

# Irregular Bipolar Fuzzy Graphs

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Date of Submission: 12-03-2023

Date of Acceptance: 22-03-2023

## ABSTRACT

In this paper, we define irregular bipolar fuzzy graphs and its various classifications. Size of regular bipolar fuzzy graphs is derived. The relation between highly and neighbourly irregular bipolar fuzzy graphs are established. Some basic theorems related to the stated graphs have also been presented.

## Keywords

Fuzzy Relation, Degree, Symmetric, bipolar fuzzy graph, Total Degree, Totally Irregular bipolar fuzzy graph, Induced sub graph, Complement, Crisp Graph

## PRELIMINARIES OF IRREGULAR BIPOLAR FUZZY GRAPHS

In this section, we first review some definitions of graphs that are necessary for this paper

**Definition 1.1** A complete graph is a simple graph in which every pair of distinct vertices is connected by an edge. An isomorphism of graphs  $G_1^*$  and  $G_2^*$  is a bijection between the vertex sets of  $G_1^*$  and  $G_2^*$  such that any two vertices  $v_1$  and  $v_2$  of  $G_1^*$  are adjacent in  $G_1^*$  if and only if  $f(v_1)$  and  $f(v_2)$  are adjacent in  $G_2^*$ . Isomorphic graphs are denoted by  $G_1^* \cong G_2^*$ .

**Definition 1.2** In graph theory, the line graph  $L(G^*)$  of a simple graph  $G^*$  is another graph  $L(G^*)$  that represents the adjacencies between edges of  $G^*$ . Given a graph  $G^*$ , its line graph  $L(G^*)$  is a graph such that,

- each vertex of  $L(G^*)$  represents an edge of  $G^*$ ; and
- two vertices of  $L(G^*)$  are adjacent if and only if their corresponding edges share a common end point (“are adjacent”) in  $G^*$ .

Let  $G^* = (V, E)$  be an undirected graph, where  $V = \{v_1, v_2, \dots, v_n\}$ : Let  $S_i = \{v_i, x_{i1}, \dots, x_{iq_i}\}$  where  $x_{ij} \in E$  has vertex  $v_i$   $i = 1; 2; \dots; n$ ,  $j = 1, 2, \dots, q_i$ . Let  $S = \{S_1, S_2, \dots, S_n\}$ : Let  $T = \{S_i S_j \mid S_i S_j \in S, S_i \cap S_j \neq \emptyset, i \neq j\}$ . Then  $P(S) = (S, T)$  is an intersection graph and  $P(S) = G^*$ . The line graph  $L(G^*)$  is by definition the intersection graph  $P(E)$ . That is,  $L(G^*) = (Z, W)$  where  $Z = \{x\} \cup \{u_x v_x \mid x \in E, u_x v_x \in V, x = u_x v_x\}$  and  $W = \{S_x S_y \mid S_x \cap S_y \neq \emptyset, x y \in E, x \neq y\}$ , and  $S_x = \{x\} \cup \{u_x v_x\}$   $x \in E$ .

**Definition 1.3** A fuzzy set  $A$  on a set  $X$  is characterized by a mapping  $m : X \rightarrow [0,1]$ , called the membership function. A fuzzy set is denoted as  $A = (X, m)$ . A fuzzy graph  $\xi = (V, \sigma, \mu)$  is a non-empty set  $V$  together with a pair of functions  $\sigma : V \rightarrow [0,1]$  and  $\mu : V \times V \rightarrow [0,1]$  such that for all  $u, v \in V$ ,  $\mu(u, v) \leq \sigma(u) \wedge \sigma(v)$  (here  $x \wedge y$  denotes the minimum of  $x$  and  $y$ ). Partial fuzzy sub graph  $\xi' = (V, \tau, \nu)$  of  $\xi$  is such that  $\tau(v) \leq \sigma(v)$  for all  $v \in V$  and  $\nu(u, v) \leq \mu(u, v)$  for all  $u, v \in V$ . Fuzzy sub graph  $\xi'' = (P, \sigma', \mu')$  of  $\xi$  is such that  $P \subseteq V$ ,  $\sigma'(u) = \sigma(u)$  for all  $u \in P$ ,  $\mu'(u, v) = \mu(u, v)$  for all  $u, v \in P$ .

