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Elastodynamics of Stiffened Plate Subjected to Moving Multi Masses

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ABSTRACT: The dynamics of a stiffened elastic system subjected to multi-moving masses have been solved. The study attempts to investigate the dynamic behaviour given a singularity functional in the system in the context of applied elasticity following the influence of distributed moving masses. The present solutions are derived using Navier's method and Galerkin's approach. Using Navier's solution, the numerical results are presented if a moderately thick parent plate is considered. The dynamic system is solved by using Runge Kutta fourth-order method. The study reveals the effects of singularity (stiffener) and velocity on the behaviour of the system.

KEYWORDS: Dynamics, moving mass moving force system, stiffened plates, singularity function.

I. INTRODUCTION

The dynamic behaviour of a continuous elastic system under the influence of a moving system is a subject of considerable engineering importance, from structural to mechanical to marine and aerospace engineering. The dynamic behaviour of a beam subjected to moving load or moving masses have been extensively studied in connection with bridges, guideways, overhead cranes, rails, roadways, runways, tunnels and pipelines, other machining processes, and guide-way systems [1-10] and [12-15]; the specifics chosen in this article are motivated by bridge transportation, this research domain is still very active as mathematicians and engineers are seriously engaged in developing a deeper understanding of the structures behaviour to moving mass, moving force system.

Studies so far have been restrained to homogenous elastic systems, for which mathematical representations and complications are rather simplified. In the case of stiffened plates under moving masses, studies are few, especially for bridge-like structures. Application of finite element approaches such as in [1] showed that high computational effort and time is required to solve such problem. The fundamental mathematical complexity encountered in this problem lies in the discretised coupled system to the varying time function. This step function represents the interplay of inertial forces due to moving mass inertia.

A vast majority of the analytical studies dealing with the moving load moving mass problems utilized the Fourier transformation method to solve the governing differential equations. To mention a few, Stanisic et. al. [12] used the Fourier transform technique to develop a coupled resultant differential equation to solve for plate system, he applied definite cosine series to deal with Dirac function that describes the motion of moving masses, although his approach is constrained by assumptions of thin plate theory. Gbadeyin and Oni [4] further solved the problem with variable coefficient in the resultant coupled transformed differential equation using modified Struble's method. Mofid, et. al. [13] transformed the differential equation into series of eigenfunctions as modal vibrational responses. Ho & Tham [7], Ng & Chen [14] investigated the dynamic response of single and multi-span plates subjected to a moving mass by using numerical method. In the case of stiffened plates, Chueng, et al. [2] used the method of the finite strip to analyse continuous slab girder bridge decks. His approach requires the derivation of the dynamic stiffness matrix of the stiffening rib and its coupling formulation with strips, in such a way that the thickness varies longitudinally.

The interest of this study is a semi-analytical development in the context of applied elasticity which often highlights stringent information concerning the physical system when subjected to a distributed moving mass.

II. PROBLEM FORMULATION

A rectangular plate shown in figure 1 stiffened in the direction parallel to one of the edges, is examined. The singularities in the stiffness and mass distribution of the plate has been built in applying elasticity principle described below.

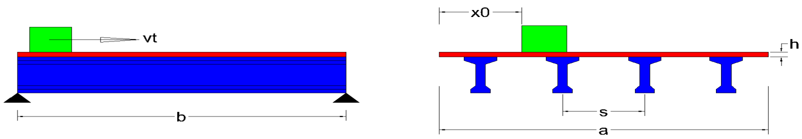

Fig. 1. Bridge Deck

The relationships between the stresses and strains and displacement fields and accounting for the stiffeners are given as

$$\begin{aligned}
&\sigma_x = \frac{E}{1-v^2}\left(\varepsilon_x + v\varepsilon_y\right),\ \sigma_y = \frac{E}{1-v^2}\left(\varepsilon_y + v\varepsilon_x\right),\\
&\sigma_y^s = E_0\varepsilon_y,\ \left(\tau_{yz},\gamma_{xz},\gamma_{xy}\right) = \frac{E}{2(1+v)}\left(\gamma_{yz},\gamma_{xz},\gamma_{xy}\right),
\end{aligned} \tag{1}$$

