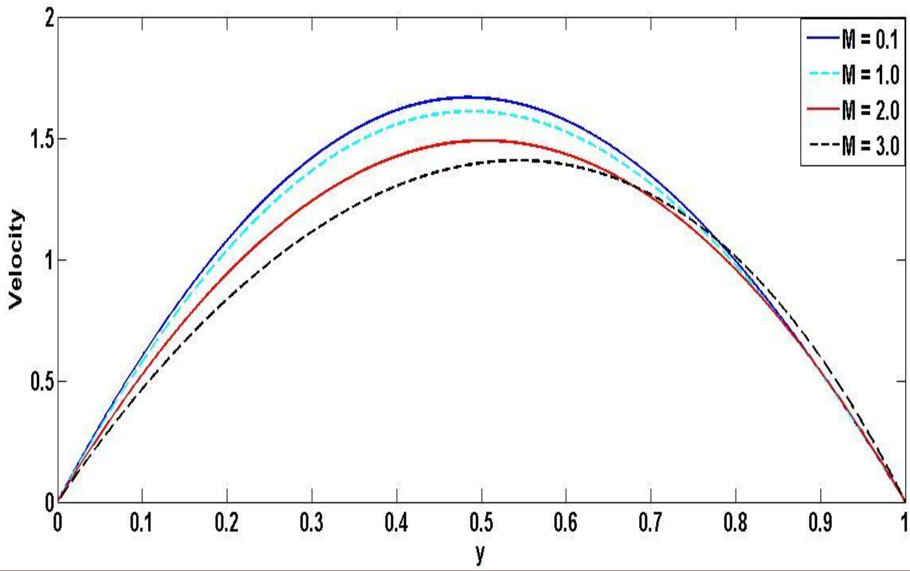

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Figure 4.6: Influence of Magnetic number on Velocity of the fluid.

Figure 4.7 displayed the effect of suction/injection on velocity, and it can be deduce from the graph that the velocity of the fluid increases in terms of suction and opposite behavior were observed in case of injection as illustrated in the figure.

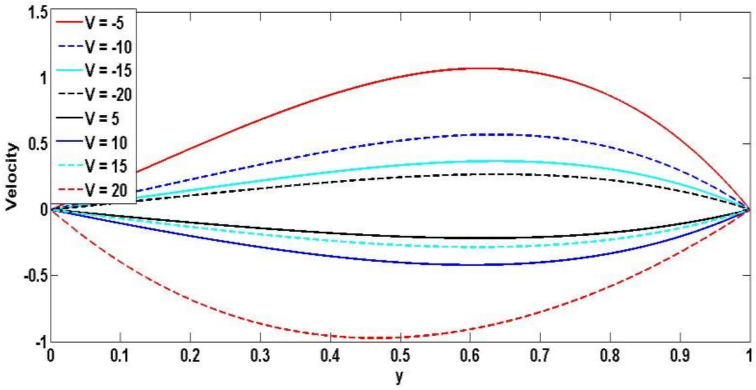

Figure 4.7: the influence of suction/injection on the velocity of the fluid.

The dependence of the rate of heat transfer on Frank-Kamenetskii parameter for different values of γ_t has been displayed in figure 4.8, and it reveals that the rate of heat transfer increases with increase in the value of γ_t at the lower and upper plates as can be seen in 4.8(a) and 4.8(b). Figure 4.9 reveals the influence of symmetric wall concentration γ_c on the rate of mass transfer, as can be seen from 4.9(a) the rate of mass transfer increases by increasing the value of γ_c and the same behavior also observed at the upper plate of the reaction, see 4.9(b). The effect of Darcy number Da on skin friction is shown in figure 4.10 and it reveals that the skin friction rate increases with increase in the value of Da at the lower plate as illustrated in 4.10(a) and increases also at the upper plate check 4.10(b). Figure 4.11 and 4.12 indicates the influences of magnetic number (M) and Prandtl number (Pr) respectively on Skin Friction, it can be deduce from 4.11(a) that the Skin friction rate decreases as the value of M increases at the lower plate $(y=0)$ and decreases also at the upper plate $(y=1)$, see 4.11(b), it is also observed that the Skin Friction rate increases with increase in the value of Pr at the lower and upper plate until it reaches the steady state as illustrated in figure 4.12(a) and (b).