**Definition 1.4** A fuzzy graph is complete if  $\mu(u, v) = \sigma(u) \wedge \sigma(v)$  for all  $u, v \in V$ . The degree of vertex  $u$  is  $d(u) = \sum_{(u,v) \in E} \mu(u, v)$ . The minimum degree of  $\xi$  is  $\delta(\xi) = \wedge \{d(u) \mid u \in V\}$ . The maximum degree of  $\xi$  is  $\Delta(\xi) = \vee \{d(u) \mid u \in V\}$ . The total degree of a vertex  $u \in V$  is  $td(u) = d(u) + \sigma(u)$ .

**Definition 1.5** A fuzzy graph  $\xi = (V, \sigma, \mu)$  is said to be regular if  $d(v) = k$ , a positive real number, for all  $v \in V$ . If each vertex of  $\xi$  has same total degree  $k$ , then  $\xi$  is said to be a totally regular fuzzy graph.

**Definition 1.6** A fuzzy graph is said to be irregular, if there is a vertex which is adjacent to vertices

with distinct degrees. A fuzzy graph is said to be neighbourly irregular, if every two adjacent vertices of the graph have different degrees.

**Definition 1.7** A fuzzy graph is said to be totally irregular, if there is a vertex which is adjacent to vertices with distinct total degrees. If every two adjacent vertices have distinct total degrees of a fuzzy graph then it is called neighbourly total irregular.

**Definition 1.8** A fuzzy graph is called highly irregular if every vertex of  $G$  is adjacent to vertices with distinct degrees.

The complement of fuzzy graph  $\xi = (V, \sigma, \mu)$  is the fuzzy graph  $\xi' = (V, \sigma', \mu')$  where  $\sigma'(u) = \sigma(u)$  for all  $u \in V$  and  $\mu'(u, v) = \begin{cases} 0, & \text{if } \mu(u, v) > 0, \\ \sigma(u) \wedge \sigma(v), & \text{otherwise.} \end{cases}$

### IRREGULAR BIPOLAR FUZZY GRAPHS

**Definition 2.1** Let  $G = (A, B)$  be a bipolar fuzzy graph where  $A = (m_1^+, m_1^-)$  and  $B = (m_2^+, m_2^-)$  be two bipolar fuzzy sets on a non-empty finite set  $V$  and  $E \subseteq V \times V$  respectively.

The graph  $G$  is called complete bipolar fuzzy graph if  $m_2^+(u, v) = \min \{ m_1^+(u), m_1^+(v) \}$  and  $m_2^-(u, v) = \max \{ m_1^-(u), m_1^-(v) \}$  for all  $u, v \in V$ .

**Definition 2.2** Let  $G = (A, B)$  be a bipolar fuzzy graph where  $A = (m_1^+, m_1^-)$  and  $B = (m_2^+, m_2^-)$  be two bipolar fuzzy sets on a non-empty finite set  $V$  and  $E \subseteq V \times V$  respectively.

If  $d^+(u) = k_1$ ,  $d^-(u) = k_2$  for all  $u \in V$ ,  $k_1, k_2$  are two real numbers, then the graph is called  $(k_1, k_2)$ -regular bipolar fuzzy graph.

