International Journal of Advances in Engineering and Management (IJAEM)
Volume 4, Issue 3 Mar 2022, pp: 923-927 www.ijaem.net ISSN: 2395-5252

Effect of Variable Viscosity and Applied Magnetic Field over a Steady Laminar Falkner-Skan Wedge Flow

Ashwini G
Department of Mathematics, Government College for Women (Autonomous), Mandya, Karnataka.

Submitted: 10-03-2022 Revised: 21-03-2022 Accepted: 25-03-2022

ABSTRACT: In this paper, Falkner-Skan wedge flow and the heat transfer of an incompressible electrically conducting fluid is analyzed in the presence of variable viscosity and applied magnetic field. The system of partial differential governing equations are reduced to a system of non-linear ordinary differential equations by similarity transformations and then solved by an implicit finite difference scheme along with quasilinearization technique. Numerical calculations are carried out for different values of dimensionless parameters, along with wedge angle ranging from 0.0 to 1.0 for water (Pr = 7.0). It is observed that the skin friction co-efficient at the surface increases by increasing of the magnetic parameter. Where as it decreases with increase of variable viscosity parameter. Further the heat transfer co-efficient increases with the increase of both magnetic field as well as variable viscosity. However, both velocity and thermal fields are appreciably affected by temperature dependent viscosity.

KEYWORDS: Variable viscosity, Falkner-Skan flow, Magnetic field

I. INTRODUCTION

Historically, the steady laminar flow passing a fixed wedge was first analyzed in the early 1930's by Falkner and Skan [1] to illustrate the applications of Ludwig Prandtl's boundary layer theory.This pioneering work created interest among researchers in the field of fluid dynamics. Later, several investigators [2 – 4], have studied the classical Falkner-Skan problem employing various analytical and numerical methods for different flow as well as heat transfer situations. In all these studies, the fluid properties were assumed to be constant. However, in many engineering applications this assumption is not obeyed. As such, we have to consider such problems by assuming variable viscosity, since this property of the fluid varies significantly when large temperature difference exists. In such situations, both momentum and energy equations are coupled and each equation affects the other. The first attempt to solve the Falkner-Skan problem including the variation of viscosity with temperature was made by Herwing and Wickern [5]. Hossain et al. [6] studied the flow of a fluid with variable viscosity past a permeable wedge with uniform surface heat flux. A Pantokratoras [7] presented Falkner-Skan flow with constant wall temperature and variable viscosity. Mourad F. Dimian [8] considered the effect of the magnetic field on forced convection flow along a wedge with variable viscosity. M. A. Seddeek.et.al [9] studied similarity solutions for a steady MHDfalkner-skan flow and heat transfer over a wedge considering the effects of variable viscosity and thermal conductivity. The objective of the present paper is to present the solutions of Falkner-Skan wedge flow with variable viscosity in the presence of an applied magnetic field for water (Pr = 7.0).

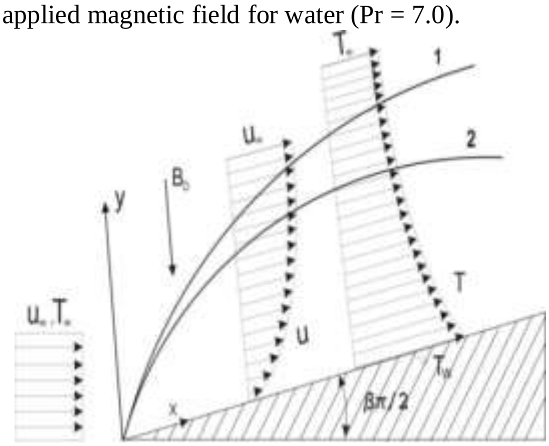


Fig.1. Velocity and thermal boundary layers for the Falkner-Skan wedge flow, where 1 and 2 represents thermal and momentum boundary layers respectively.

