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Fixed Point Theorems For Quasi- Contraction with Applications

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ABSTRACT - The purpose of this topic is to establish fixed point theorems For quasi-contraction and theorems on contraction mapping. After this Result will be use multivalued contraction mappings. Introduced the concept of fixed point theorems for uqasi-contraction and established some results.
A mappings $T : M \longrightarrow M$ of a matric space M into itself is said to be a quasi-contraction iff there exists a number $q, 0 \leq q < 1$,such that
$$d(tx,ty) \leq q.\max\{d(x,y);d(x,tx);d(y,ty);d(x,ty);d(y,tx)\}$$
holds for every x,y ϵ M.

KEYWORDS – fixed point, Unique fixed point, Quasi-contraction,Banach contraction principle.

I. INTRODUCTION –

fixed point theory is an important area in fast growing fields of non-linear analysis and non-linear opretors. A fixed point (also known as an invariant point) of a function is a point that is mapped to itself by the function. That is ,x is a fixed point of the functionfixed point theory is an important area in fast growing fields of non-linear analysis and non-linear opretors. A fixed point (also known as an invariant point) of a function is a point that is mapped to itself by the function. That is ,x is a fixed point of the function f if and only if $f(x) = x$.

Example if f is defined on the real number by $f(x) = 2x - 2$, then 2 is a fixed point of f, because $f(2) = 2$. In graphical terms a fixed point means the point $(x, f(x))$ is on the line $Y = x$ or in other words the graph of f has a point in common with that line.

A fixed point theorem is a result saying that a function f will have at least one fixed point , under some conditions on f that can be stated in general term. Result of this kind are amongst the most generally useful in mathematics. Fixed point theorm concerning the existenct and nature of fixed points used to give solutions to equations.

Every continuous function f from a closed disk to itself has at least one fixed point.

Fixed point property . A topological space x is said to have the fixed point property if for every continuous mapping T form X into itself, there exist a point x in X such that x = Tx.

Clearly a contraction mapping is continuous but the converse meed not be true.

The Banach contraction principle state that a contraction mapping if a complete metric space into itself has a Unique fixed points.

Quasi-contractions – Let T be a mapping of metric space M into itself for A C M let

$$\delta(A) = \sup\{d(a,b): a,b \in A\}$$

and for each x ϵ M,

let

$$0(x,n) = \{x,tx\ldots\ldots,t^n x\},\ n = 1,2\ldots..,$$

$$0(x,\infty) = \{x,tx,\ldots\ldots\ldots\}.$$

A space M is said to be T orbitally complete iff every cauchy sequence which is contained in $0(x,\infty)$ for some x ϵ M converges in M.

Defination – A mappings $T : M \rightarrow M$ of a matric space M into itself is said to be a quasi-contraction iff there exists a number $q, 0 \leq q < 1$,such that

$$d(Tx,ty) \leq q.\max\{d(x,y);d(x,tx);d(y,ty);d(x,ty);d(y,tx)\}$$

Holds for every x,y ϵ M.

Before stating the fixed –point theorem for quasi-contractions we will prove two lemmas on these mapping. First lemma is fundamental.

Lemma 1.- let T be a quasi-contraction on M and let η be any positive integer. Then for each x ϵ M and all positive integers i and j,i,j ϵ $\{1,2\ldots\ldots.,\eta\}$ implies $d(T^i x, T^j x) \leq q.\delta[0(x,n)]$.

Proof. Let x ϵ M be arbitrary,let η by any positive integers and let i and j satisfy the condition of lemma 1 . then $T^{i-1}x, T^i x, T^{j-1}x, T^j x \in 0(x,n)$(where it is understood that $T^0x=x$) and since T is a quasi contraction, we have

$$d(t^i x, T^j x) = d(TT^{i-1}x, TT^{j-1}x)$$
$$\leq q.\max\{d(T^{i-1}x,T^{j-1}x);d(T^{i-1}x,T^j x);d(T^{j-1}x,T^j x);d(T^{i-1}x,T^j x);d(T^i x,T^{j-1}x)\}$$
$$\leq q.\delta[0(x,n)].$$

Which proves the lemma.

Lemma 2. - If T is a quasicontraction on M, then

$$\delta[0(x,\infty)] \leq (1/(1-q))\, d(x,tx)$$

holds for all x ϵ M.

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Proof. Let x ϵ M be arbitrary. Since $\delta[o(x,1)] \leq \delta[0(x,2)] \leq \ldots\ldots$,we see that $\delta[0(x,\infty)] = \sup\{\delta[0(x,n)] : n \in N\}$. The lemma will follow if we show that $\delta[0(x,n)] \leq (1/1\text{-}q)\ d(x,Tx)$ for all n ϵ N.