**Definition 2.3** Let  $G = (A, B)$  be a bipolar fuzzy graph where  $A = (m_1^+, m_1^-)$  and  $B = (m_2^+, m_2^-)$  be two bipolar fuzzy sets on a non-empty finite set  $V$  and  $E \subseteq V \times V$  respectively. The total degree of a vertex  $u \in V$  is denoted by  $td(u)$  and studied as  $td(u) = (td^+(u), td^-(u))$  Where  $td^+(u) = \sum_{(u,v) \in E} m_2^+(u, v) + m_1^+(u)$ ,  $td^-(u) =$

$\sum_{(u,v) \in E} m_2^-(u, v) + m_1^-(u)$ . If all the vertices of a bipolar fuzzy graph are of total degree, then the graph is said to be totally regular bipolar fuzzy graph.

Degree of a vertex of a bipolar fuzzy graph is studied below,

**Definition 2.4** Let  $G = (A, B)$  be a bipolar fuzzy graph where  $A = (m_1^+, m_1^-)$  and  $B = (m_2^+, m_2^-)$  be two bipolar fuzzy sets on a non-empty finite set  $V$  and  $E \subseteq V \times V$  respectively.

The positive degree of a vertex  $u \in G$  is  $d^+(u) = \sum_{(u,v) \in E} m_2^+(u, v)$ . Similarly negative degree of a vertex  $u \in G$  is  $d^-(u) = \sum_{(u,v) \in E} m_2^-(u, v)$ . The degree of a vertex  $u$  is  $d(u) = (d^+(u), d^-(u))$ .

**Example 2.5** We consider a bipolar fuzzy graph shown in Figure 0. Here  $d^+(v_1) = 0.4 + 0.5 = 0.9$ ,  $d^-(v_1) = (-0.3) + (-0.2) = -0.5$ . so  $d(v_1) = (0.9, -0.5)$ . similarly  $d(v_2) = (0.9, -0.7)$  and  $d(v_3) = (1, -0.8)$ . order and size of a bipolar fuzzy graph is an important term in bipolar fuzzy graph theory. They are studied below.

**Definition 2.6** Let  $G = (A, B)$  be a bipolar fuzzy graph where  $A = (m_1^+, m_1^-)$  and  $B = (m_2^+, m_2^-)$  be two bipolar fuzzy sets on a non-empty finite set  $V$  and  $E \subseteq V \times V$  respectively. The order of  $G$  is denoted by  $O(G)$  and studied as  $O(G) = (O^+(G), O^-(G))$  where

$$O^+(G) = \sum_{u \in V} m_1^+(u) \text{ and } O^-(G) = \sum_{u \in V} m_1^-(u)$$

**Definition 2.7** Let  $G = (A, B)$  be a bipolar fuzzy graph where  $A = (m_1^+, m_1^-)$  and  $B = (m_2^+, m_2^-)$  be two bipolar fuzzy sets on a non-empty finite set  $V$  and  $E \subseteq V \times V$  respectively. The size of  $G$  is studied by  $S(G) = (S^+(G), S^-(G))$  where  $(S^+(G)) = \sum_{(u,v) \in E, u \neq v} m_2^+(u, v)$   
 $(S^-(G)) = \sum_{(u,v) \in E, u \neq v} m_2^-(u, v)$

