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Investigation on Exact Solutionsin the Course of Mathematical and Physical Equations

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ABSTRACT: In this paper, a generalized sub-ODE method is proposed toconstruct exact solutions ofBoussinesqequation and (2+1) dimensional Boussinesq equation in the teaching of the course of mathematical and physical equations. As a result, some new exact tra-veling wavesolutions are found.

KEYWORDS:sub-ODE method, traveling wave solutions,exact solution, evolution equation, Boussinesq equation, (2+1) dimensional Bouss-inesq equation

I. INTRODUCTION

In the teaching of the course of mathematical and physical equations, seeking the exact solutions of no-nlinearequations is a hot topic. Many approaches have beenpresented so far. Some of these approaches are the homogeneous balancemethod], the hyperbolic tangent expansion method , the trialfunction method, the tanh-method, the nonlinear transform method, the inverse scattering transform, the Backlund transform, theHirotas bilinear method, the generalized Riccati equation, the theta function method, the sine-Ccosine method, the Jacobi elliptic function expa-nsion, the complex hyperbolic function method andso on[1-7].

In this paper, we proposed a sub-ODE method to constructexact traveling wave solutions for NLEES.

The rest of the paper is organized as follows. In Section 2, we describethe sub-ODE method for finding traveling wave solutions of nonlinearevolution equations, and give the main steps of the method. In thesubsequent sections, we will apply the method to find exact traveling wave solutionsof the Boussinesq equation and (2+1) dimensional Boussinesq equation.In the last Section, some conclusions are presented.

II. DESCRIPTION OF THE SUB-ODE METHOD

In this section we present the solutions of the following ODE:

$$G'+\lambda G=\mu G^2 \quad (2.1)$$

where $\lambda\neq 0, G=G(\xi)$

The solution of Eq.(2.1) is denoted as follows

$$G=\frac{1}{\frac{\mu}{\lambda}+de^{\lambda\xi}} \quad (2.2)$$

where d is an arbitrary constant.

Suppose that a nonlinear equation, say in two or three indep-endentvariablesx, y andt , is given by

$$P(u,u_{t,}u_x,u_y,u_{tt},u_{xt},u_{yt},u_{xx},u_{yy}......)=0 \quad (2.3)$$

whereu = u(x, y, t) is an unknown function,P isa polyno-mial in u = u(x, y, t) and its various partialderivatives, in which the highest order derivatives andnonlinear terms are involved. By using the solutions of Eq.(2.1),we can constr-uct a serials of exact solutions of nonlinear equations:.

Step 1.We suppose that

$$u(x,y,t)=u(\xi),\xi=\xi(x,y,t) \quad (2.4)$$

the travelling wave variable (2.4) permits us reducing Eq.(2.3) to an ODE for $u=u(\xi)$

$$P(u,u',u'',......)=0 \quad (2.5)$$

Step 2. Suppose that the solution of (2.5) can be expre-ssedby a polynomial in G as follows:

$$u(\xi)=\alpha_m G^m+\alpha_{m-1}G^{m-1}+...... \quad (2.6)$$

where $G=G(\xi)$ satisfies Eq.(2.1), and $\alpha_m,\alpha_{m-1}...$ are constants to be determined later, $\alpha_m\neq 0$ .The positive integer m can be determined by considering thehomogen-eous balance between

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the highest order derivativesand non-linear terms appearing in (2.5).

Step 3. Substituting (2.6) into (2.5) and using (2.1), collecting all terms with the sameorder of G together, the left-hand side of Eq. (2.5)is converted into another polynomial in G. Equatingeach coefficient of this polynomial to zero, yields a set ofalgebraic equations for $\alpha_m, \alpha_{m-1}, ...\lambda, \mu$.

Step 4. Solving the algebraic equations system in Step 3, and by usingthe solutions of Eq.(2.1), we can construct the traveling wave solutions of thenonlinear evolution equation (2.5)..

In the subsequent sections we will illustrate the proposedmethod in detail by applying it to Boussinesq equation and (2+1) dimensional Boussinesq equation.

III.APPLICATION FOR BOUSSINESQ EQUATION

In this section, we will consider the following Boussinesq equation:

$$u_{tt} + \alpha u_{xx} + \beta(u^2)_{xx} + \gamma u_x^{(4)} = 0, \alpha < 0 \quad (3.1)$$

Suppose that

$$u(\xi), \xi = k(x - ct) \quad (3.2)$$

where the constants c, k can be determined later.