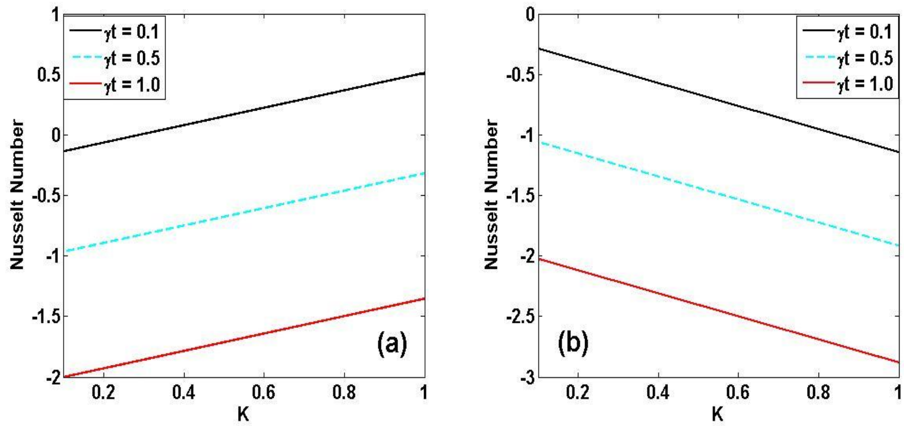

Figure 4.8: influence of γ_t on the rate of heat transfer.

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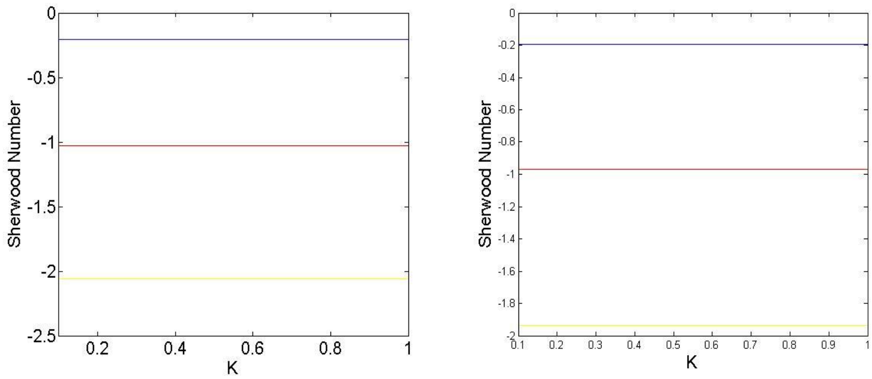

Figure 4.9: influence of γ_c on Sherwood number.

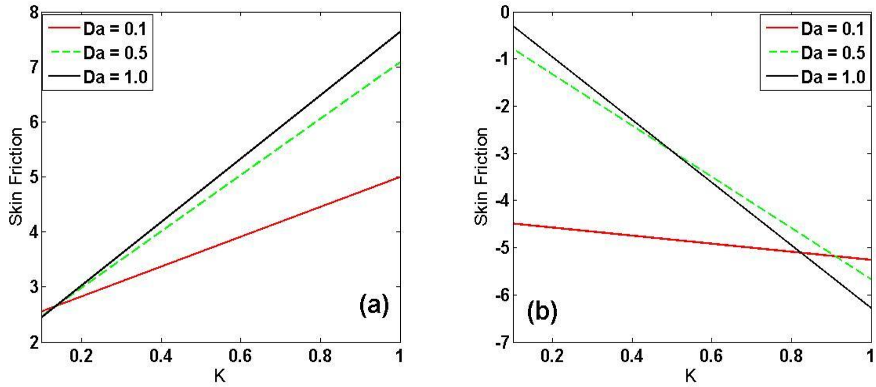

Figure 4.10: The effect of Darcy number on Skin Friction.