**Example 2.8** In Figure 1, a bipolar fuzzy graph is shown. Here  $O(G) = (1.9, -1.4)$  and  $S(G) = (1.4, -1)$

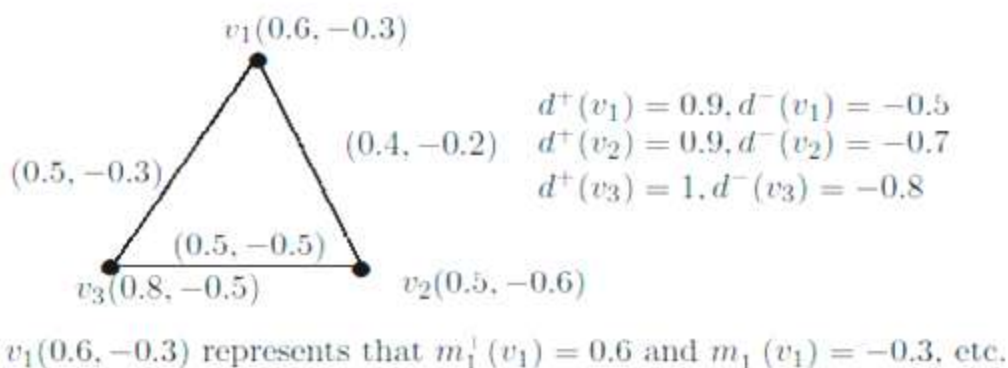


Figure 1 Degrees of the vertices of a bipolar fuzzy graph

**Definition 2.9** Let  $G = (A, B)$  be a bipolar fuzzy graph where  $A = (m_1^+, m_1^-)$  and  $B = (m_2^+, m_2^-)$  be two bipolar fuzzy sets on a non-empty finite set  $V$  and  $E \subseteq V \times V$  respectively. The underlying crisp graph of  $G$  is the crisp graph  $G' = (V', E')$  where  $V' = \{v \mid m_1^+(v) > 0 \text{ or } m_1^-(v) < 0\}$  and  $E' = \{(u, v) \mid m_2^+(u, v) > 0 \text{ or } m_2^-(u, v) < 0\}$ .

**Definition 2.10** A bipolar fuzzy graph is said to be connected if its underlying crisp graph is connected.

**Example 2.11** A crisp underlying graph is shown in Figure 2. It is also an example of connected bipolar fuzzy graph.

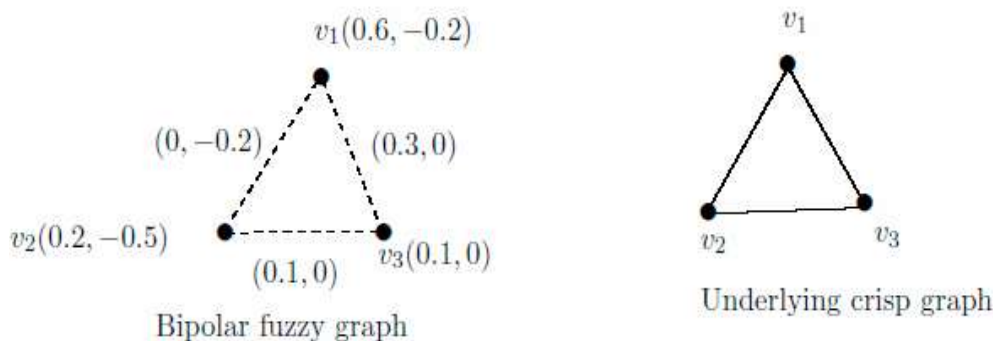


Figure 2 Example of connected regular bipolar fuzzy graph

**Theorem 2.12** Let  $G$  be a regular bipolar fuzzy graph where induced crisp graph  $G'$  is an even cycle. Then  $G$  is regular bipolar fuzzy graph if and only if either  $m_2^+$  and  $m_2^-$  are constant functions or alternate edges have same positive membership values and negative membership values.

$$m_2^+(e_i) = \begin{cases} c_1, & \text{if } i \text{ is odd} \\ k_1 - c_1, & \text{if } i \text{ is even} \end{cases}$$

$$m_2^-(e_i) = \begin{cases} c_2, & \text{if } i \text{ is odd} \\ k_2 - c_2, & \text{if } i \text{ is even} \end{cases}$$

**Proof.** Let  $G = (A, B)$  be a regular bipolar fuzzy graph where  $A = (m_1^+, m_1^-)$  and  $B = (m_2^+, m_2^-)$  be two bipolar fuzzy sets on a non-empty finite set  $V$  and  $E \subseteq V \times V$  respectively and underlying crisp graph  $G'$  of  $G$  be an even cycle. If either  $m_2^+, m_2^-$  are constant functions or alternate edges have same positive and negative membership values, then  $G$  is a regular bipolar fuzzy graph. Conversely, suppose  $G$  is a  $(k_1, k_2)$  regular bipolar fuzzy graph. Let  $e_1, e_2, \dots, e_n$  be the edges of  $G'$  in order. As in the theorem 3

If  $c_1 = k_1 - c_1$ , then  $m_2^+$  is constant.  $C_1 \neq k_1 - c_1$ , then alternate edges have same positive and negative membership values. Similarly for  $m_2^-$ . Hence the results.

