

Evaluation of Heat Transfer in Aluminium Alloy Sand Casting

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ABSTRACT: The quality of a cast product is largely determined by the solidification process. However, poor management of the process can lead to defect in production and ultimately amount to waste. In recent times, simulation of the casting design is known to increase optimization of output and reduce defect since it can be used to obtain temperature distribution of various parts of the mold. It is therefore necessary to evaluate using finite element analysis of the heat transfer process in Aluminum alloy sand-casting as well as the experimental investigation of the rate of solidification in various depth and time interval. This study was carried out such that the casting of Aluminum alloy was produced while the parameters for pouring temperature, mould temperature and depth of mold and cast was also applied to the finite element analysis for congruency in the experimental and analytical methods of design. The heat exchange with time variation of casting was recorded from which the solidification time table and cooling curves were obtained for the determination of solidification time of the cast and mold. The analysis of the solidification process of the cubic casting was presented in two, four, six and sixteen elements simulation with temperature distribution recorded every six minutes. The simulation results were compared with the experimental reading which showed good agreement and the direction of heat transfer from the core of the molten metal to the mold. Also, the temperature of molten metal reduces drastically, close to the mold wall due to the chilling effect of the mold.

Keywords- Sand Casting, Finite Element Method, Heat Transfer, Discretization, Conduction

I. INTRODUCTION

The critical and continual need for basic and sophisticated material products has necessitated the need for castings but the unending waste as a result of poor design and implementation in the course of production calls for optimization of the casting process. According to [1], sand casting is an inexpensive method for the production of materials. Raw castings are transformed into finished products or components. The least costly of all casting methods is sand casting. Sand casting is used to manufacture a large range of metal parts of different sizes and weights in complicated geometries. These casting products are found in automobile parts and airspace [2], communication and power generation parts [3], petrochemicals [4], construction parts [5] and many more. [6] explained that, sand casting which is also known as sand molded casting, involves pouring of molten metal into the mold cavity through the pouring basin and then the gate. The molten metal is seen to mount quickly at the riser when the cavity is full.

[7] differentiated the sand mold and metal mold conditions. The sand mold contains a dissipation structure, air filled capillary-permeable body that forms the sand mold layer. All three methods of heat transfer are shared: radiation, convection and conduction. These basic heat transfer systems are theoretically disconnected to allow for evaluation, although the events occur and interrelate at varying intensity at the same time. Also, [8] relayed that certain processes optimum impact are within a certain heat range. This definition shows why sand mold deposition has a reduced thermal value than a metal mold. Thus, solidification happens faster in cast metal mold since it is a denser conductive system. [9] noted that, the variation of heat throughout the mold and

cast represents the thermal energy situations of the Fourier differential equation.

[10] opined that sand casting necessitates the use of replica that forms the cavity of the required product to be made so that molten substance poured into the mold can take the desired shape. However, [11] were of the opinion that, the aforementioned assertions of [10] causes some amount of heat flow to be released from the molten substance to the environment, which is the mold wall. The metal solidifies and it is separated from the mold after very much cooling. [12], emphasizes some factors that influence the quality of the required product such as melting temperature, heat transfer, stress concentration, shrinkage and other factors. However, [13], considered an equilibrium environment instead of seeking a generic composite structure out of equilibrium. [14], defined metal solidification as the change of metal from its molten state to solid state after it has been poured into the mold. This procedure starts rapidly after pouring the molten metal into a mold and followed by chain of events. [15] stressed that the main principle of solidification is the conversion phase of the liquid to solid along a transient interface, which is followed by the liberation and redistribution of latent heat energy at both stages. This shift in the phase of the substance varies according to the content of the material, that is, either a pure or alloyed. [16] exclaimed that pure metallic substance solidifies at a specific temperature. An alloy solidifies at over a temperature spectrum, based on the characteristics that make up the metal. The cooling feature that results in solidification has a major influence on the size of the grain. According to [17], a rapidly cooled casting region would have a small grain size, and a zone steadily cooled would have a rough grain size. Also, [18], added that the quality of the finished casting depends primarily on the solidification pace, as faster solidification produces better microstructure, finer grain size with better mechanical properties as well as certain metallurgical characteristics. [15], also described the mold as a hollow for which the liquid metal is filled to solidify and create the required appearance. Therefore, among several factors influencing the casting product is the mold. [19], stated that the solidification of molten metal in the mold is a result of the extraction of heat from the metal by the mold that surrounds it. This process of heat extraction is called heat transfer. [20], therefore concluded that the solidification challenge of casting is a transient heat transfer problem and hence the temperature distribution in the metal casting and the mold