Let ɳ be any positive integer. From the remark to the previous lemma, there exists $T^k x \in 0(x,n)(1 \leq k \leq n)$ such that $d(x,T^k x) = \delta[0(x,n)]$.

Applying a triangle inequality and lemma 1, we get

$$d(x,T^k x) \leq d(x,tx) + d(Tx,T^k x)$$
$$\leq d(x,tx) + q.\delta[0(x,n)]$$
$$= d(x,tx) + q.d(x,Tx).$$

Thererfore,$\delta[0(x,n)] = d(x,T^k x) \leq (\ 1/(1\text{-}q)\ d(x,Tx)$.
Since ɳ was arbitrary, the proof is completed.

II. MAIN RESULT

Theorem. Let T be a quasi-contraction on a metric space M and let M be T –orbitally complete. Then

(1) T has a unique fixed point u in M,
(2) $\text{Lim}_n T^n x = u$,and
(3) $d(T^n x,u) \leq (\ q^n/(1\text{-}q)\ d(x,tx)$ for every x ϵ M.

Proof. Let x be an arbitrary point of M . we shall show that the sequence of iterates $\{T^n x\}$ is a Cauchy sequence.

Let n and m (n < m) be any positive integers. Since T is a quasi-contraction, it follows from lemma 1 that

$$d(T^n x,T^m x) = d(TT^{n-1}x,\ T^{m-n+1}T^{n-1}x)$$
$\leq q.\delta[0(T^{n-1}x,m\text{-}n+1)]$.

According to the remark to lemma 1, there exists an integer k_1, $1 \leq k \leq m\text{-}n+1$, such that

$$\delta[0(T^{n-1}x,\ m\text{-}n+1)] = d(T^{n-1}x,\ T^{k1}T^{n-1}x).$$

Again, by lemma 1, we have

$d(T^{n-1}x,T^{k1}T^{n-1}x) = d(TT^{n-2}x,T^{k1}+{}^1T^{n-2}x)$
$$d(T^{n-1}x,T^{k1}T^{n-1}x) \leq q.\delta[0(T^{n-2}x,k_1+1)$$
$$d(T^{n-1}x,T^{k1}T^{n-1}x) \leq q.\delta[0(T^{n-2}x,m\text{-}n+2)].$$

Therefore, we have the following system of inequalities.

$$d(T^n x,T^m x) \leq q.\delta[0(T^{n-1}x,m\text{-}n+1)]$$
$$d(T^n x,T^m x) \leq q^2.\delta[0(T^{n-2}x,\ m\text{-}n+2)].$$

Proceeding in this manner ,we obtain
$d(T^n x,T^m x) \leq q.\delta[0(T^{n-1}x,m\text{-}n+1)] \leq \ldots \leq q^n.\delta[o(x,m)]$.

Then it follows from lemma 2 that

$$d(T^n x,T^m x) \leq (q^n/1\text{-}q)\ d(x,Tx).$$

Since $\lim_n q^n = 0$,$\{T^n x\}$ is a cauchy sequence.

Again, M being T – orbitally complete, $\{T^n x\}$ has a limit u in M. To prove that Tu=u, let us conside r the following inequalities.

$$d(u,Tu) \leq d(u,T^{n+1}x) + d(TT^n x,Tu)$$
$\leq d(u,T^{n+1}x) + q.\max\{d(T^n x,u);d(T^n x,T^{n+1}x);d(u,tu);d(T^n x,Tu);d(T^{n+1}x,u)\}$
$\leq d(u,t^{n+1}x) + q.[d(T^n x,T^{n+1}x) + d(T^n x,u) + d(u,tu) + d(T^{n+1}x.u)]$.

Hence

$d(u,Tu) \leq ⅟1—q[(1+q)d(u,T^{n+1}x) + q.d(u,T^n x,T^{n+1}x)]$.

Since $\lim_n T^n x = u$, this shows that d(u,tu) = 0. The uniqueness follows from the quasi-contractivity of T.So we have (1) and (2) as x was arbitrary. Letting m tend to infinity so we obtain the inequality(3).

The proof is completed.

III. APPLICATION.

The application of fixed point theorem enocompass diverse disciplines of mathematics ,statics ,engineering and economics in dealing with problems arising in approximation theory, potential theory, game theory, mathematical economics ,theory of differential equations etc.

To use the Banach contraction principle for Banach theorem.

Theorem [Banach]

let T be a contraction on a complete metric space X. Then T has a unique fixed point x ϵ X.

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