II. GOVERNING EQUATIONS

Consider the flow of an incompressible viscous electrically conducting fluid over a wedge, as shown in Fig.1. The temperature of the wall T_w is uniform and constant and is greater than the free stream temperature. It is assumed that the flow in the laminar boundary layer is two-dimensional with a constant uniform velocity u_∞. A transverse magnetic field B_0 is applied in $y-$direction normal to the body surface and it is assumed that the magnetic Reynolds is small, so that the induced magnetic field can be neglected. The fluid is assumed to have constant physical properties except for the fluid viscosity (μ) which is assumed to be an inverse linear function of the temperature (T) (see Lai and Kulacki [10]), viz.,

$$\frac{1}{\mu}=\frac{1}{\mu_\infty}\left[1+\gamma(T-T_\infty)\right]; \qquad \frac{1}{\mu}=a(T-T_e)$$

where $a=\dfrac{\gamma}{\mu_\infty}$; $T_e=T_\infty-\dfrac{1}{\gamma}$ (1)

Here, both $'a'$ and T_e are constants and their values depend on the reference state and the thermal property of the fluid i.e., (γ). In general, $a>0$ for liquids and $a<0$ for gases.Under the aforesaid assumptions, the boundary layer equations governing the steady, MHD Falkner-Skan wedge flow are given by

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad (2)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=U\frac{\partial U}{\partial x}+\frac{1}{\rho}\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right)-\frac{\sigma B_0^2}{\rho}(u-U) \qquad (3)$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2} \qquad (4)$$

where u and v are respectively, velocity components in x and y -directions of the flow, U is the inviscid flow velocity at the edge of the boundary layer and is a function of x; α is the thermal diffusivity of the fluid; T is the temperature in the vicinity of the wedge; μ, σ, ρ are respectively, dynamic viscosity, electrical conduction, and density

The boundary conditions are given by

$$\begin{aligned}&at\ \ y=0: u=v=0\ \ and\ \ T=T_w\\ &as\ y\to\infty: u\to U(x)=u_\infty (x/L)^m\ \ and\ \ T=T_\infty \qquad (5)\\ &at\ \ x=0: u=u_\infty\ \ and\ \ T=T_\infty\end{aligned}$$

where L is the length of the wedge, m is the Falkner-Skan power-law parameter and x is measured from the tip of the wedge. The subscript ∞ denotes conditions at infinity.

Introducing the transformations

$$u=\frac{\partial\psi}{\partial y}; \quad v=-\frac{\partial\psi}{\partial x};$$

$$f(\eta)=\sqrt{\frac{1+m}{2}\ \frac{L^m}{\upsilon u_\infty}}\ \left(\frac{\psi}{x^{(1+m)/2}}\right)$$

$$\eta=\sqrt{\frac{1+m}{2}\ \frac{u_\infty}{\upsilon L^m}}\ \left(\frac{y}{x^{(1-m)/2}}\right);$$

$$G(\eta)=\frac{T-T_\infty}{T_w-T_\infty} \qquad (6)$$

to Eqns. (1) - (3) we observe that continuity Eqn.(2) is identically satisfied and Eqns.(3) and (4) are respectively transformed to:

$$F''+\left(1-\frac{G}{G_e}\right)fF'+\beta\left(1-\frac{G}{G_e}\right)[1-F^2]+M\left(1-\frac{G}{G_e}\right)[1-F]+\frac{G'F'}{(G_e-G)}=0 \qquad (7)$$

$$Pr^{-1}G''+fG'=0 \qquad (8)$$

where

$$\frac{u}{U}=f'=F;$$

$$v=-\sqrt{\frac{2}{1+m}\ \frac{\upsilon u_\infty}{L^m}}\ x^{(m-1)/2}\left\{\frac{m+1}{2}f+\eta\frac{m-1}{2}F\right\}$$

$$;\quad v=\frac{\mu}{\mu_\infty};\ \Pr=\frac{\upsilon}{\alpha};\ \mathrm{Re}_L=\frac{u_\infty L}{\upsilon};$$

$$M=\frac{2L\sigma B_0^2}{\rho u_\infty(m+1)}\ ;\ G_e=\frac{T_e-T_\infty}{T_w-T_\infty} \qquad (9)$$

The transformed boundary conditions are:

$$\begin{aligned}&F=0; \quad G=1 \quad at \quad \eta=0\\ &F=1; \quad G=0 \quad as \quad \eta\to\infty\end{aligned} \qquad (10)$$

Here, ψ and f are dimensional and dimensionless stream functions respectively; F is velocity function; M is dimensionless magnetic parameter; Re_L is the local Reynolds number, $\Pr$ is the Prandtl number; η is the transformed coordinate. Here prime (')denotes derivate with respect to η

We also note that in Eqn.(6), the parameter β and m are related through the expression $\beta=2m/(1+m)$