during the solidification process is to be tracked. [21] Describe finite element method as a very momentous tool capable of simulating the heat transfer in the solidification process and also the casting, mold and air gap formation. It is in fact important to understand the sand casting procedure to be able to model the temperature distribution process. According to [22], the molten metal fills the mold immediately it is poured but the air gap created between the mold wall and the molten metal increases due to metal shrinkage as solidification takes place. Meanwhile, [23] opined that, the capacity for heat to migrate through the casting, interface and mold strongly impacts the emergence of solidification which plays a prominent role in deciding the freezing state and then the soundness of the casting. The goals of finite element analysis in the manufacture of casting products are to have an oversight of the casting process at every stage of the solidification and transformation of the product, to predict defects in its design stage and proactively control the process in order to obtain the most desired products.

[24], rationalized the need for treating each casting production process as special and that they require precision planning and execution. They bemoaned that discrepancies during manufacturing, such as inappropriate pouring temperature, may have detrimental effect on the final output. Even though the pattern and mold are made with extreme accuracy, flaws during the process of molding can result in significant waste of resources. They however discussed few of the defects that occur frequently during casting processes, which are shifts, warpage, fin, swell, blowholes, drop, dirt, honeycombing or sponginess, metal penetration and rough surface, sand holes, pin holes, scabs, shrinkage cavity, hot tears (Pulls), cold shut and misrun, poured sort, internal air pocket. [25] also stressed that casting rejection is the dilemma of the foundry industry as energy, materials and time can be saved should the casting defects be predicted and corrected in the casting design prior to molding. Since the temperature distribution in sand casting molten metal and mold can be predicted, monitored and analyzed appropriately as the process forms the properties of the resulting cast then, one must employ an analytical method for the optimization of the sand-casting process.

[26] emphasized the distinctiveness of the finite element method as the division of a given domain into a set of simple sub-domains, called finite elements. He proceeded to state that any geometric shape that allows computation of the

solution or its approximation or provides necessary relations among the values of the solution at selected points, called nodes, of the sub-domain, qualifies as a finite element. [27] referred to the finite element method (FEM) or finite element analysis (FEA), as an analytical technique used to obtain approximate solutions of boundary value problems in engineering. [28], itemized the various numerical solution methods to complex problems and they include; least-squares method, the Rayleigh-Ritz method, methods of weighted-residuals, finite element method, the Petrov-Galerkin method, the Galerkin method and the collocation method. [29], expressed fears of the threat of porosity, voids in metals and gas bubbles that remain inherent byproduct of the casting process and added that the various effects can be studied by generating the model of internal void with the help of finite element methods. [30] asserted the high cost of analyzing and solving solidification challenges in casting using experimental design analysis and the geometries and complex boundary conditions that sometimes defy analytical solutions. The report of [30] maintained that, the heat conduction coupled with thermal stress analysis packages were considered in order to forecast the mechanical behaviour such as cracking and eventual failure of the cast. The report also included the phase change interface and latent heat effects. [31], studied the solidification process of metal during casting using Finite element method. It was analyzed and named a non-linear transient phenomenon. They described the solidification period in which the hottest region inside the casting is solidifying as the most important instant of time.