**Theorem 2.13** The size of a  $(k_1, k_2)$  -regular bipolar fuzzy graph is  $(\frac{pk_1}{2}, \frac{pk_2}{2})$  where  $p = |V|$ .

**Proof.** Let  $G = (A, B)$  be a bipolar fuzzy graph where  $A = (m_1^+, m_1^-)$  and  $B = (m_2^+, m_2^-)$  be two bipolar fuzzy sets on a non-empty finite set  $V$  and  $E \subseteq V \times V$  respectively. The size of  $G$  is  $S(G) = (\sum_{u \neq v} m_2^+(u, v), \sum_{u \neq v} m_2^-(u, v))$

we have  $\sum_{v \in V} d(v) = 2[\sum_{(u,v) \in E} m_2^+(u,v) + \sum_{(u,v) \in E} m_2^-(u,v)] = 2S(G)$ . so  $2S(G) = \sum_{v \in V} d(v)$ .  
 i.e  $2S(G) = (\sum_{v \in V} (k_1), \sum_{v \in V} (k_2))$ . This gives  $2S(G) = (pk_1, pk_2)$ . Hence the result.

**Theorem 2.14** If  $G$  is  $(k, k')$  –totally regular bipolar fuzzy graph, then  $2S(G) + O(G) = (pk, pk')$  where  $p = |V|$ .

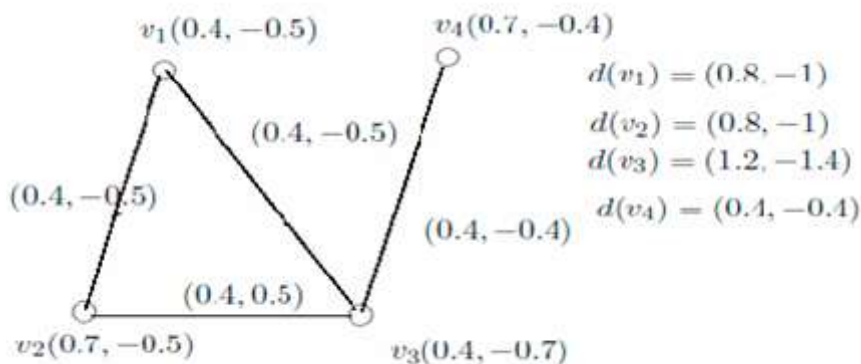
**Proof.** Let  $G = (A, B)$  be a bipolar fuzzy graph where  $A = (m_1^+, m_1^-)$  and  $B = (m_2^+, m_2^-)$  be two bipolar fuzzy sets on a non-empty finite set  $V$  and  $V \times V$  respectively. Since  $G$  is a  $(k, k')$  –totally regular fuzzy graph. So  $k = td^+(v) = d^+(v) + m_1^+(v)$  and  $k' = td^-(v) = d^-(v) + m_1^-(v)$  for all  $v \in V$ . Therefore  $\sum_{v \in V} (k) = \sum_{v \in V} d^+(v) + \sum_{v \in V} m_1^+(v)$  and  $\sum_{v \in V} (k') = \sum_{v \in V} d^-(v) + \sum_{v \in V} m_1^-(v)$ .  $pk = 2S^+(G)$  and  $pk' = 2S^-(G)$ .

So  $pk + pk' = 2(S^+(G) + S^-(G)) + O^+(G) + O^-(G)$ . Hence  $2S(G) + O(G) = (pk, pk')$ .

