International Journal of Advances in Engineering and Management (IJAEM)
Volume 2, Issue 6, pp: 518-520 www.ijaem.net ISSN: 2395-5252

(Σ, Τ) Generalized Derivations in Prime Rings

DR. K.L. Kaushik
Associate Professor in Mathematics Aggarwal College, Ballabgarh Faridabad , INDIA

ABSTRACT: We prove that $[x,yz]_{\sigma,\tau}= \tau(y)[x,y]_{\sigma,\tau}+ [x,y]_{\sigma,\tau}\sigma(z)$, where σ,τare the automorphisms of R, R is 2-torsion free Prime ring.
Finally, we have proved a Theorem from which Mohammad Ashraf and Nadeem UrRehman [2] Lemma 2.2 on Page 260 can be derived immediately.
Keywords 2-torsion free, Prime Ring, (σ,τ) generalized derivation, Automorphism.

Date of Submission: 15-09-2020 Date of Acceptance: 26-09-2020

I. INTRODUCTION

We have used Havala [1] definition, "LetR be a ring. The additive map f :R → R is said to be generalized derivation if

$$f(xy) = f(x)y + xd(y)$$

where dis derivation of R and x,y∈ R. We have defined an additive mapping f :R → R is said to be (σ,τ) generalized derivation if

$$f(xy) = f(x)\sigma(y) + \sigma(x)d(y)\ \forall\ x,y\in R$$

where σ,τare automorphisms of R. We have proved that

$$[x,yz]_{\sigma,\tau}= \tau(y)[x,y]_{\sigma,\tau}+ [x,y]_{\sigma,\tau}\sigma(z)$$

Finally we proved a theorem "LetR be a 2-torsion free ring, I be a non zero ideal of R. If R admits a (σ,τ) generalized derivation f such that $f^2(I) = (0)$ and f commutes with both σ,τThen f = 0,d = 0. "

From this Theorem we immediately derive Mohammad Ashraf and Nadeem Ur-Rehman [2] Lemma 2.2 on Page 260 as corollary.

1 (σ,τ) Generalized Derivation

1.1 Definition(Prime Ring): Let A be any ring. Then its is said to be a Prime ring iff

xay= 0 a ∈ A ⇒ x = 0 or y = 0

1.2 Definition(2-Torsion free Prime Ring): A prime Ring A with characteristic different from 2 is called 2-Torsion free Prime Ring.

1.3 Definition $[x,y]_{\sigma,\tau}$**:** Let R be a 2-torsion free Prime ring. Let σ,τare automorphisms of R. We set

$$[x,y]_{\sigma,\tau}= x\sigma(y) - \tau(y)x\ \forall\ x,y\in R$$

1.4 Definition ((σ,τ) Generalised Derivation): Havala [1] defined "An additive mapping f : R → R" is called Generalised Derivation if

$$f(xy) = f(x)y + xd(y)$$

where d is the derivation of R. We define an additive mapping f :R → R" is called (σ,τ) Generalised Derivation if

$$f(xy) = f(x)\sigma(y) + \tau(x)d(y)\ \forall\ x,y\in R$$

Lemma 1.5. Prove that

$$[xy,z]_{\sigma,\tau}= x[y,z]_{\sigma,\tau}+ [x,\tau(z)]y = x[y,\sigma(z)] + [x,z]_{\sigma,\tau}y$$

Proof Now

$$\begin{aligned}[xy,z]_{\sigma,\tau} &= xy\sigma(z) - \tau(z)xy\\ &= xy\sigma(z) - x\tau(z)y + x\tau(z)y - \tau(z)xy\\ &= x(y\sigma(z) - \tau(z)y) + (x\tau(z) - \tau(z)x)y\\ &= x[y,z]_{\sigma,\tau}+ [x,\tau(z)]y\end{aligned}$$

Hence 1st part is proved.

Now 2nd part

$$\begin{aligned}[xy,z]_{\sigma,\tau} &= xy\sigma(z) - \tau(z)xy\\ &= xy\sigma(z) - x\sigma(z)y + x\sigma(z)y - \tau(z)xy\\ &= x(y\sigma(z) - x\sigma(z)) + (x\sigma(z) - \tau(z)x)y\\ &= x[y,\sigma(z)] + [x,z]_{\sigma,\tau}y\end{aligned}$$

Hence proved.

Lemma 1.6. Prove that

$[x,yz]_{\sigma,\tau} = \tau(y)[x,y]_{\sigma,\tau} + [x,y]_{\sigma,\tau}\sigma(z)$

Proof Now

$$
\begin{aligned}
[x,yz]_{\sigma,\tau} &= x\sigma(yz) - \tau(yz)x \\
&= x\sigma(y)\sigma(z) - \tau(y)\tau(z)x \quad (\because \sigma,\tau \text{ are automorphisms}) \\
&= x\sigma(y)\sigma(z) - \tau(y)x\sigma(z) + \tau(y)x\sigma(z) - \tau(y)\tau(z)x \\
&= (x\sigma(y) - \tau(y)x)\sigma(z) + \tau(y)(x\sigma(z) - \tau(z)x) \\
&= [x,y]_{\sigma,\tau}\sigma(z) + \tau(y)[x,z]_{\sigma,\tau}
\end{aligned}
$$

Hence proved.