In many simulation techniques where comparison is made between foundry experiment and simulation methods, attention is given to the

analytical result while neglecting the several effects of the immediate sudden freezing of the molten metal during the pouring process. Therefore, in this research, the mold was preheated and the same was incorporated into the analysis. Also, the molten metal must be poured at optimum melting temperature since according to [32], mis-run can ensue from pouring molten metal at very low temperature which can lead to a discontinued fluid metal flow. Furthermore, [33] relayed the causes, features and consequent implication of sand burning. This occurs due to pouring of molten metal at very high temperature such that the thin sand crusts firmly glue to the casting surface. Taking temperature measurements around this region can impair the result and relevance of the research. The basic objective of this research is to clarify, collect, and establish the quantitative relationships between the heat transfer process associated with the Aluminium alloy sand casting experiment and the finite element analysis simulation.

II. METHODOLOGY

This research work made use of the Linear Lagrange interpolation function of the finite element methods to analyze the temperature distribution in a thin solidifying Aluminum alloy cast governed by the 1-D transient heat conduction equation. Therefore, the Linear Lagrange interpolation function of the finite element methods was employed. The variables that were obtained while performing the experiment such as the initial temperature of the mold and pouring temperature of the molten Aluminum alloy were inputted into the equation and the MATLAB programme so as to obtain comparable results with the Finite element methods.

The Governing Equation

The governing equation is represented by:

$$\rho_{Aluminium} C_{PAluminium} \frac{dT}{dt} = K_{Aluminium} \frac{d^2T}{dx^2} + \frac{dK_{Aluminium}}{dT} \left(\frac{dT}{dx} \right)^2$$

$$\rho C_p \frac{dT}{dt} = K \frac{d^2T}{dx^2} + \frac{dK}{dT} \left(\frac{dT}{dx} \right)^2 \quad 1$$

Expansion of equation 1 gives;

$$\rho C_p \frac{dT}{dt} = K \frac{d^2T}{dx^2} + \frac{dK}{dT} \cdot \frac{dT}{dx} \cdot \frac{dT}{dx} \quad 2$$

Applying the chain rule of the form; $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$ 3

According to Barnes (1999), the function notation can be written as; $\frac{dy}{dx} = f'(g(x)) \cdot g'(x)$

Equation 2 can then be written as

$$\rho C_p \frac{dT}{dt} = K \frac{d^2T}{dx^2} + \frac{d}{dx} \left(K \frac{dT}{dx} \right) - K \frac{d^2T}{dx^2} \quad 4$$

The model equation is finally reduced to;

$$\rho C_p \frac{dT}{dt} = \frac{d}{dx} \left(K \frac{dT}{dx} \right) \quad 5$$

The Boundary Conditions

In other to prevent the melt from causing thermal damage during pouring due to instantaneous and sudden increase in the temperature of the mold [34], the mold was preheated to a considerable temperature and the

initial temperature in the casting, mold and mold-metal interface were:

$$T_c = T_p$$

$$T_g = T_m = T_M$$

The temperature (T_i) at the interface between the two materials that is the cast and the mold on pouring the molten metal are as follows:

$$T_i = \frac{(\rho_m C_m T_m) + (\rho_c C_c T_c) + (\rho_c \Delta H)}{\rho_m C_m + \rho_c C_c} \quad 6$$

$$T_B = \frac{T_P + T_A}{2}$$

$$Q_{1st} = h_1 (T_P - T_B)$$

$$Q_{last} = h_2 (T_M - T_A)$$

Weak Formulation

Rearrange equation 5 (the derived equation) equals zero, that is:

$$\frac{d}{dx} \left(K \frac{dT}{dx} \right) - \rho C_p \frac{dT}{dt} = 0 \quad 7$$

Multiplying equation 5 through with a weight function $w(x)$ to introduce it into the problem

$$\text{results into: } w(x) \frac{d}{dx} \left(K \frac{dT}{dx} \right) - \rho C_p w(x) \frac{dT}{dt} = 0 \quad 8$$

Therefore, derive the weak form of the equation using the method of Integration by parts, introducing the notations:

$$\int w(x) \frac{d}{dx} \left(K \frac{dT}{dx} \right) - \int \rho C_p w(x) \frac{dT}{dt} = 0 \quad 9$$