$$\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \varepsilon_z\\ \gamma_{xy}\\ \gamma_{yz}\\ \gamma_{xz}\end{Bmatrix} = z\begin{Bmatrix}\varepsilon_x^1\\ \varepsilon_y^1\\ 0\\ \gamma_{xy}^1\\ 0\\ 0\end{Bmatrix} + f(z)\begin{Bmatrix}\varepsilon_x^2\\ \varepsilon_y^2\\ 0\\ \gamma_{xy}^2\\ 0\\ 0\end{Bmatrix} + g(z)\begin{Bmatrix}0\\ 0\\ 0\\ 0\\ \gamma_{yz}^s\\ \gamma_{xz}^s\end{Bmatrix} = -z\begin{Bmatrix}w_{b,xx}\\ w_{b,yy}\\ 0\\ 2w_{b,xy}\\ 0\\ 0\end{Bmatrix} - f(z)\begin{Bmatrix}w_{s,xx}\\ w_{s,yy}\\ 0\\ 2w_{s,xy}\\ 0\\ 0\end{Bmatrix} + g(z)\begin{Bmatrix}0\\ 0\\ 0\\ 0\\ w_{s,y}\\ w_{s,x}\end{Bmatrix} \tag{2}$$

The function maybe a sinusoidal function according to Zenkour [19]

$$where,\ f(z) = \frac{h}{\pi}\sin\left(\frac{\pi z}{h}\right),\ \text{and}\ \ g(z) = 1 - f'(z) \tag{3}$$

Eq. (1) can better be represented as

$$\begin{aligned}
\sigma_x &= \frac{E}{1-v^2}\left\{-zw_{b,xx} - f(z)\,w_{s,xx} + v\left\{-zw_{b,yy} - f(z)\,w_{s,yy}\right\}\right\}\\
\sigma_y &= \frac{E}{1-v^2}\left\{-zw_{b,yy} - f(z)\,w_{s,yy} + v\left\{-zw_{b,xx} - f(z)\,w_{s,xx}\right\}\right\}\\
\tau_{yz} &= \frac{E}{2(1+v)}g(z)\,w_{s,y},\qquad \tau_{xz} = \frac{E}{2(1+v)}g(z)\,w_{s,x}\\
\tau_{xy} &= \frac{E}{2(1+v)}\left[-2zw_{b,xy} - 2f(z)\,w_{s,xy}\right]
\end{aligned} \tag{4}$$

Kinetics of the system are developed by integrating the biaxial stresses through the plate thickness and uniaxial stresses over the stiffeners to give

$$\begin{Bmatrix}M_x,S_x\\ M_y,S_y\\ M_{xy},S_{xy}\\ Q_x,Q_y\end{Bmatrix} = \int_{\frac{h}{2}}^{-\frac{h}{2}}\begin{Bmatrix}\sigma_x\left[z,f(z)\right]\\ \sigma_y\left[z,f(z)\right]\\ \tau_{xy}\left[z,f(z)\right]\\ g(z)\left(\tau_{xz},\tau_{yz}\right)\end{Bmatrix}dz + \begin{Bmatrix}0\\ \frac{1}{s}\int_{A_s}\sigma_y^s\left[z,f(z)\right]dA_s\\ 0\\ 0,\frac{1}{s}\int_{A_s}\tau_{yz}dA_s\end{Bmatrix} \tag{5}$$

Which becomes

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$$
\begin{aligned}
&M_x=-D_1 w_{b,xx}-F_1 w_{s,xx}-D_2 w_{b,yy}-F_2 w_{s,yy},\ S_x=-F_1 w_{b,xx}-G_1 w_{s,xx}-F_2 w_{b,yy}-G_2 w_{s,yy},\\
&M_y=-\left(D_1+\frac{E_s I_s}{s}\right)w_{b,yy}-\left[F_1+\frac{E_s h^3}{\pi^3 s}\sin\left(\frac{\pi A_s}{h}\right)-\frac{E_s h^2 A_s}{\pi^2 s}\cos\left(\frac{\pi A_s}{h}\right)\right]w_{s,yy}-D_2 w_{b,xx}-F_2 w_{s,xx},\\
&S_y=-\left[F_1+\frac{E_s h^3}{\pi^3 s}\sin\left(\frac{\pi A_s}{h}\right)-\frac{E_s h^2 A_s}{\pi^2 s}\cos\left(\frac{\pi A_s}{h}\right)\right]w_{b,yy}-\left[G_1+\frac{E_s h^2 A_s}{2\pi^2 s}-\frac{E_s h^3}{4\pi^3 s}\sin\left(\frac{2\pi A_s}{h}\right)\right]w_{s,yy}-F_2 w_{b,xx}-G_2 w_{s,xx},\\
&M_{xy}=-2D_3 w_{b,xy}-2F_3 w_{s,xy},\ S_{xy}=-2F_3 w_{b,xy}-2G_3 w_{s,xy},\ Q_x=H_3 w_{s,x},\ Q_y=\left\{H_3+\frac{1}{s}G_s\left[A_s-\frac{h}{\pi}\sin\left(\frac{\pi A_s}{h}\right)\right]\right\}w_{s,y}
\end{aligned}
\tag{6}
$$

