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Exact Solutions for BKDV Equation by a Sub-ODE Method

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ABSTRACT: In this paper, we derive exact traveling wave soluti-ons of1D-BKDV equation and 2D-BKDV equation by a proposed Bernoulli sub-ODE method.The method appears to be efficient in seeking exact solutionsof nonlinear equations.
KEYWORDS: Bernoulli sub-ODE method, traveling wave solutio-ns,exact solution, evolution equation,BKDV equation

I. INTRODUCTION

During the past four decades or so searching for explicit solutions of nonlinear evolutionequations by using various different methods have been the main goal for many resea-rchers,and many powerful methods for constructing exact solutions of nonlinear evolution equationshave been estab-lished and developed such as the inverse scattering transform, theDarboux transform, the tanh-function expansion and its various extension, the Jacobi ellipticfunction expansion, the homogeneous balance method, the sine-cosine method, the rank analysismethod, the exp-function expansion meth-od and so on [1-6]. In this paper, we proposed a Bernoulli sub-ODE method to constructexact traveling wave solution-ns for NLEES.

The rest of the paper is organized as follows. In Section 2, we describethe Bernoulli sub-ODE method for finding traveling wave solutions of nonlinearevolution equations, and give the main steps of the method. In thesubsequent sections, we will apply the method to find exact traveling wave solutionsof the 1D-BKDV equation and the 2D-BKDV equation.In the last Section, some conclus-ions are presented.

II. DESCRIPTION OF THESUB-ODE METHOD

In this section we present the solutions of the following ODE:

$$G'+\lambda G=\mu G^2 \quad (2.1)$$

where $\lambda\neq 0, G=G(\xi)$

The solution of Eq.(2.1) is denoted as follows

$$G=\frac{1}{\frac{\mu}{\lambda}+de^{\lambda\xi}} \quad (2.2)$$

where d is an arbitrary constant.

Suppose that a nonlinear equation, say in two or three indep-endentvariablesx, y andt , is given by

$$P(u,u_{t,}u_x,u_y,u_{tt},u_{xt},u_{yt},u_{xx},u_{yy}......)=0 \quad (2.3)$$

whereu = u(x, y, t) is an unknown function,P isa polyno-mial in u = u(x, y, t) and its various partialderivatives, in which the highest order derivatives andnonlinear terms are involved. By using the solutions of Eq.(2.1),we can constr-uct a serials of exact solutions of nonlinear equations:.

Step 1.We suppose that

$$u(x,y,t)=u(\xi),\xi=\xi(x,y,t) \quad (2.4)$$

the travelling wave variable (2.4) permits us reducing Eq.(2.3) to an ODE for $u=u(\xi)$

$$P(u,u',u'',......)=0 \quad (2.5)$$

Step 2. Suppose that the solution of (2.5) can be expre-ssedby a polynomial in G as follows:

$$u(\xi)=\alpha_m G^m+\alpha_{m-1}G^{m-1}+...... \quad (2.6)$$

where $G=G(\xi)$ satisfies Eq.(2.1), and $\alpha_m,\alpha_{m-1}...$ are constants to be determined later, $\alpha_m\neq 0$ .The positive integer m can be determined by considering thehomogen-eous balance between the highest order derivativesand non-linear terms appearing in (2.5).

Step 3. Substituting (2.6) into (2.5) and using (2.1), collecting all terms with the sameorder of G together, the left-hand side of Eq. (2.5)is converted

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into another polynomial in G. Equatingeach coefficient of this polynomial to zero, yields a set ofalgebraic equations for $\alpha_m, \alpha_{m-1}, \ldots \lambda, \mu$.

Step 4. Solving the algebraic equations system in Step 3, and by usingthe solutions of Eq.(2.1), we can construct the traveling wave solutions of thenonlinear evolution equation (2.5)..

In the subsequent sections we will illustrate the proposedmethod in detail by applying it to Boussinesq equation and (2+1) dimensional Boussinesq equation.

III. APPLICATION FOR1D-BKDVEQUATION

In this section, we will consider the following 1D-BKDV equation:

$$u_t + \alpha u u_x + \beta u_{xx} + \gamma u_{xxx} = 0 \quad (3.1)$$

Suppose that

$$u(x,t) = u(\xi), \xi = k(x - ct) \quad (3.2)$$

c, k are constants that to be determined later.