In order to weaken the form, recall the part $w(x) \frac{d}{dx} \left(K \frac{dT}{dx} \right)$ from equation 7

$$\int_{x_i}^{x_{i+1}} U \frac{dV}{dx} dx = [UV]_{x_i}^{x_{i+1}} - \int_{x_i}^{x_{i+1}} V \frac{dU}{dx} dx \quad 10$$

$$\text{If } U = w(x) \text{ and } \frac{dV}{dx} = \frac{d}{dx} \left(K \frac{dT}{dx} \right) \text{ then } \frac{dU}{dx} = \frac{dw(x)}{dx} \text{ and } V = K \frac{dT}{dx} \quad 11$$

$$\int_{x_i}^{x_{i+1}} w(x) \frac{d}{dx} \left(K \frac{dT}{dx} \right) dx = \left[w(x) K \frac{dT}{dx} \right]_{x_i}^{x_{i+1}} - \int_{x_i}^{x_{i+1}} k \frac{dT}{dx} \frac{dw(x)}{dx} dx \quad 12$$

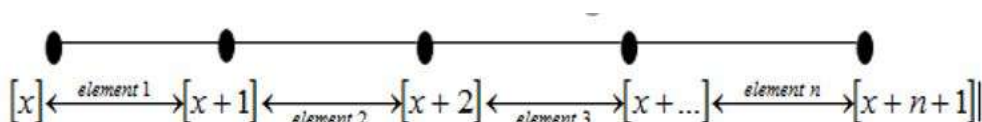


Fig 1. The structural representation of the difference between two successive nodes such as x_i and x_{i+1} and its relationship with other elements

combined. [26], An Introduction to Finite Element Method. Second Edition, McGraw-Hill, Inc.

Assembling the whole equation 7 again with the part $w(x) \frac{d}{dx} \left(K \frac{dT}{dx} \right)$ being replaced by equation 10, it

becomes:

$$\int_0^h w(x) \rho C_p \frac{dT}{dt} - \left[w(x) K \frac{dT}{dx} \right]_{x_i}^{x_{i+1}} + \int_{x_i}^{x_{i+1}} k \frac{dT}{dx} \frac{dw(x)}{dx} dx = 0 \quad 13$$

Derivation of the Finite Element Interpolation Functions

Equation 13 is the integral variable, $\left[-w(x) K \frac{dT}{dx} \right]$ is the natural variable while $-K \frac{dT}{dx}$ is the natural boundary condition and secondary variable and T is the essential boundary condition that is the primary variable assuming a finite element solution of the form:

$$T^e(x, t) \approx T^e(x, t) = \sum_{j=1}^n T_j^e(t) \psi_j^e(x) \quad 14$$

The approximate solution is a decoupled formulation where $T_j^e(t)$ is the value of the solution at the nodes of the finite element, it is a function of time and $w(x)$ is replaced by $\psi_j^e(x)$, it is the approximation function over the element in x direction. Therefore, $w(x)$ is substituted from equation 13 to give:

$$\int_0^h \psi_j^e \rho C_p \frac{dT}{dt} - \left[\psi_j^e K \frac{dT}{dx} \right]_{x_i}^{x_{i+1}} + \int_{x_i}^{x_{i+1}} k \frac{dT}{dx} \frac{d\psi_j^e}{dx} dx = 0 \quad 15$$

From equation 15 $\frac{dT}{dx}$ and $\frac{dT}{dt}$ are evaluated

$$\frac{dT}{dt} = \sum_{j=1}^n \frac{dT}{dt}(t) \psi_j^e(x) \quad 16$$

$$\frac{dT}{dx} = \sum_{j=1}^n T_j^e(t) \frac{d\psi_j^e}{dx}(x) \quad 17$$

In element notation $T^e(x, t) \approx T^e(x, t) = \sum_{j=1}^n T_j^e(t) \psi_j^e(x)$ where (s = 1, 2, 3, ... n) time counter.