**Definition 2.15** Let  $G = (A, B)$  be a bipolar fuzzy graph where  $A = (m_1^+, m_1^-)$  and  $B = (m_2^+, m_2^-)$  be two bipolar fuzzy sets on a non-empty finite set  $V$  and  $E \subseteq V \times V$  respectively.

$G$  is said to be irregular bipolar fuzzy graph if there exists a vertex which is adjacent to a vertex with distinct degrees.

**Example 2.16** Let  $G = (A, B)$  be a bipolar fuzzy graph where  $A = (m_1^+, m_1^-)$  and  $B = (m_2^+, m_2^-)$  be two bipolar fuzzy sets on a non-empty finite set  $V$  and  $E \subseteq V \times V$  respectively, where  $V = \{v_1, v_2, v_3, v_4\}$ ,  $d(v_1) = (0.8, -1)$ ,  $d(v_2) = (0.8, -1)$ ,  $d(v_3) = (1.2, -1.4)$ ,  $d(v_4) = (0.4, -0.4)$ . Here  $d(v_2) \neq d(v_3)$ . So this graph is an example of irregular bipolar fuzzy graph.



**Figure 3: Example of irregular bipolar fuzzy graph.**

Neighbourly irregular bipolar fuzzy graph is a special case of irregular bipolar fuzzy graph.

**Definition 2.17** Let  $G$  be a connected bipolar fuzzy graph. Then  $G$  is called neighbourly irregular bipolar fuzzy graph if for every two adjacent vertices of  $G$  have distinct degrees.

**Definition 2.18** Let  $G = (A, B)$  be a bipolar fuzzy graph where  $A = (m_1^+, m_1^-)$  and  $B = (m_2^+, m_2^-)$  be two bipolar fuzzy sets on a non-empty finite set  $V$  and  $E \subseteq V \times V$  respectively.  $G$  is said to be totally irregular bipolar fuzzy graph if there exists a vertex which is adjacent to a vertex with distinct total degrees.

**Example 2.19** Let  $G = (A, B)$  be a bipolar fuzzy graph where  $A = (m_1^+, m_1^-)$  and  $B = (m_2^+, m_2^-)$  be

two bipolar fuzzy sets on a non-empty finite set  $V$  and  $E \subseteq V \times V$  respectively.

Let  $V = \{v_1(0.5, -0.4), v_2(0.4, -0.6), v_3(0.3, 0.2)\}$ .  $d(v_1) = (0.7, -1.1)$ ,  $d(v_2) = (0.6, -1.3)$ , and  $d(v_3) = (0.5, -1.2)$ . Here  $td(v_1) = (1.2, -1.5)$   $td(v_2) = (1, -1.9)$ . So this graph is an example of totally irregular bipolar fuzzy graph.

**Definition 2.20** Let  $G$  be a connected bipolar fuzzy graph. Then  $G$  is called neighbourly totally irregular bipolar fuzzy graph if for every two adjacent vertices of  $G$  have distinct total degrees.

**Definition 2.21** Let  $G$  be a connected bipolar fuzzy graph. Then  $G$  is called highly irregular bipolar fuzzy graph if every vertex of  $G$  is adjacent to vertices with distinct degrees.



**Example 2.22** Let  $G = (A, B)$  be a bipolar fuzzy graph where  $A = (m_1^+, m_1^-)$  and  $B = (m_2^+, m_2^-)$  be two bipolar fuzzy sets on a non-empty finite set  $V$  and  $E \subseteq V \times V$  respectively Where  $V = \{v_1, v_2, v_3\}$ .  $d(v_1) = (0.6, -0.5)$ ,  $d(v_2) = (0.8, -0.4)$  and  $d(v_3) = (0.5, -0.4)$ . The graph is an example of highly irregular bipolar fuzzy graph.