By (3.2), (3.1) is converted into an ODE

$$-cu' + \alpha u u' + \beta k u'' + \gamma k^2 u''' = 0 \quad (3.3)$$

Integrating the ODE (3.3) once, we obtain

$$-cu + \frac{\alpha}{2}u^2 + \beta k u' + \gamma k^2 u'' = g \quad (3.4)$$

where g is the integration constant that can be determinedlater.

Suppose that the solution of (3.4) can be expressed by apolynomial inGas follows:

$$u(\xi) = \sum_{i=0}^{m} a_i G^i \quad (3.5)$$

where a_i are constants, and $G = G(\xi)$ satisfies Eq.(2.1).

Balancing the order of u^2 and u'' in Eq.(3.4), we have $2m = m + 2 \Rightarrow m = 2$ .So Eq.(3.5) can berewritten as

$$u(\xi) = a_2 G^2 + a_1 G + a_0, a_2 \neq 0 \quad (3.6)$$

a_2, a_1, a_0 are constants to be determined later.

Substituting (3.6) into (3.4) and collecting all the termswith the same power of G together, equating eachcoefficient to zero, yields a set of simultaneous algebraicequations as follows:

$$G^0: -ca_0 - g + \frac{1}{2}\alpha a_0^2 = 0$$

$$G^1: -ca_1 + \alpha a_1 a_0 + \beta k a_1 \mu + \gamma k^2 a_1 \mu^2 = 0$$

$$G^2: -ca_2 + 2\beta k a_2 \mu + \alpha a_0 a_2 - \beta k a_1 \lambda + \frac{1}{2}\alpha a_1^2 + 4\gamma k^2 a_2 \mu^2$$

$$-3\gamma k^2 a_1 \mu \lambda = 0$$

$$G^3: -2\beta k a_2 \lambda - 10\gamma k^2 a_2 \mu \lambda + \alpha a_1 a_2 + 2\gamma k^2 a_1 \lambda^2 = 0$$

$$G^4: \frac{1}{2}\alpha a_2^2 + 6\gamma k^2 a_2 \lambda^2 = 0$$

Solving the algebraic equations above, yields:
Case 1:

$$a_2 = -\frac{12}{25}\frac{\beta^2\lambda^2}{\alpha\gamma\mu^2}, a_1 = 0, a_0 = a_0, k = -\frac{1}{5}\frac{\beta}{\gamma\mu},$$

$$c = \frac{1}{25}\frac{-6\beta^2 + 25\alpha a_0 \gamma}{\gamma}, g = -\frac{1}{50}\frac{a_0(-12\beta^2 + 25\alpha a_0\gamma)}{\gamma}$$

(3.7)

where a_0 is an arbitrary constants.

Substituting (3.7) into (3.6), we get that

$$u_1(\xi) = -\frac{12}{25}\frac{\beta^2\lambda^2}{\alpha\gamma\mu^2}G^2 + a_0$$

$$\xi = -\frac{1}{5}\frac{\beta}{\gamma\mu}\left(x - \frac{1}{25}\frac{-6\beta^2 + 25\alpha a_0\gamma}{\gamma}t\right) \quad (3.8)$$

where k, c are defined as in (3.7).

Combining with Eq. (2.2), wecan obtain the traveling wave solutions of (3.1) as follows:

$$u_1(\xi) = -\frac{12}{25}\frac{\beta^2\lambda^2}{\alpha\gamma\mu^2}\left(\frac{1}{\frac{\mu}{\lambda} + de^{\lambda\xi}}\right)^2 + a_0 \quad (3.9)$$

where a_0, d are arbitrary constants,

$$\xi = -\frac{1}{5}\frac{\beta}{\gamma\mu}\left(x - \frac{1}{25}\frac{-6\beta^2 + 25\alpha a_0\gamma}{\gamma}t\right)$$

Case 2:

$$a_2 = -\frac{12}{25}\frac{\beta^2\lambda^2}{\alpha\gamma\mu^2}, a_1 = \frac{24}{25}\frac{\beta^2\lambda}{\alpha\gamma\mu}, a_0 = a_0, k = \frac{1}{5}\frac{\beta}{\gamma\mu},$$

$$c = \frac{1}{25}\frac{6\beta^2 + 25\alpha a_0\gamma}{\gamma}, g = -\frac{1}{50}\frac{a_0(12\beta^2 + 25\alpha a_0\gamma)}{\gamma}$$

(3.10)

where a_0 is an arbitrary constants.