Note $T_j^s(t)$ is the value of T(x, t) at time $t = t_s$ for node j of the element n.

Thus, we have:

$$\int_0^h \rho C_p \left(\sum_{j=1}^n \frac{dT}{dt}(t) \psi_j^e(x) \right) \psi_j^e(x) + \int_0^h \left(\frac{d\psi_j^e(x)}{dx} \right) K \left(\sum_{j=1}^n T_j^e(t) \frac{d\psi_j^e}{dx}(x) \right) - \left[K \sum_{j=1}^n T_j^e(t) \frac{d\psi_j^e}{dx}(x) \psi_j^e(x) \right]_0^h = 0 \quad 18$$

$$\int_0^h \rho C_p \left(\sum_{j=1}^n \frac{dT}{dt}(t) \psi_j^e(x) \right) \psi_j^e(x) + \int_0^h \left(\frac{d\psi_j^e(x)}{dx} \right) K \left(\sum_{j=1}^n T_j^e(t) \frac{d\psi_j^e}{dx}(x) \right) = \left[K \sum_{j=1}^n T_j^e(t) \frac{d\psi_j^e}{dx}(x) \psi_j^e(x) \right]_0^h$$

Application of the α - Family of Approximation

Considering the time aspect in the problem, to resolve this, we introduce an appropriate time approximate scheme which is the α - family of approximation. The model developed being a one-dimensional time dependent problem will describe time approximation scheme and convert the differential equation to algebraic equation with respect to time.

$$(1 - \alpha) \dot{T}_s + \alpha \dot{T}_{s+1} = \frac{T_{s+1} - T_s}{\Delta t_{s+1}} \quad 0 \leq \alpha \leq 1 \quad 19$$

$$\left([M^e] + \alpha \Delta t [K^e] \right) \{T^e\}_{s+1} = \left([M^e] + \Delta t (1 - \alpha) [K^e] \right) \{T^e\}_s + \Delta t (\alpha \{Q^e\}_{s+1}) + (\alpha) \{Q^e\}_s \quad 20$$

The equation 18 is however reduced to equation 20 using the Backward Difference Scheme

Evaluation of the Finite Elemental Matrices

The system model is of the form: $[M^e] \left\{ \dot{T}_s \right\} + [K^e] \{T_s\} = \{Q_s^e\}$ 21

Where the conductivity matrix $M_{ij} = \rho C_p \int_0^h (\psi_i \psi_j) dx$

The Enthalpy matrix $K_{ij} = \int_0^h \left(K \frac{d\psi_i}{dx} \frac{d\psi_j}{dx} \right) dx$ and

The one-dimensional Lagrange quadratic interpolation function for the equation becomes: $\psi_1 = 1 - \frac{x}{h}$,

$$\psi_2 = \frac{x}{h} \quad 22$$

The interpolation properties ψ_1^e is equal to one at node one and zero at node two while ψ_2^e is equal to zero at node one and one at node two [26].

Using the Linear Lagrange interpolation function in equation 22 to obtain all the M_{ij} and K_{ij} in the matrix in equation 21

To obtain M-matrix for M_{ij}

$$M_{11} = \rho C_p \int_0^h (\psi_1 \psi_1) dx$$

$$M_{11} = \rho C_p \int_0^h \left(1 - \frac{x}{h} \right) \left(1 - \frac{x}{h} \right) dx$$

$$M_{11} = \rho C_p \left[\frac{h}{3} \right] \quad 23$$

where $M_{11} = M_{22}$

$$M_{11} = \frac{\rho C_p h}{3}$$

$$M_{12} = \rho C_p \int_0^h (\psi_1 \psi_2) dx$$

$$M_{12} = \rho C_p \int_0^h \left(1 - \frac{x}{h} \right) \left(\frac{x}{h} \right) dx$$

$$M_{12} = \rho C_p \left[\frac{h}{6} \right] \quad 24$$

where $M_{12} = M_{21}$

$$M_{12} = \frac{\rho C_p h}{6}$$

To obtain K-matrix for K_{ij}

$$\text{where } \psi_1 = 1 - \frac{x}{h}$$

$$\frac{d\psi_1}{dx} = -\frac{1}{h}$$

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where $\psi_2 = \frac{x}{h}$