By using (3.2), (3.1) is converted into an ODE

$$(\alpha + c^2)u'' + \beta(u^2)'' + \gamma k^3 u^{(5)} = 0 \quad (3.3)$$

Integrating (3.3) twice, and take the integration constant for zero, then we have

$$(\alpha + c^2)u + \beta u^2 + \gamma k^3 u''' = 0 \quad (3.4)$$

Suppose that the solution of (3.4) can be expressed by apolynomial in G as follows:

$$u(\xi) = \sum_{i=0}^{m} a_i G^i$$

where a_i are constants, and $G = G(\xi)$ satisfiesEq. (2.1).

Balancing the order of u^2 and u''' in Eq.(3.4), we obtain that $2m = m + 3 \Rightarrow m = 3$. So Eq.(3.4) can berewritten as

$$u(\xi) = a_3 G^3 + a_2 G^2 + a_1 G + a_0, a_3 \neq 0 \quad (3.5)$$

a_3, a_2, a_1, a_0 are constants to be determined later.

Substituting (3.5) into (3.3) and collecting all the termswith the same power of G together, the left-handside of Eq.(3.3) is converted into another polynomial in G.

Equating eachcoefficient to zero, yields a set of simul-taneous algebraic equations as follows:

$$G^0 : \beta a_0^2 + \alpha a_0 + c^2 a_0 = 0$$

$$G^1 : 2\beta a_0 a_1 + \alpha a_1 + c^2 a_1 + \gamma k^3 a_1 \mu^3 = 0$$

$$G^2 : -7\gamma k^3 \lambda a_1 \mu^2 + 2\beta a_0 a_2 + (c^2 + \alpha)a_2 + 8\gamma k^3 a_2 \mu^3 + \beta a_1^2 = 0$$

$$G^3 : -38\gamma k^3 a_2 \mu^2 \lambda + (c^2 + \alpha)a_3 + 2\beta a_0 a_3 + 2\beta a_1 a_2 + 12\gamma k^3 \lambda^2 a_1 \mu + 27\gamma a_3 k^3 \mu^3 = 0$$

$$G^4 : 54\gamma a_2 \mu k^3 \lambda^2 + \beta a_2^2 + 2\beta a_1 a_3 - 111\gamma a_3 \lambda k^3 \mu^2 - 6\gamma a_1 k^3 \lambda^3 = 0$$

$$G^5 : 144\gamma a_3 \mu k^3 \lambda^2 + 2\beta a_2 a_3 - 24\gamma a_2 k^3 \lambda^3 = 0$$

$$G^6 : -60\gamma a_3 k^3 \lambda^3 + \beta a_3^2 = 0$$

Solving the algebraic equations above, yields:

Case 1:

$$a_3 = \frac{60\gamma k^3 \lambda^3}{\beta}, a_2 = a_1 = a_0 = 0, k = k,$$

$$c = \sqrt{-\alpha}, \lambda = \lambda, \mu = 0 \quad (3.6)$$

Substituting (3.6) into (3.5), we have

$$u_1(\xi) = \frac{60\gamma k^3 \lambda^3}{\beta} G^3 \quad (3.7)$$

$$\xi = k(x - \sqrt{-\alpha} t)$$

Combining with Eq. (2.2) and wecan obtain the traveling wave solutions of (3.1) as follows:

$$u_1(x,t) = \frac{60\gamma k^3 \lambda^3}{\beta}[de^{-\lambda k(x - \sqrt{-\alpha} t)}]^3 \quad (3.8)$$

where d isan arbitrary constant.

Case 2:

$$a_3 = \frac{60\gamma k^3 \lambda^3}{\beta}, a_2 = a_1 = a_0 = 0, k = k,$$

$$c = -\sqrt{-\alpha}, \lambda = \lambda, \mu = 0 \quad (3.9)$$

Substituting (3.6) into (3.5), we have

$$u_1(\xi) = \frac{60\gamma k^3 \lambda^3}{\beta} G^3 \quad (3.10)$$

$$\xi = k(x + \sqrt{-\alpha} t)$$

Combining with Eq. (2.2) and wecan obtain the traveling wave solutions of (3.1) as follows:

$$u_1(x,t) = \frac{60\gamma k^3 \lambda^3}{\beta}[de^{-\lambda k(x + \sqrt{-\alpha} t)}]^3 \quad (3.11)$$

where d isan arbitrary constant.