While observing the following definitions

$$
\begin{aligned}
&(A,B,D,E,F,G)_i=\int_{-h/2}^{h/2} C_i\left(1,z,z^2,f(z),zf(z),f(z)^2\right)dz,\\
&\mathrm{H}_i=\int_{-h/2}^{h/2} C_i\ g(z)^2\,dz,\ \mathrm{i}=1,2,3\quad\text{and,}\quad C_1=\frac{E}{\left(1-v^2\right)},\ C_2=\frac{vE}{\left(1-v^2\right)},\ C_3=\frac{E}{2(1+v)}
\end{aligned}
\tag{7}
$$

Equilibrium Equation

The equations of equilibrium and boundary conditions can be derived using Hamilton's principle as:

$$
\int_0^t(\delta U-\delta W-\delta K)\,dt=0 \tag{8}
$$

And,

$$
\delta U=\int_{\Omega}\left(\mathrm{M}_x\delta\varepsilon_x^1+\mathrm{S}_x\delta\varepsilon_x^2+\mathrm{M}_y\delta\varepsilon_y^1+\mathrm{S}_y\delta\varepsilon_y^2+\mathrm{M}_{xy}\delta\gamma_{xy}^1+\mathrm{S}_{xy}\delta\varepsilon_{xy}^2+\mathrm{Q}_x\delta\gamma_{xz}^0+\mathrm{Q}_y\delta\gamma_{yz}^0\right)dA \tag{9}
$$

$$
\delta W_e=\int_{\Omega}P(x,y,t)\ \delta\left(w_b+w_s\right)dA \tag{10}
$$

$$
\delta K=\int_0^t\int_v\left[\left(-z\ddot{w}_{b,x}-f(z)\ddot{w}_{s,x}\right)\delta\left(-zw_{b,x}-f(z)w_{s,x}\right)+\left(-z\ddot{w}_{b,y}-f(z)\ddot{w}_{s,y}\right)\delta\left(-zw_{b,y}-f(z)w_{s,y}\right)+\left(\ddot{w}_b+\ddot{w}_s\right)\delta\left(w_b+w_s\right)\right]dVdt \tag{11}
$$

Integrating Eq. (8) by parts after substituting (9), (10), and (11), and putting them in terms of coefficients of $\delta u,\delta v,\delta w,$ and $\delta\varphi$ to equal zero, separately. Then, one can obtain

$$
\begin{aligned}
&\delta w_b:\quad M_{x,xx}+2M_{xy,xy}+M_{y,yy}-P=I_0\left(\ddot{w}_b+\ddot{w}_s\right)-I_2\left(\ddot{w}_{b,xx}+\ddot{w}_{b,yy}\right)-I_4\left(\ddot{w}_{s,xx}+\ddot{w}_{s,yy}\right)\\
&\delta w_s:\quad S_{x,xx}+2S_{xy,xy}+S_{y,yy}+Q_{xz,x}+Q_{yz,y}-P=I_0\left(\ddot{w}_b+\ddot{w}_s\right)-I_4\left(\ddot{w}_{b,xx}+\ddot{w}_{b,yy}\right)-I_5\left(\ddot{w}_{s,xx}+\ddot{w}_{s,yy}\right)
\end{aligned}
\tag{12}
$$

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where

$$I_i = \int_0^t \rho\left\{1, z, z^2, f(z), zf(z), f(z)^2\right\} dz, \quad \text{for } i = 0,1,2,3,4,5. \tag{13}$$