The dimensionless temperature G and viscosity

International Journal of Advances in Engineering and Management (IJAEM)
Volume 4, Issue 3 Mar 2022, pp: 923-927 www.ijaem.net ISSN: 2395-5252

ratio $\frac{\mu}{\mu_\infty}$ are re-defined as follows:

$$G = \frac{T - T_e}{T_w - T_\infty} + G_e \text{ and } \frac{\mu}{\mu_\infty} = \frac{G_e}{G_e - G} \quad (11)$$

where G_e is constant, called viscosity variation parameter, which is defined by

$$G_e = \frac{T_e - T_\infty}{T_w - T_\infty} = \frac{-1}{\gamma(T_w - T_\infty)} = \text{constant} \quad (12)$$

And its value is determined by viscosity characteristics of the fluid under consideration and operating temperature difference $\Delta T = T_w - T_\infty$.

It may be remarked here that, if G_e is large (i.e., $G_e \to \infty$), the effect of variable viscosity can be neglected. On the other hand, for a smaller value of G_e, either the fluid viscosity changes markedly with temperature or operating temperature difference is high. In either case, the variable viscosity effect is expected to become very significant. Also, it may be noted here that, liquid viscosity varies differently with temperature than that of gas and therefore, it is important to note that $G_e <$ 0 for liquids and G_e >0 for gases when the temperature difference ΔT is positive.It is worth mentioning here that when $\gamma \to 0$ i.e., $\mu = \mu_\infty$, then $G_e \to \infty$ and Eqns. (7) and (8) reduces to

$$F'' + f F' + \beta(1 - F^2) + M(1 - F) = 0$$

$$\Pr^{-1} G'' + f G' = 0$$

which are exactly same as those of Jayakumar.et.al.[11], who have obtained a finite difference solution of MHD Falkner-Skan wedge flow, for constant fluid properties in the absence of viscous dissipation for air(Pr = 0.7).

The skin friction and, heat transfer coefficient in the form of Nusselt number can be expressed as

$$C_f(\mathrm{Re}_L)^{1/2} = \sqrt{1/(2-\beta)}\,(G_e/(G_e - 1))(F')_{\eta=0} \quad (15)$$

$$Nu(\mathrm{Re}_L)^{-1/2} = -\sqrt{1/(2-\beta)}\,(G')_{\eta=0}$$

III. RESULTS AND DISCUSSION

In order to analyze the results, numerical computations have been carried out by using implicit finite difference scheme along withquasilinearization technique [12] for solving the partial differential equations (7) and (8) along with boundary conditions (10). The skin friction and heat transfer coefficients has been obtained for various values of variable viscosity [Ge] and magnetic parameter [M]. In order to validate the accuracy of the numerical method used, our skin friction coefficient and heat transfer coefficient results are compared with those of Jayakumar.et.al [11] for air (Pr = 0.7)by solving Equations (13) and (14) [See Fig.2 (a) and 2(b)]. It is easy to observe that the computed results are in good agreement, with those of [11]. In subsequent paragraphs, we present our numerical results for water(Pr = 7.0).

Fig.3. displays the effect of magnetic field [M] and variable viscosity parameter [Ge] on the skin friction coefficient [$C_f(\mathrm{Re}_L)^{1/2}$] at various values of wedge angle parameter (β).

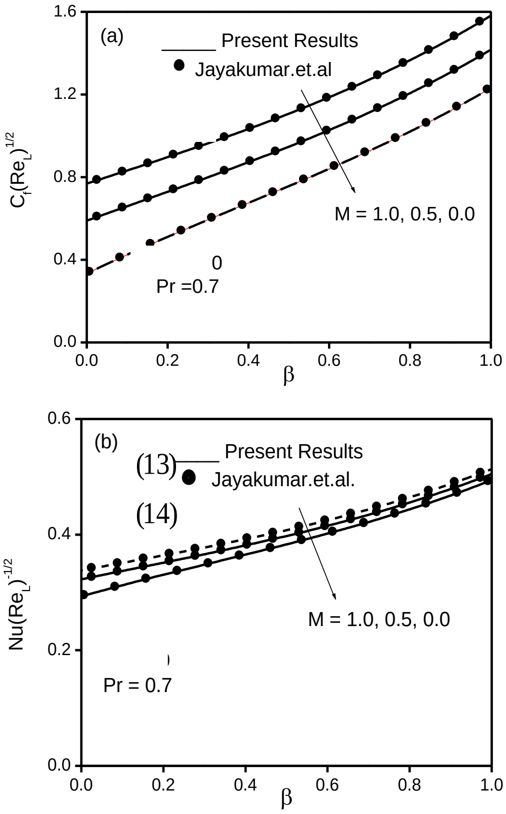


Fig.2. Comparison of (a) skin friction coefficient and (b) heat transfer coefficientwith those of Jayakumar.et.al.[11]