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IV.APPLICATION OF THE BERNOULLI SUB-ODE METHOD FOR (2+1) DIMENSIONAL BOUSSINESQ EQUATION

In this section, we will consider the following (2+1) dimensional Boussinesq equation:

$$u_{tt}-u_{xx}-u_{yy}-(u^2)_{xx}-u_{xxxx}=0 \quad (4.1)$$

Suppose that

$$u(x,y,t)=u(\xi),\xi=kx+ly+mt+d \quad (4.2)$$

l,k,m,d are constants that to be determined later.

By (4.2), (4.1) is converted into an ODE

$$(m^2-k^2-l^2)u''-2k^2(u'^2+uu'')-k^4u''''=0$$

(4.3)

Integrating (4.3) once we obtain

$$(m^2-k^2-l^2)u'-2k^2uu'-k^4u'''=g \quad (4.4)$$

where g is the integration constant.

Suppose that the solution of (4.4) can be expressed by apolynomial in G as follows:

$$u(\xi)=\sum_{i=0}^{m}a_iG^i \quad (4.5)$$

where a_i are constants, and $G=G(\xi)$ satisfies Eq.(2.1).Balancing the order of uu' and u''' in Eq.(4.4), we have $2m+1=m+3\Rightarrow m=2$ .So Eq.(4.5) can berewritten as

$$u(\xi)=a_2G^2+a_1G+a_0,a_2\neq 0 \quad (4.6)$$

a_2,a_1,a_0 are constants to be determined later.

Substituting (4.6) into (4.4) and collecting all the termswith the same power of G together,equating eachcoefficient to zero, yields a set of imultaneous algebraicequations as follows:

$$G^0:-g=0$$

$$G^1:(k^2+l^2-m^2)a_1\lambda+k^4a_1\lambda^3+2k^2a_0a_1=0$$

$$G^2:-7k^4\mu a_1\lambda^2+8k^4a_2\lambda^3+2(k^2+l^2-m^2)a_2\lambda$$
$$+(m^2-k^2-l^2)\mu a_1-a_1^2+2k^2a_1^2\lambda+4k^2a_0a_2\lambda-2k^2a_0a_1\mu=0$$

$$G^3:2(m^2-k^2-l^2)a_2\mu-4k^2a_0a_2\mu+6k^2a_1a_2\lambda$$
$$-38a_2\mu k^4\lambda^2-2k^2a_1^2\mu+12a_1\lambda k^4\mu^2=0$$

$$G^4:-6k^2a_1a_2\mu+54a_2\lambda k^4\mu^2-6a_1k^4\mu^3+4k^2a_2^2\lambda=0$$

$$G^5:-24a_2k^4\mu^3-4k^2a_2^2\mu=0$$

Solving the algebraic equations above, yields:

$$a_2=-6k^2\mu^2,a_1=-6k^2\mu\lambda,a_0=-\frac{1}{2}\frac{l^2+k^2-m^2+\lambda^2k^4}{k^2}$$

$$k=k,l=l,m=m,d=d \quad (4.7)$$

where k,l,m,d are arbitrary constants.

Substituting (4.7) into (4.6), we get that

$$u(\xi)=-6k^2\mu^2G^2-6k^2\mu\lambda G-\frac{1}{2}\frac{l^2+k^2-m^2+\lambda^2k^4}{k^2}$$

$$\xi=kx+ly+mt+d \quad (4.8)$$

Combining with Eq. (2.2), wecan obtain the traveling wave solutions of (4.1) as follows:

$$u(\xi)=-6k^2\mu^2\left(\frac{1}{\frac{\mu}{\lambda}+de^{\lambda\xi}}\right)^2-6k^2\mu\lambda\left(\frac{1}{\frac{\mu}{\lambda}+de^{\lambda\xi}}\right)$$
$$-\frac{1}{2}\frac{l^2+k^2-m^2+\lambda^2k^4}{k^2} \quad (4.9)$$

Remark : Our result (4.9) is new exact traveli-ngwave solutions for Eq.(4.1).

V.CONCLUSIONS

In the present work,we propose a new sub-ODE method, and then test its power by finding some new traveling wave solutions sof Boussinesq equation and (2+1) dimensional Boussinesq equation Tthis method is one of the most effective approaches handling nonlinear evolu-tion equations. One can see the method is concise and effective. Also this method can be used to many other nonlinear problems.

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