After some re-arrangements,

$$\begin{gathered}
-D_1 w_{b,xxxx} - 2(D_2 + 2D_3) w_{b,xxyy} - \left(D_1 + \frac{E_s I_s}{s}\right) w_{b,yyyy} - F_1 w_{s,xxxx} - 2(F_2 + 2F_3) w_{s,xxyy} - \left[F_1 + \frac{E_s h^3}{\pi^3 s}\sin\left(\frac{\pi A_s}{h}\right) - \frac{E_s h^2 A_s}{\pi^2 s}\cos\left(\frac{\pi A_s}{h}\right)\right] w_{s,yyyy} \\
- P = I_0(\ddot{w}_b + \ddot{w}_s) - I_2(\ddot{w}_{b,xx} + \ddot{w}_{b,yy}) - I_4(\ddot{w}_{s,xx} + \ddot{w}_{s,yy}) \\
-F_1 w_{b,xxxx} - 2(F_2 + 2F_3) w_{b,xxyy} - \left[F_1 + \frac{E_s h^3}{\pi^3 s}\sin\left(\frac{\pi A_s}{h}\right) - \frac{E_s h^2 A_s}{\pi^2 s}\cos\left(\frac{\pi A_s}{h}\right)\right] w_{b,yyyy} - G_1 w_{s,xxxx} - 2(G_2 + 2G_3) w_{s,xxyy} \\
-\left[G_1 + \frac{E_s h^2 A_s}{2\pi^2 s} - \frac{E_s h^3}{4\pi^3 s}\sin\left(\frac{2\pi A_s}{h}\right)\right] w_{s,yyyy} + H_3 w_{s,xx} + \left\{H_3 + \frac{1}{s}G_s\left[A_s - \frac{h}{\pi}\sin\left(\frac{\pi A_s}{h}\right)\right]\right\} w_{s,yy} - P \\
= I_0(\ddot{w}_b + \ddot{w}_s) - I_4(\ddot{w}_{b,xx} + \ddot{w}_{b,yy}) - I_5(\ddot{w}_{s,xx} + \ddot{w}_{s,yy})
\end{gathered} \tag{14}$$

Eq. (14) are used to investigate the vibration and dynamic stability of stiffened plates. The moving-force problem is described below, with associated convective terms as approximations to the moving mass system

$$P(x,y,t) = \left| \begin{array}{l} moving\ load:\ P_0\ \left[H(x - x_0) - H(x - x_0 - \mathrm{c})\right]\delta(\mathrm{y} - \mathrm{vt}) \\ moving\ mass:\ M\left[g - \left(w_{,tt} + 2vw_{,yt} + v^2 w_{,yy}\right)\right] \times \left[H(x - x_0) - H(x - x_0 - \mathrm{c})\right]\delta(\mathrm{y} - \mathrm{vt}) \end{array} \right. \tag{15}$$

III. CLOSE-FORM SOLUTION

The simply- supported boundary conditions are:

$$w_b = w_s = N_y = M_y = S_y = 0, \quad at \ \ y = 0, b \tag{16}$$

The free edge boundary conditions are:

$$N_x = N_{xy} = M_x = \mathrm{S}_x = M_{x,x} = \mathrm{S}_{x,x} = 2M_{xy,y} = 2\mathrm{S}_{xy,y} = Q_x = 0, \quad at \quad x = 0, a \tag{17}$$

The following approximate assumptions applies to solve Eq. (14) with the consideration of the boundary conditions in Eq. (16) and (17)

$$\begin{Bmatrix} \mathrm{w}(x,y,t) \\ \varphi(x,y,t) \end{Bmatrix} = \begin{Bmatrix} W(t) \\ \Psi(t) \end{Bmatrix} X_m(\alpha_m x) Y_n(\beta_n y) \tag{18}$$

According to sobhy [17] and Hadji & Avcar [5],

$$\begin{gathered} X_m(\alpha_m x) = \cos^2(\alpha_m x)\left[\sin^2(\alpha_m x) + 1\right], \ Y_n(\beta_n y) = \sin\beta_n y, \\ \text{and} \ \ \alpha_m = \frac{m\pi}{a}, \ \ \beta_n = \frac{n\pi}{b} \end{gathered} \tag{19}$$