A highly irregular bipolar fuzzy graph need not be neighbourly irregular bipolar fuzzy graph. As for example we consider a bipolar fuzzy graph of vertices  $v_1(0.5, -0.4)$ ,  $v_2(0.6, -0.5)$ ,  $v_3(0.5, -0.4)$ ,  $v_4(0.4, -0.4)$  with  $m_2^+(v_1, v_2) = 0.4$ ,  $m_2^-(v_1, v_2) = -0.3$ ,  $m_2^+(v_2, v_3) = 0.2$ ,  $m_2^-(v_2, v_3) = -0.4$ ,  $m_2^+(v_2, v_4) = 0.2$ ,  $m_2^-(v_2, v_4) = -0.4$ ,  $m_2^+(v_3, v_4) = 0.4$ ,  $m_2^-(v_3, v_4) = -0.3$ ,

So  $d(v_1) = (0.4, -0.3)$ ,  $d(v_2) = (0.8, -1.1)$ ,  $d(v_3) = (0.6, -0.7)$ ,  $d(v_4) = (0.6, -0.7)$ .

Here the bipolar fuzzy graph is highly irregular but not neighbourly irregular as  $d(v_3) = d(v_4)$ .

**Theorem 2.23** Let  $G$  be a bipolar fuzzy graph. Then  $G$  is highly irregular bipolar fuzzy graph and neighbourly irregular bipolar fuzzy graph if and only if the degrees of all vertices of  $G$  are distinct.

**Proof:** Let  $G = (A, B)$  be a bipolar fuzzy graph where  $A = (m_1^+, m_1^-)$  and  $B = (m_2^+, m_2^-)$  be two bipolar fuzzy sets on a non-empty finite set  $V$  and  $V \times V$  respectively. Let  $V = \{v_1, v_2, v_3 \dots v_n\}$ . We assume that  $G$  is highly irregular and neighbourly irregular bipolar fuzzy graphs. Let the adjacent vertices of  $u_1$  be  $u_2, u_3, \dots u_n$  with degrees  $(k_2^+, k_2^-), (k_3^+, k_3^-), \dots (k_n^+, k_n^-)$  respectively. As  $G$  is highly and neighbourly irregular,  $d(u_1) \neq d(u_2) \neq d(u_3) \neq \dots \neq d(u_n)$ . So it is obvious that all vertices are of distinct degrees.

Conversely, assume that the degrees of all vertices of  $G$  are distinct. This means that every two adjacent vertices have distinct degrees and to every vertex the adjacent vertices have distinct degrees. Hence  $G$  is neighbourly irregular and highly irregular fuzzy graphs. Complement of a bipolar fuzzy graph is studied below.

**Definition 2.24** Let  $G = (A, B)$  be a bipolar fuzzy graph where  $A = (m_1^+, m_1^-)$  and  $B = (m_2^+, m_2^-)$  be two bipolar fuzzy sets on a non-empty finite set  $V$  and  $E \subseteq V \times V$  respectively. The complement of a bipolar fuzzy graph  $G$  is  $\bar{G} = (\bar{A}, \bar{B})$  where  $\bar{A} = (\bar{m}_A^+, \bar{m}_A^-)$  is a bipolar fuzzy set on  $\bar{V}$  and  $\bar{B} = (\bar{m}_B^+, \bar{m}_B^-)$  is a bipolar fuzzy set on  $\bar{E} \subseteq \bar{V} \times \bar{V}$  such that

- (1)  $\bar{V} = V$
- (2)  $\bar{m}_A^+(x) = m_A^+(x)$  and  $\bar{m}_A^-(x) = m_A^-(x)$  for all  $x \in V$ ,

$$(3) \quad \bar{m}_B^+(x, y) = \begin{cases} 0, & \text{if } m_B^+(x, y) > 0. \\ m_A^+(x) \wedge m_A^+(y), & \text{otherwish.} \end{cases}$$

$$\bar{m}_B^-(x, y) = \begin{cases} 0, & \text{if } m_B^-(x, y) < 0, \\ m_A^-(x) \vee m_A^-(y), & \text{otherwise.} \end{cases}$$

If a bipolar fuzzy graph  $G$  is neighbourly irregular, then  $\bar{G}$  is not neighbourly irregular. Let us consider a bipolar fuzzy graph whose non adjacent vertices are of same degree. Then clearly, adjacent vertices of  $\bar{G}$  are of same degree. Hence the statement is true.