$$\frac{d\psi_2}{dx} = \frac{1}{h}$$

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$$K_{11} = K \int_0^h \left(\frac{d\psi_1}{dx} \right) \left(\frac{d\psi_1}{dx} \right) dx$$

$$K_{11} = K \int_0^h \left(-\frac{1}{h} \right) \left(-\frac{1}{h} \right) dx$$

$$K_{11} = K \left[\frac{1}{h} \right]$$

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where $K_{11} = K_{22}$

$$K_{11} = \frac{K}{h}$$

$$K_{12} = K \int_0^h \left(\frac{d\psi_1}{dx} \right) \left(\frac{d\psi_2}{dx} \right) dx$$

$$K_{12} = K \int_0^h \left(-\frac{1}{h} \right) \left(\frac{1}{h} \right) dx$$

$$K_{12} = K \left[-\frac{1}{h} \right]$$

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where $K_{12} = K_{21}$

$$K_{12} = -\frac{K}{h}$$

$$Q = \sum h \left[\psi T \frac{d\psi}{dx} \right]_0^h$$

$$Q_{11} = \left[\psi_1(x) T_1 \frac{d\psi_1(x)}{dx} \right]_0^h$$

$$Q_{12} = \left[\psi_1(x) T_2 \frac{d\psi_2(x)}{dx} \right]_0^h$$

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$$Q_{21} = \left[\psi_2(x) T_1 \frac{d\psi_1(x)}{dx} \right]_0^h$$

$$Q_{22} = \left[\psi_2(x) T_2 \frac{d\psi_2(x)}{dx} \right]_0^h$$

The assembled Enthalpy matrix K_e is given as:

$$\begin{bmatrix} \frac{\rho C_p h}{3} & \frac{\rho C_p h}{6} \\ \frac{\rho C_p h}{6} & \frac{\rho C_p h}{3} \end{bmatrix} \begin{Bmatrix} \dot{T}_1 \\ \dot{T}_2 \end{Bmatrix}$$

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The assembled Conductivity matrix M_e is given as:

$$\begin{bmatrix} \frac{K}{h} & -\frac{K}{h} \\ -\frac{K}{h} & \frac{K}{h} \end{bmatrix} \begin{Bmatrix} T_1 \\ T_2 \end{Bmatrix} \quad 31$$

The Global System of Algebraic Equations

The Conductivity matrix and the Enthalpy matrix are derived and assembled by substituting the Linear Lagrange interpolation function represented in equation 23 to 31 to become:

$$\begin{bmatrix} M_{11}^1 & M_{12}^1 \\ M_{21}^1 & M_{22}^1 \end{bmatrix} \begin{Bmatrix} \dot{T}_1 \\ \dot{T}_2 \end{Bmatrix} + \begin{bmatrix} K_{11}^1 & K_{12}^1 \\ K_{21}^1 & K_{22}^1 \end{bmatrix} \begin{Bmatrix} T_1 \\ T_2 \end{Bmatrix} = \begin{Bmatrix} Q_1 \\ Q_2 \end{Bmatrix} \quad 32$$

$$\begin{bmatrix} \frac{\rho ch}{3} & \frac{\rho ch}{6} \\ \frac{\rho ch}{6} & \frac{\rho ch}{3} \end{bmatrix} \begin{Bmatrix} \dot{T}_1 \\ \dot{T}_2 \end{Bmatrix} + \begin{bmatrix} \frac{K}{h} & -\frac{K}{h} \\ -\frac{K}{h} & \frac{K}{h} \end{bmatrix} \begin{Bmatrix} T_1 \\ T_2 \end{Bmatrix} = \begin{Bmatrix} Q_1 \\ Q_2 \end{Bmatrix}$$