Introducing eq. (19) into (14) and applying Galerkin method becomes

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$$\left[-D_1a_{11}-2\left(D_2+2D_3\right)a_{12}-\left(D_1+\frac{E_sI_s}{s}\right)a_{13}\right]W+\left\{-F_1a_{11}-2\left(F_2+2F_3\right)a_{12}-\left[F_1+\frac{E_sh^3}{\pi^3 s}\sin\left(\frac{\pi A_s}{h}\right)-\frac{E_sh^2A_s}{\pi^2 s}\cos\left(\frac{\pi A_s}{h}\right)\right]a_{13}\right\}\Psi$$
$$-P=\left[I_0a_{14}-I_2\left(a_{15}+a_{17}\right)\right]\ddot{W}+\left[I_0a_{14}-I_4\left(a_{15}+a_{17}\right)\right]\ddot{\Psi}$$

$$\left\{-F_1a_{11}-2\left(F_2+2F_3\right)a_{12}-\left[F_1+\frac{E_sh^3}{\pi^3 s}\sin\left(\frac{\pi A_s}{h}\right)-\frac{E_sh^2A_s}{\pi^2 s}\cos\left(\frac{\pi A_s}{h}\right)\right]a_{13}\right\}W+\left\{\begin{array}{l}-G_1a_{11}-2\left(G_2+2G_3\right)a_{12}-\left[\begin{array}{l}G_1+\dfrac{E_sh^2A_s}{2\pi^2 s}\\ -\dfrac{E_sh^3}{4\pi^3 s}\sin\left(\dfrac{2\pi A_s}{h}\right)\end{array}\right]a_{13}\\ +H_3a_{15}+\left\{H_3+\dfrac{1}{s}G_s\left[A_s-\dfrac{h}{\pi}\sin\left(\dfrac{\pi A_s}{h}\right)\right]\right\}a_{17}\end{array}\right\}\Psi \qquad (20)$$
$$-P=\left[I_0a_{14}-I_4\left(a_{15}+a_{17}\right)\right]\ddot{W}+\left[I_0a_{14}-I_5\left(a_{15}+a_{17}\right)\right]\ddot{\Psi}$$

The dynamics of the elastic system is investigated by solving eq. (20) using Runge Kutta fourth-order method with the initial conditions w(0) = dw(0)/dt = 0. The determinant of the gives the natural frequencies (ω) of the plate as

$$\begin{vmatrix}A_{11}+\omega^2M_{11} & A_{12}+\omega^2M_{12}\\ A_{21}+\omega^2M_{21} & A_{22}+\omega^2M_{22}\end{vmatrix}=0 \qquad (21)$$

The moving body force can be rewritten in the form

$$P\Rightarrow\left|\begin{array}{l}P_0a_{23}\\ M\left\{ga_{23}-\left[\ddot{w}(t)\,\mathrm{a}_{24}+2v\dot{w}(t)\,\mathrm{a}_{25}+v^2w(t)\,\mathrm{a}_{26}\right]\right\}\end{array}\right. \qquad (22)$$

Where the following definitions applies

$$\begin{aligned}&\left(a_{11},a_{12},a_{13},a_{14},a_{15},a_{17}\right)=\int_0^a\int_0^b\left(X_m^{IV}Y_n,X_m^{II}Y_n^{II},X_mY_n^{IV},X_mY_n,X_m^{II}Y_n,X_mY_n^{II}\right)X_pY_q\,dxdy\\ &\left(a_{23},\mathrm{a}_{24},\mathrm{a}_{25},\mathrm{a}_{26}\right)=\int_{x_0}^{x_0+c}\left(1,X_mY_n,X_mY_n^{I},X_mY_n^{II}\right)X_pY_q\left(\beta_q vt\right)dx\end{aligned} \qquad (23)$$

IV. NUMERICAL VERIFICATION AND DISCUSSIONS

A rectangular stiffened plate of size a = b = 10 m, and thickness h = 0.02 m. The material properties are E = 2.0 × 10^{11} Pa, ν = 0.3, and ρ = 25 kg/m^3. The material properties of the stiffeners are same as the parent plate, with geometrical configuration of AASHTO girder VI; c = 2.6 m, area A = 0.70 m^2, moment of area I = 0.3052 m^4, polar moment J = 0.0164 m^4. The present paper introduces an inert body of mass M = 31.9 × 10^3 kg with applied velocities 10, 20, 30, and 50 m/s, and x_0 = 3.75 m. The dimensionless frequency parameter used is $\Omega=\frac{\omega b^2}{h}\sqrt{\frac{\rho}{E}}$ and in good agreement with references in Table 1.