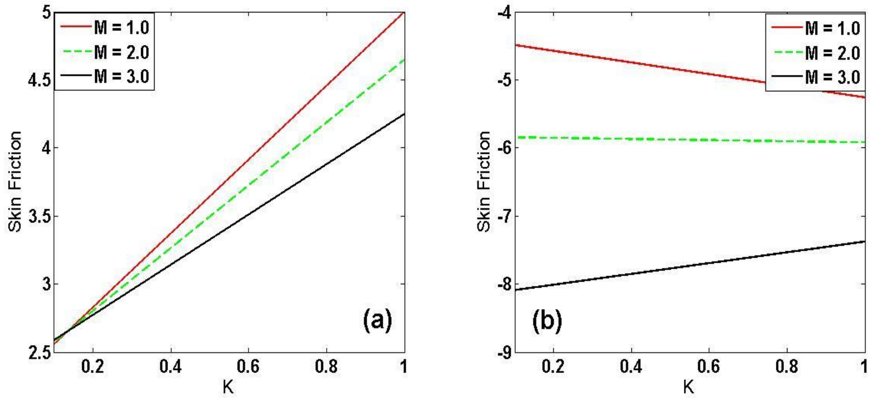

Figure 4.11: effect of magnetic number M on the Skin Friction rate.

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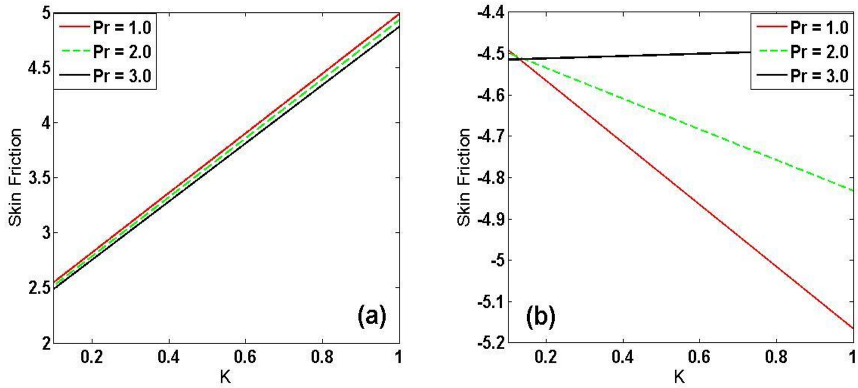

Figure 4.12: Showing the influence of Prandtl number (Pr) on the Skin Friction.

V. SUMMARY AND CONCLUSION

The effect of Suction/Injection on mixed convection flow of an exothermic fluid through a vertical porous channel were studied, Differential Transformation Method were utilized and obtained the solutions of the governing equations. The influences of magnetic number (M), Suction/Injection parameter and some other pertinent parameters on the fluid concentration, temperature, velocity, rate of heat transfer and skin friction were considered, analyzed and represented graphically and arrive at the following conclusions from the obtained result.

(i) That the velocity of the fluid increases interms of Suction, while Injection decelarates velocity of the fluid.

(ii) The temperature of the fluid increases by increasing the value of Frank-Kamenetskii number.

(iii) The presence of Schmidt number (Sc) accelarate the fluid concentration whenever the value is increase until the steady state is attained.

(iv) The concentration of the fluid increases at the lower plate when the value of the symmetric wall concentration is increase and opposite behavior at the upper plate of the cahnnel.

(v) While as the value of magnetic number (M) increases, its enhances the rate of skin friction of the fluid.

REFERENCES

Afify, A.A. (2004). The effect of radiation on free convective flow and mass transfer past a vertical isothermal cone surface with chemical reaction in the presence of a transverse magnetic field. Canadian journal of physics, 82(6), 447-458.

Ahmad, S.K., Bello, Y, and Hamza, M.M. (2017). Effect of Mass Transfer on Mixed Convection Flow of an Exothermic Fluid in a Vertical Channel. Asian Journal of Mathematics and Computer Research, 245-255.