For cast $\rho = 2650, c = 921, K = 230, h = 0.3$

For mold $\rho = 1495, c = 1172.304, K = 0.519, h = 0.1$

$$\begin{bmatrix} M_{11}^1 & M_{12}^1 & 0 & 0 \\ M_{21}^1 & M_{22}^1 + M_{11}^2 & M_{11}^2 & 0 \\ 0 & M_{21}^2 & M_{22}^2 + M_{11}^3 & M_{12}^3 \\ 0 & 0 & M_{21}^3 & M_{22}^3 \end{bmatrix} \begin{Bmatrix} \dot{T}_1 \\ \dot{T}_2 \\ \dot{T}_3 \\ \dot{T}_4 \end{Bmatrix} + \begin{bmatrix} K_{11}^1 & K_{12}^1 & 0 & 0 \\ K_{21}^1 & K_{22}^1 + K_{11}^2 & K_{12}^2 & 0 \\ 0 & K_{21}^2 & K_{22}^2 + K_{11}^3 & K_{12}^3 \\ 0 & 0 & K_{21}^3 & K_{22}^3 \end{bmatrix} \begin{Bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \end{Bmatrix} = \begin{Bmatrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \end{Bmatrix}$$

$$\begin{bmatrix} \frac{\rho ch}{3} & \frac{\rho ch}{6} & 0 & 0 \\ \frac{\rho ch}{6} & \frac{\rho ch}{3} + \frac{\rho ch}{3} & \frac{\rho ch}{6} & 0 \\ 0 & \frac{\rho ch}{6} & \frac{\rho ch}{3} + \frac{\rho ch}{3} & \frac{\rho ch}{6} \\ 0 & 0 & \frac{\rho ch}{6} & \frac{\rho ch}{3} \end{bmatrix} \begin{Bmatrix} \dot{T}_1 \\ \dot{T}_2 \\ \dot{T}_3 \\ \dot{T}_4 \end{Bmatrix} + \begin{bmatrix} \frac{K}{h} & -\frac{K}{h} & 0 & 0 \\ -\frac{K}{h} & \frac{K}{h} + \frac{K}{h} & -\frac{K}{h} & 0 \\ 0 & -\frac{K}{h} & \frac{K}{h} + \frac{K}{h} & -\frac{K}{h} \\ 0 & 0 & -\frac{K}{h} & \frac{K}{h} \end{bmatrix} \begin{Bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \end{Bmatrix} = \begin{Bmatrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \end{Bmatrix}$$

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Numerical Results and Discussion

This study required that, the thermo physical properties of the mold and cast used to be the same with the exact design and operating parameters applied to the numerical method and the results of the finite element under consideration are

presented. The convergence characteristics of the results investigated using the finite element method and the same was compared with that obtained using the casting design experiment carried out in the laboratory.

Table 1.0: The experimental and simulation results for the Aluminium alloy casting with varying depth and time interval.