Table 1. Natural frequency of unstiffened plate with opposite-sides simply supported
Take h = 0.01 × a; b / a = 1.0; ν = 0.3

Mode (m,n)	Theory			
	Hash & Arsanjani [6]	Leissa A. [11]	Present	
			Unstiffened	Stiffened
(1,1)	9.6945	9.87	9.9103	41.3275

Figure 2 analyses the dynamic response of stiffened and unstiffened plate, at 15 m/s velocity and stiffener spacing 2.5 m. Obviously, the result shown is evident that the stiffener's decrease the vibration amplitude of the plate. This condition is with respect to the coefficient of rigidity of the stiffening ribs.

Figure 3a, 3b, 3c, and 3d analyses the dynamics response of the distributed moving masses for velocities 10, 20, 30, and 50 m/s respectively. It showed how maximum displacement is attained at shorter time with increase in velocity. It also showed that there exists increase in displacement of moving masses of convective terms for a given natural frequency and velocity when compared with that of moving loads only. In figure 3a the convective effects were minimal, it shows that these terms directly affect the amplitude of displacements in the system. In figure 3b the moving masses increased appreciably than the moving force, this was evident in the interplay in maximum amplitude between the moving mass only and the mass with convective terms. In figure 3(c-d), it shows that velocity directly increases the amplitude of displacement of the moving mass system, and must be taken note of in any system of moving body especially that with very large mass.

Figure 4a, 4b, and 4c analysis the effect of stiffener's spacing for the distributed mass responses of the plates for velocities of 10, 20, and 30 m/s respectively and stiffener spacing 2.5 m. It is showed that increase in spacing between stiffeners decreases the rigidity of the plate and hence an increase in deformation of the system when subjected to convective masses. It also showed that maximum deformation is obtained in a shorter time as velocity of body masses increases.

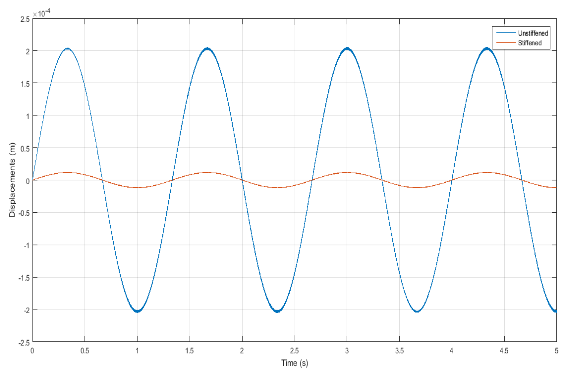

Fig. 2. Displacement relation between stiffened and unstiffened elastic system by moving force at velocity of 15 m/sec.