2. Now we prove a Theorem 2.2 which would be useful in getting results generalized derivation. We will use the following Lemma to prove Theorem 2.2.

Lemma 2.1 Let R be a prime ring, I 6= 0 Ideal of R and a ∈ R. If R admits a (σ,τ) derivation d such that ad(I) = 0 or d(I)a = 0

Then either d = 0 or a = 0.

Theorem 2.2 Let R be a 2-Torsion free ring, I be a nonzero Ideal of R. If R admits a (σ,τ) generalized derivation f such that $f^2(I) = (0)$ and f commutes with both σ,τThen f = 0, d = 0.

Proof For any x ∈ I, we have $f^2(x) = 0$

Replacing x by xy, we get $f^2(xy) = 0$

$$
\begin{aligned}
&\Rightarrow f(f(xy)) = 0 \\
&\Rightarrow f\left(f(x)\sigma(y) + \tau(x)d(y)\right) = 0 \\
&\Rightarrow f\left(f(x)\sigma(y)\right) + f\left(\tau(x)d(y)\right) = 0 \\
&\Rightarrow f\left(f(x)\right)\sigma\left(\sigma(y)\right) + \tau(f(x))d(\sigma(y)) + f(\tau(x))\sigma(d(y)) + \tau(\tau(x))d(d(y)) = 0 \\
&\Rightarrow \underbrace{f^2(x)}_{=0}\sigma^2(y) + \tau(f(x))d(\sigma(y)) + f(\tau(x))\sigma(d(y)) + \tau^2(x)\underbrace{d^2(y)}_{=0} = 0 \\
&\Rightarrow \tau(f(x))d(\sigma(y)) + f(\tau(x))\sigma(d(y)) = 0 \\
&\Rightarrow \tau(f(x))\sigma(d(y)) + \tau(f(x))\sigma(d(y)) = 0
\end{aligned}
$$

(∵ f and d both commutes σ,τ)

⇒ $2\tau(f(x))\sigma(d(y)) = 0$

⇒ $\tau(f(x))\sigma(d(y)) = 0$

⇒ $\sigma^{-1}(\tau(f(x)))d(y) = 0$ ∀ x,y∈ I

⇒ By Lemma 2.1 either d = 0 or $\sigma^{-1}(\tau(f(x))) = 0$

If $\sigma^{-1}(\tau(f(x))) = 0$ ∀ x ∈ I

⇒ $\tau(f(x)) = 0$ ∵ σ = automorphism

⇒ $f(x) = 0$ ∀ x ∈ I, ∵ τ = automorphism

Replacing xbyxr, r ∈ R

$$
\begin{aligned}
&f(xr) = 0 \\
&\Rightarrow \underbrace{f(x)}_{=0}\sigma(r) + \tau(x)d(r) = 0 \\
&\Rightarrow \tau(x)d(r) = 0 \\
&\Rightarrow \tau^{-1}(\tau(x))\tau^{-1}(d(r)) = 0 \\
&\Rightarrow x\tau^{-1}(d(r)) = 0 \\
&\Rightarrow \tau^{-1}(d(r)) = 0 \\
&\Rightarrow d(r) = 0 \\
&\Rightarrow d = 0 \\
&\Rightarrow f = 0 \qquad d = 0
\end{aligned}
$$

Hence proved.

Corollary 2.2.1 Replacing f by d, we get Mohd. Asraf and Nadeem-Ur-Rahman [2] Lemma 2.2 on page 260.

II. CONCLUSION

In this Paper we proved that $[x,yz]_{\sigma,\tau}= \tau(y)[x,y]_{\sigma,\tau}+ [x,y]_{\sigma,\tau}\sigma(z)$, where σ,τare the automorphisms of R. Using this we proved our theorem

”LetR be a 2-torsion free ring, I be a non zero ideal of R. If R admits a (σ,τ) generalized derivation f such that $f^2(I) = (0)$ and f commutes with both σ,τThen f = 0,d = 0,” from which Mohammad Ashraf and Nadeem Ur-Rehman [2] Lemma 2.2 on Page 260 comes out as a corollary.

REFERENCES

[1]. Havala, B. generalized Derivations in rings, Communication in Algbera 26 (4), 11471166 (1998).
[2]. Mohammad Ashraf and Nadeem Ur-Rehman, “On (σ,τ)-Derivations in Prime rings” Archivum Mathematicum (BRNO) Tomus 38 (2002) 259-264.
[3]. Jacobson Nathan, Lie algebras. Interscience Publishers, New York (1992)
[4]. I. N. Herstein, Jordan Derivation of Prime rings, Proc. Amer. Math. Soc. 8 (1957) 1104-1110.