Time	Depth	Experiment	Two Elements	Four Elements	Six Elements	Sixteen Elements
10 seconds	0.15	1015.82	1010.6219354872	1011.408649723	1011.328087	1011.845740639
	0.30	953.34	3	4	69438	24
	0.40	379.15	972.83436706285 9 366.885868037	961.5823694102 82 367.9796339110 67	986.5823694 10282 368.7498010 35115	989.0081970309 79 369.3837919607 45
360 seconds	0.15		1002.6012815494	1000.841333875	1006.391172	998.4850298269
	0.30		7	9	16883	62
	0.40		877.16289321656 394.65213661188 3	882.1909740916 72 365.1861275682 29	886.1909740 91672 365.6502209 55691	887.1330569618 85 357.7673024583 77
720 seconds	0.15		965.26058259233	991.0513959878	989.0513959	984.7549192733
	0.30		1	55	87855	65
	0.40		7416.2580057869 02 339.16223226819 5	767.4048353591 05 362.5688643843	757.4048353 59105 354.6112178 01332	764.3972938519 35 351.1034005249 78
1080 seconds	0.15		974.74920195200	969.9556519967	971.9556519	977.3583430872
	0.30		7	47	96747	21
	0.40		972.94157872564 8 353.33848276195 1	955.6796154657 66 342.9164080069 07	954.7455242 85497 345.9164080 06907	973.4096034522 9 345.9444653211 36
1440 seconds	0.15		945.35849302335	949.4793088702	968.4793088	971.0570228020
	0.30		8	57	70257	6
	0.40		869.63472298489 5 366.96103550994 1	947.3507886466 81 334.9889517278 76	949.5479817 61712 334.9889517 27876	968.0675388745 31 341.8448493065 16
1800 seconds	0.15	945.05	942.03032466360	944.4263450321	961.4263450	965.4063941929
	0.30	872.46	3	96	32196	91
	0.40	347.02	866.38520687475 9 360.02541581238 2	942.7513813370 14 331.4992881976 03	942.7513813 37014 331.4992881 97603	963.0647561702 68 339.5545117019 17
2160 seconds	0.15		940.76130927234	939.6947322505	956.6947322	960.2050158663
	0.30		7	37	50537	32
	0.40		863.19193278106 8 352.55145951252 2	938.3735369175 47 331.8848686105 73	938.3735369 17547 331.8848686 10573	958.3390833271 16 339.5328579542
2520 seconds	0.15		959.54926798503	935.2153376768	951.2153376	955.3381177422
	0.30		2	01	76801	65
	0.40		860.05287125427 1 354.55877746047	934.1771571680 21 335.3126345184 46	934.1771571 68021 335.3126345 18446	930.8401248543 82 341.7652161142 14
2880 seconds	0.15		956.39212373362	930.9387486796	950.9387486	950.7324332323
	0.30		4	11	79611	28
	0.40		856.96604677231	930.1317784967	930.1317784	949.5294542804

			9 336.06627599830 7	85 341.0014890799 03	96785 341.0014890 79903	61 345.9726211258 03
3240 seconds	0.15 0.30 0.40		936.28787561678 6 853.92955540390 3 337.09217257998 9	936.8286061607 06 926.2141555094 4 348.3080545410 91	946.8286061 60706 926.2141555 0944 348.3080545 41091	946.3376994687 01 945.3776789172 76 351.7868129635 71
3600 seconds	0.15 0.30 0.40	918.37 834.93 340.76	920.23459567170 9 850.94156260375 7 337.65402067262 3	922.8576655977 59 922.4064499090 48 356.7303220490 26	941.8576655 97759 922.4064499 09048 356.7303220 49026	942.1176817110 18 941.3620249064 96 358.8430003363 87

Source: Author's Field Work (2019)

III. CONCLUSIONS AND RECOMMENDATIONS

Table 1, showed the analytical results retrieved using the Linear Lagrange interpolation function of the Finite element analysis in comparison with the experimental outcome obtained from the laboratory indicated slight variations in different time interval. This is an indication that plugging in the exact laboratory conditions into the mathematical formulation of the physical process will yield comparable result and so evaluation of heat transfer in casting process can be achieved using finite element analysis. More so, due to the complexity of the solidification process, the compactness and intricacies present in the casting designs, it is usually difficult to take readings of the temperature distribution in the casting design. Therefore, the finite element simulation method affords the whole casting material into sub domains and analyzed the mid-section of the cast, the cast-mold interface and the mold wall.

The study therefore revealed the essence of preheating as seen in Table 1.0 that immediately after pouring the molten metal, there is a sudden drop in the temperature of the casting. Finally, this analytical result easily provided temperature history of the casting, mold wall and the mold every three (3) minutes after pouring the molten metal in the mold. This evidently shows that the finite element method is an efficient and accurate method in the simulation of heat transfer in sand casting.

The study thereby recommends the following based on the outcome of the analysis:

1. Casting Engineers and various Researchers in manufacturing sectors should prioritize the knowledge and the use of finite element analysis in casting.

2. Government should encourage the various engineering professions about the use of finite element analysis tool through optimization of casting designs.
3. The casting engineers should ensure that they improve their knowledge of finite element analysis for effectiveness.

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