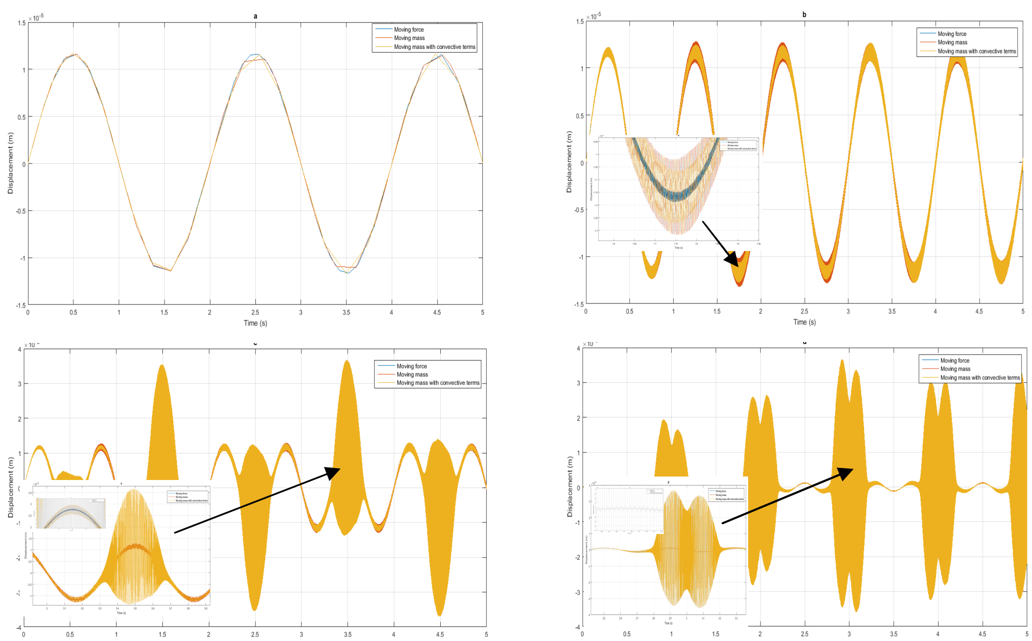

Fig. 3. Effect of convective terms with velocities (10, 20, 30, and 50 m/s) in the displacement of the system.

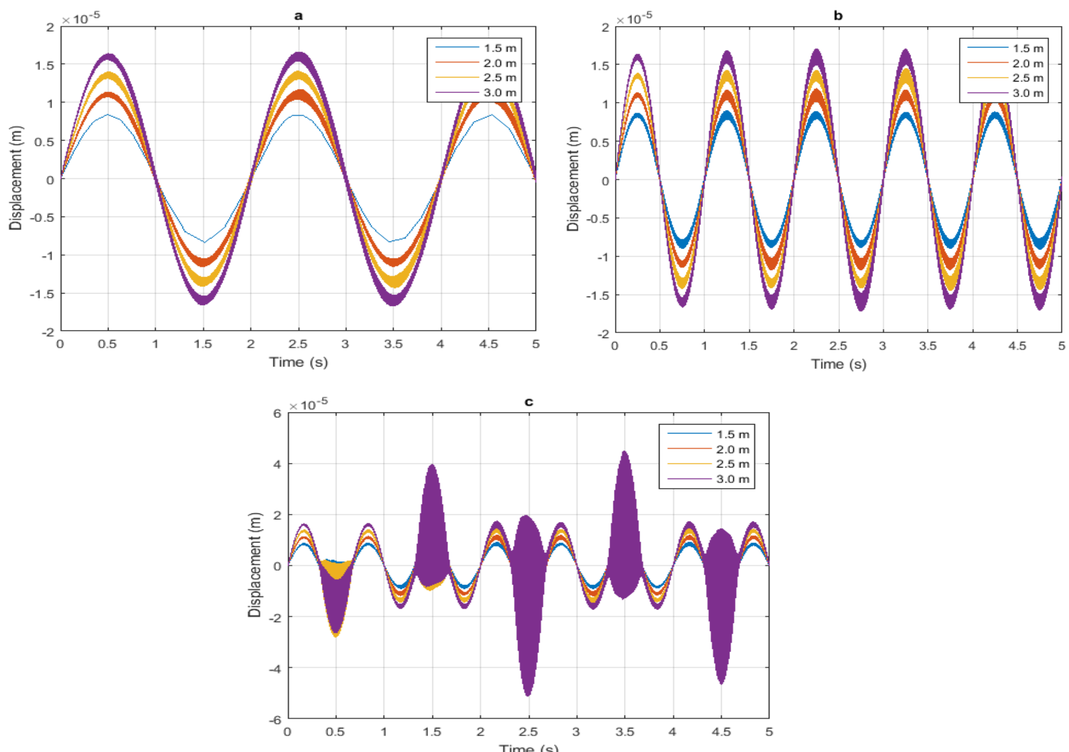

Fig. 4. Effect of stiffener spacing in the displacement relation of the system for 10, 20, and 30 m/s velocity

V. CONCLUSION

The study presented a close form solution to the stiffened plate model of bridge-like boundary conditions. It reported a solution to moving mass problem using Galerkin's approach and Runge Kutta algorithm to deal with the complexity of the discretised coupled time dependent moving masses. The effect of stiffener spacing, and constant velocity variant has been studied. The deflection profile proves to be more stable at close-range stiffeners. The results obtained authenticates the theoretical assertion that high speed condition is detrimental to solid structure like highway bridges, and the importance of mass considerations in bridge design.

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