Substituting (3.10) into (3.6), we get that

$$u_2(\xi) = -\frac{12}{25}\frac{\beta^2\lambda^2}{\alpha\gamma\mu^2}G^2 + \frac{24}{25}\frac{\beta^2\lambda}{\alpha\gamma\mu}G + a_0$$

$$\xi = \frac{1}{5}\frac{\beta}{\gamma\mu}\left(x - \frac{1}{25}\frac{6\beta^2 + 25\alpha a_0\gamma}{\gamma}t\right) \quad (3.11)$$

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where k, c are defined as in (3.7).

Combining with Eq. (2.2), wecan obtain the traveling wave solutions of (3.1) as follows:

$$u_2(\xi) = -\frac{12}{25}\frac{\beta^2\lambda^2}{\alpha\gamma\mu^2}\left(\frac{1}{\frac{\mu}{\lambda}+de^{\lambda\xi}}\right)^2 + \frac{24}{25}\frac{\beta^2\lambda}{\alpha\gamma\mu}\left(\frac{1}{\frac{\mu}{\lambda}+de^{\lambda\xi}}\right)+a_0 \quad (3.12)$$

where a_0, d are arbitrary constants,

$$\xi = \frac{1}{5}\frac{\beta}{\gamma\mu}\left(x - \frac{1}{25}\frac{6\beta^2+25\alpha a_0\gamma}{\gamma}t\right)$$

Remark : Our results (3.9) and (3.12) are new families of exact travelingwave solutions for Eq.(3.1).

IV.APPLICATION OF THE BERNOULLI SUB-ODE METHOD FOR 2D-BKDV EQUATION

In this section, we will consider the following 2D-BKDV equation:

$$(u_t + \alpha u u_x + \beta u_{xx} + s u_{xxx})_x + \gamma u_{yy} = 0 \quad (4.1)$$

Suppose that

$$u(x,y,t) = u(\xi), \xi = k(x+y-ct) \quad (4.2)$$

c, k are constants that to be determined later.

By (4.2), (4.1) is converted into an ODE

$$(-cu' + \alpha uu' + \beta k u'' + sk^2 u''')' + \gamma u'' = 0 \quad (4.3)$$

Integrating the ODE (4.3) once, we obtain

$$-cu' + \alpha uu' + \beta k u'' + sk^2 u''' + \gamma u' = g \quad (4.4)$$

where g is the integration constant that can be determinedlater.

Suppose that the solution of (4.4) can be expressed by apolynomial in G as follows:

$$u(\xi) = \sum_{i=0}^{m} a_i G^i \quad (4.5)$$

where a_i are constants, and $G = G(\xi)$ satisfies Eq.(2.1).

Balancing the order of uu' and u''' in Eq.(4.4), we have

$$m+m+1 = m+3 \Rightarrow m = 2.$$

So Eq.(4.5) can berewritten as

$$u(\xi) = a_2 G^2 + a_1 G + a_0, a_2 \neq 0 \quad (4.6)$$

a_2, a_1, a_0 are constants to be determined later.

Substituting (4.6) into (4.4) and collecting all the termswith the same power of G together, equating eachcoefficient to zero, yields a set of simultaneous algebraicequations as follows:

$G^0: -g = 0$

$G^1: -\alpha a_1 a_0 \lambda + \beta k a_1 \lambda^2 - sk^2 a_1 \lambda^3 - \gamma a_1 \lambda - c a_1 \lambda = 0$

$G^2: 2ca_2\lambda - ca_1\mu - \alpha a_1^2\lambda - 3\beta k a_1 \mu\lambda - 2\alpha a_0 a_2 \lambda + \alpha a_0 a_1 \mu = 0$

$4\beta k a_2 \lambda^2 + \gamma a_1 \mu + 7sk^2 \mu a_1 \lambda^2 - 2\gamma a_2 \lambda - 8sk^2 a_2 \lambda^3 = 0$

$G^3: -3\alpha a_1 a_2 \mu - 2ca_2\mu + 38sk^2 a_2 \mu\lambda^2 + 2\beta k a_1 \mu^2 + 2\alpha a_0 a_2 \mu$
$+2\gamma a_2 \mu - 12sk^2\mu^2 a_1 \lambda - 10\beta k a_2 \mu\lambda + \alpha a_1^2 \mu = 0$

$G^4: -54sa_2k^2\mu^2\lambda - 2\alpha a_2^2\lambda + 6sk^2\mu^3 a_1 + 3\alpha a_2 a_1 \mu + 6\beta k\mu^2 a_2 = 0$

$G^5: 24sa_2k^2\mu^3 + 2\alpha a_2^2 \mu = 0$

Solving the algebraic equations above, yields:

Case 1:

$$a_2 = -\frac{12}{25}\frac{\beta^2\mu^2}{s\lambda^2\alpha}, a_1 = \frac{24}{25}\frac{\beta^2\mu}{s\lambda\alpha}, a_0 = a_0,$$

$$c = \frac{1}{25}\frac{25\alpha a_0 s + 6\beta^2 + 25\gamma s}{s}, g = 0, k = -\frac{1}{5}\frac{\beta}{s\lambda} \quad (4.7)$$

where a_0 is an arbitrary constants.