The skin friction coefficient increases as magnetic field increases, whereas it decreases with increase of variable viscosity parameter. Further, the percentage of increase of skin friction coefficient is about 30.14 % for an increase Min the range $0.0 \le M \le 1.0$ whereas the percentage of its decrease is about 8.13 % for an increase of Gein the range and -1.5 ≤ Ge ≤ -3.5 at β = 0.6.

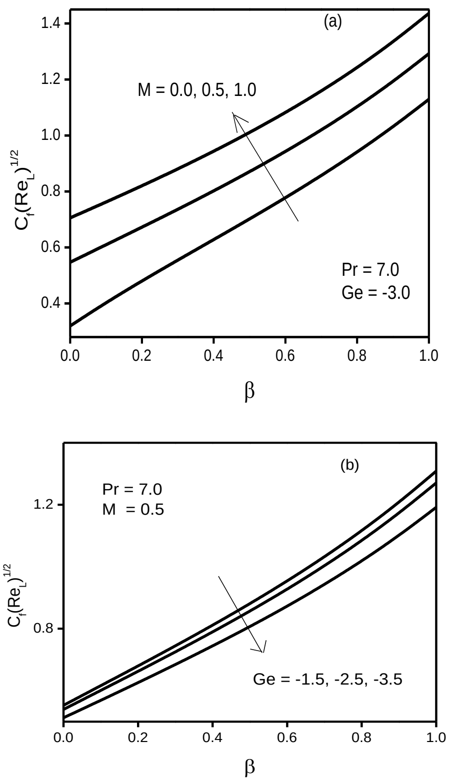
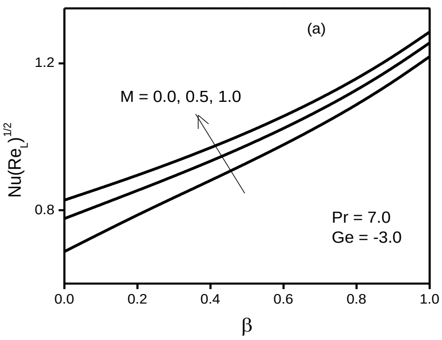


Fig.3. Effect of (a) magnetic field [M] and (b) variable viscosity [Ge] onskin friction coefficient

Fig.4 shows the effect of magnetic filed [M]and variable viscosity parameter [Ge] on the heat transfer coefficient $[N_u(Re_L)^{-1/2}]$ at various values of β. It is clear that the heat transfer coefficient increases with the increase of both magnetic field and variable viscosity. Indeed, the percentage of increase of heat transfer coefficient is about 7.77% for an increase of M in the range 0.0 ≤ M≤ 1.0 and the percentage of its increaseis about 1.56% for an increase of Ge in the range -1.5 ≤ Ge ≤ -3.5 at β = 0.6



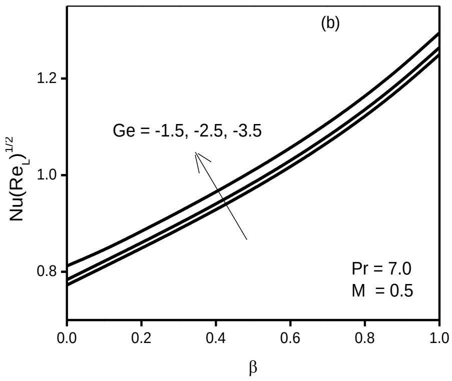


Fig.4. Effect of (a) magnetic field [M] and (b) variable viscosity [Ge] on heattransfer coefficient

The effect of magnetic parameter [M] on velocity [F] and temperature [G] profiles is shown in Fig.5. It is seen that the velocity gradients increases and temperature gradient decreaseswith the increase of magnetic parameter. This results in the decrease of both momentum and thermal boundary layer thickness.