**Theorem 2.25** Let  $G$  be a bipolar fuzzy graph. If  $G$  is neighbourly irregular and  $m_1^+, m_1^-$  are constant functions, then  $G$  is a neighbourly total irregular bipolar fuzzy graph.

**Proof:** Let  $G = (A, B)$  be a bipolar fuzzy graph where  $A = (m_1^+, m_1^-)$  and  $B = (m_2^+, m_2^-)$  be two bipolar fuzzy sets on a non-empty finite set  $V$  and  $E \subseteq V \times V$  respectively. Assume that  $G$  is a neighbourly irregular bipolar fuzzy graph. i.e. the degrees of every two adjacent vertices are distinct. Consider two adjacent vertices  $u_1$  and  $u_2$  with distinct degrees  $(k_1^+, k_1^-)$  and  $(k_2^+, k_2^-)$  respectively. Also let,  $m_1^+(u) = c_1$  for all  $u \in V$ ,  $m_1^-(u) = c_2$  for all  $u \in V$  where  $c_1 \in (0, 1]$ ,  $c_2 \in [-1, 0)$  are constant. Therefore  $td(u_1) = (d^+(u_1) + c_1, d^-(u_1) + c_2) = (k_1^+ + c_1, k_1^- + c_2)$ ,  $td(u_2) = (d^+(u_2) + c_1, d^-(u_2) + c_2) = (k_2^+ + c_1, k_2^- + c_2)$ , clearly  $td(u_1) \neq td(u_2)$ . Therefore for any two adjacent vertices  $u_1$  and  $u_2$  with distinct degrees, its total degrees are also distinct, provided  $m_1^+, m_1^-$  are constant functions. The above argument is true for every pair of adjacent vertices in  $G$ .

**Theorem 2.26** Let  $G$  be a bipolar fuzzy graph. If  $G$  is neighbourly total irregular and  $m_1^+, m_1^-$  are constant functions, then  $G$  is a neighbourly irregular bipolar fuzzy graph.

**Proof:** Let  $G = (A, B)$  be a bipolar fuzzy graph where  $A = (m_1^+, m_1^-)$  and  $B = (m_2^+, m_2^-)$  be two bipolar fuzzy sets on a non-empty finite set  $V$  and  $V \times V$  respectively. Assume that  $G$  is a neighbourly total irregular bipolar fuzzy graph. i.e. the total degree of every two adjacent vertices are distinct. Consider two adjacent vertices  $u_1$  and  $u_2$  with degrees  $(k_1^+, k_1^-)$  and  $(k_2^+, k_2^-)$  respectively. Also assume that  $m_1^+(u) = c_1$  for all  $u \in V$ ,  $m_1^-(u) = c_2$  for all  $u \in V$  where  $c_1 \in (0, 1]$ ,  $c_2 \in [-1, 0)$  are constants. Also  $td(u_1) \neq td(u_2)$ . We are to prove that  $d(u_1) \neq d(u_2)$ .  $k_1^+ + c_1 \neq k_2^+ + c_1$  and  $k_1^- + c_2 \neq k_2^- + c_2$ . So  $k_1^+ \neq k_2^+$  and  $k_1^- \neq k_2^-$ . Hence the degrees of

adjacent vertices of  $G$  are distinct. This is true for every pair of adjacent vertices in  $G$ .  
Hence the result.

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