Substituting (4.7) into (4.6), we get that

$$u_1(\xi) = -\frac{12}{25}\frac{\beta^2\mu^2}{s\lambda^2\alpha}G^2 + \frac{24}{25}\frac{\beta^2\mu}{s\lambda\alpha}G + a_0$$

$$\xi = -\frac{1}{5}\frac{\beta}{s\lambda}\left(x+y-\frac{1}{25}\frac{25\alpha a_0 s + 6\beta^2 + 25\gamma s}{s}t\right) \quad (4.8)$$

where k, c are defined as in (4.7).

Combining with Eq. (2.2), wecan obtain the traveling wave solutions of (4.1) as follows:

$$u_1(\xi) = -\frac{12}{25}\frac{\beta^2\mu^2}{s\lambda^2\alpha}\left(\frac{1}{\frac{\mu}{\lambda}+de^{\lambda\xi}}\right)^2 + \frac{24}{25}\frac{\beta^2\mu}{s\lambda\alpha}\left(\frac{1}{\frac{\mu}{\lambda}+de^{\lambda\xi}}\right)+a_0 \quad (4.9)$$

where a_0, d are arbitrary constants.

$$\xi = -\frac{1}{5}\frac{\beta}{s\lambda}\left(x+y-\frac{1}{25}\frac{25\alpha a_0 s + 6\beta^2 + 25\gamma s}{s}t\right)$$

Case 2:

$$a_2 = -\frac{12}{25}\frac{\beta^2\mu^2}{s\lambda^2\alpha}, a_1 = 0, a_0 = a_0,$$

$$c = \frac{1}{25}\frac{25\alpha a_0 s - 6\beta^2 + 25\gamma s}{s}, g = 0, k = \frac{1}{5}\frac{\beta}{s\lambda}, \quad (4.10)$$

where a_0 is an arbitrary constants.

Substituting (4.10) into (4.6), we get that

$$u_2(\xi) = -\frac{12}{25}\frac{\beta^2\mu^2}{s\lambda^2\alpha}G^2 + \frac{24}{25}\frac{\beta^2\mu}{s\lambda\alpha}G + a_0$$

$$\xi = \frac{1}{5}\frac{\beta}{s\lambda}(x + y - \frac{1}{25}\frac{25\alpha a_0 s - 6\beta^2 + 25\gamma s}{s}t) \quad (4.11)$$

where k, c are defined as in (4.7).

Combining with Eq. (2.2), wecan obtain the traveling wave solutions of (4.1) as follows:

$$u_2(\xi) = -\frac{12}{25}\frac{\beta^2\mu^2}{s\lambda^2\alpha}\left(\frac{1}{\frac{\mu}{\lambda} + de^{\lambda\xi}}\right)^2 + \frac{24}{25}\frac{\beta^2\mu}{s\lambda\alpha}\left(\frac{1}{\frac{\mu}{\lambda} + de^{\lambda\xi}}\right) + a_0$$

(4.12)

where a_0, d are arbitrary constants.

$$\xi = \frac{1}{5}\frac{\beta}{s\lambda}(x + y - \frac{1}{25}\frac{25\alpha a_0 s - 6\beta^2 + 25\gamma s}{s}t)$$

Remark : Our results (4.9) and (4.12) are new families of exact traveli-ngwave solutions for Eq.(4.1).

CONCLUSIONS

We have seen that some new traveling wave solutions of 1D and 2D-BKDV equation are successfully found by using the Bernoulli sub-ODE method. The main pointsof the method are that assuming the solution of the ODE reduced by usingthe traveling wave variable as well as integrating can be expressed by an m -th degree polynomialin G , where $G = G(\xi)$ is the general solutions of aBernoulli sub-ODE equation. The positive integer m can be determined bythe general homogeneous balance method, and the coefficients of the polynomialcan be obtained by solving a set of simultaneous algebraic equations.Also this method can be used to many other nonlinear problems.

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