Campbell, A.N., Cardoso, S.S.S., and Hayhurst, A.N. (2007). A comparison of measured temperatures with those calculated numerically and analytically for an exothermic chemical reaction inside a spherical batch reactor with natural convection. Chemical engineering science, 62(11), 3068-3082.

Das, K, and Jana, S. (2010). Heat and mass transfer effects on unsteady MHD free convection flow near a moving vertical plate in porous medium. Bull. Soc. Math. Banja Luka, 17(10), 15-32.

Hamza, M.M. (2016). Free convection slip flow of an exothermic fluid in a convectively heated vertical channel. Ain Shams Engineering Journal.

Hayat, T.A., Bilal M, Shehzad, S.A., and Alsaedi, A. (2015). Mixed Convection Flow of Casson Nanofluid over a Stretching Sheet with Convectively Heated Chemical Reaction and Heat Source/Sink. Journal of Applied Fluid Mechanics, 8(4).

Jha, B.K, Aina, B., and Ajiya, A.T. (2015). MHD natural convection flow in a vertical parallel plate microchannel. Ain Shams Engineering Journal, 6(1), 289-295.

Jha, B.K, Aina, B., and Joseph, S.B. (2014). Natural convection flow in a vertical microchannel with suction/injection. Proceedings of the Institution of Mechanical Engineers, Part E: Journal of Process Mechanical Engineering, 228(3), 171-180.

Khanafer, K.M., and Chamkha, A.J. (1999). Mixed convection flow in a lid-driven enclosure filled with a fluid-saturated porous medium. International Journal of Heat and Mass Transfer, 42(13), 2465-2481.

Makinde, O.D. (2008). Thermal criticality in viscous reactive flows through channels with a sliding wall: An exploitation of the Hermite– Padé approximation method. Mathematical and Computer Modelling, 47(3-4), 312-317.
Minto B.J., Ingham D.B., and Pop, I. (1998). Free convection driven by an exothermic reaction on a vertical surface embedded in porous media. International journal of heat and mass transfer, 41(1), 11-23.
Saleh, H, Hashim, I, and Basriati, S. (2013). Flow reversal of fully developed mixed convection in a vertical channel with chemical reaction. International Journal of Chemical Engineering, 2013.
Uwanta, I.J., and Hamza, M.M. (2014). Unsteady flow of reactive viscous, heat generating/absorbing fluid with Soret and variable thermal conductivity. International Journal of Chemical Engineering, 2014.
Weng, H.C. (2005). Natural convection in a vertical microchannel. Journal of Heat Transfer, 127(9), 1053-1056.
Yih, K.A. (1997). The effect of transpiration on coupled heat and mass transfer in mixed convection over a vertical plate embedded in a saturated porous medium. International Communications in Heat and Mass Transfer, 24(2), 265-275.

APPENDIXES

$a1 = -HP_rV_0 - Kr - NJS_cV_0 \qquad a2 = \gamma - \lambda\left[H + NJ\right] \qquad a3 = \gamma - \lambda\left[\gamma_t + N\gamma_c\right]$

$$H = \frac{Kr\left(P_r^2V_0^2 - 5P_rV_0 + 17\right) - 48\gamma_t}{24 - P_r^3V_0^3 + 4P_r^2V_0^2 - 12P_rV_0} \qquad J = \frac{-48\gamma_c}{24 - 12S_cV_0 + S_c^2V_0^2 - S_c^3V_0^3}$$

$$Z = \frac{12a3 + 4a2 - 4V_0a3 + \gamma - \lambda a1 - V_0a2 + V_0^2a3 + \alpha a3}{12V_0 - 24 - 4V_0^2 - 4\alpha + V_0^3 + 2\alpha V_0}$$

$$\alpha = M^2 + \frac{1}{Da} \qquad S_c = \frac{\nu}{D} \qquad Da = \frac{k'}{L^2}$$

$$M^2 = \frac{\sigma B_0^2 L^2}{\rho\nu}$$