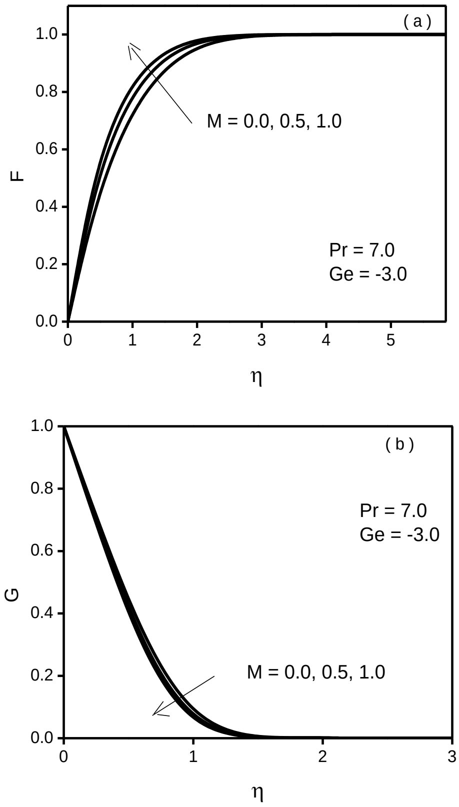


Fig.5. Effect of magnetic field [M] on (a) velocity and (b) temperature profiles.

IV.CONCLUSIONS

In this paper, we have discussedthe steady Falkner-Skan flow and heat transfer over a wedge by adopting the effects of variable viscosity and magnetic field. Numerical results showthat the skin friction coefficient increases as magnetic parameter increases whereas it decreases when viscosity variable increases. On the other hand, heat transfer coefficient increases with the increase of both magnetic field and variableviscosity parameter. This results in the decline of both momentum and thermal boundary layer thicknesseswhich in turn increases the velocity and temperature of the fluid.

ACKNOWLEDGEMENTS
The author is thankful to the reviewer for their valuable comments and suggestions.

REFERENCES

[1]. Falkner,V.M;Skan,S.W., 1931,"Some approximate solutions of the boundary layer equation," Philos. Mag.,**12,**pp.865-896.

[2]. Hartree.D.R.,1937, "On an equation occurring in Falkner and Skan's approximate treatment of the equations of the boundary layer," Part II, Proc. Cambridge Philos. Soc., **33,** pp.223 – 239.

[3]. Lin.H.T;Lin.K.L.,1987, "Similarity solutions for laminar forced convection heat transfer from wedges to fluids of any Prandtl number," Int. J. Heat Mass Transfer,**30,** pp.1111 – 1118.

[4]. Eswara,A.T.,1998, "A parametric differentiated finite-difference study of Falkner-Skan problem," Bull. Cal. Math. Soc., **90**, pp.191-196.

[5]. Herwig.H;Wickern.G., 1986, "The effect of variable properties on laminar boundary layer flow," Warme and Stoffubertragung, **20,** pp.47 – 57.

[6]. Houssain.M.A; Munir.M.Z; Hafiz.M.S; Takhar.H.S., 2000, "Flow of viscous incompressible fluid with temperature – dependent viscosity past a permeable wedge with uniforms surface heat flux," Heat Mass Transfer, **36,**pp.333 – 341.

[7]. Pantokratoras.A.,2006, "The Falkner – Skan flow with constant wall temperature and variable viscosity," Int. J of Thermal Sciences, **45**, pp.378 – 389.

[8]. Mourad F. Dimian., 2004,"Effect of the Magnetic Field on Forced Convection Flow Along a Wedge with Variable Viscosity," Mechanics and Mechanical Engineering, **7**, No. 1, pp.107-120.

[9]. M. A. Seddeek; A. A. Afify; A. M. Al-Hanaya., 2009, "Similarity Solutions for a Steady MHD Falkner-Skan Flow and Heat Transfer over a Wedge Considering the Effects of Variable Viscosity and Thermal Conductivity," Applications and Applied Mathematics: An International Journal (AAM), **4**, Issue 2, pp. 301 – 313.

[10]. F.C.Lai;F.A.Kulacki; 1990,"The effect of variable viscosity on convective heat transfer along a vertical surface in a saturated porous medium," Int. J. Heat Mass Transfer, **33,** pp.1028-1031.

[11]. Jayakumar.K.R; Srinivasa.A.H;Eswara.A.T.,2008, "A finite difference solution of MHD Falkner – Skan wedge flow," Proc. of the 35thNational conference, FMFP, pp.128 – 134.

[12]. K.Inouye; A.Tate., 1974, "Finite difference version of quasilinearization applied to boundary layer equations," A.I.A.A.J., **12,** pp.558-560.