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"Laplace transform of an integral equation with the Hypergeometric- function as its kernel"

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ABSTRACT: The object of this paper is to solve an integral equation of convolution form having Hypergeometric function of two variable as it's kernel. Some known results are obtained as special cases

Keywords - integral equation, convolution, generalized type geometric function of two variables
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I. DEFINITION AND INTRODUCTION

The following definition and results will be required in this paper

(i) The Laplace Transform if

$$F(P)=L[f(t);p]=\int_0^\infty e^{-pt}f(t)dt\ ,\quad \mathrm{Re}(p)>0 \qquad \ldots(1.1)$$

Then F(p) is called the Laplace transform of f(t) with parameter p and is represented by F(p) = f (t) Erdelyi [(3) pp.129-131]

(ii) $L[f(t);p]=F(P)$ then $L[e^{-at}f(t)]=F(p+a)$ ...(1.2)

And if $f(0)=f^{'}(0)=f^{"}(0)=\ldots..=f^{m-1}(0)=0\quad ,\ f^{n}(t)$

Is continuous and differential, then

$$L[f^{n}(t);p]=P^{n}F(p) \qquad \ldots.(1.3)$$

(iii) If $L[f_1(t)]=F_1(p)$ then $L[f_2(t)]=F_2(p)$

Then convolution theorem for Laplace transform is

$$L\{\int_0^1 f_1(t)f_2(t-u)du\}=L\{f_1(t)\}L\{f_2(t)\}=F_1(p).F_2(p) \qquad \ldots(1.4)$$

(iv) The H-Function Defined by Saxena and kumbhat [1] is an extension of Fox's H-Function on specializing the parameters , H-Function can be reduced to almost all the known special function as well as unknown

The Fox's H-Function of one variable is defined and represented in this Paper as follows [see Srivastava et al [2] ,pp 11-13]

$$H[x]=H_{P,Q}^{M,N}\left[x\,/\begin{matrix}(a_j,\alpha_j)_{1,P}\\(b_j,\beta_j)_{1,Q}\end{matrix}\right]=\frac{1}{2\pi\omega}\int_{\theta=N-1}\theta(\xi)x^{\xi}d\xi$$

…(1.5)

$$\theta(\xi) = \frac{\prod_{i=1}^{n} \Gamma b_j - \beta_j \xi \prod_{j=1}^{N} \Gamma 1 - a_j - \alpha_j \xi}{\prod_{i=M=1}^{Q} \Gamma 1 - b_j + \beta_j \xi \prod_{j=N+1}^{P} \Gamma a_j - \alpha_j \xi} \qquad \dots(1.6)$$

For condition of the H-Function of one variable (1.5) and on the contour L we refer to srivastava et al [2]
(V) The H-Function of two variable occurring in this paper is defined and represented as follows [see Srivastava et al [2] ,pp 83-85]

$$H[x,y] = H^{0,n_1,m_2,n_2,m_3,n_3}_{p_1,q_1,p_2,q_2,p_3,q_3}\left[x_y \,/ \begin{matrix} (a_j,\alpha_j,A_j)_{1,p_1} & (c_j,z_j)_{1,p_2} & (e_j,E_j)_{1,p_3} \\ (b_j,\beta_j,B_j)_{1,q_1} & (d_j,\delta_j)_{1,q_2} & (f_j,F_j)_{1,q_3} \end{matrix} \right]$$

$$= -\frac{1}{4\pi^2} \int_{L_1}\int_{L_2} \phi_1(\xi,\eta)\psi_2(\xi)\,\psi_3(\eta)\,x(\xi)\,y(\eta)\,d\xi\,d\eta \qquad \dots(1.7)$$

Where

$$\phi_1(\xi,\eta) = \prod_{j=1}^{n_1} \Gamma(1 - a_j + \alpha_j \xi + A_j \eta)$$

$$\times \left[\prod_{j=n_1+1}^{p_1} \Gamma a_j - \alpha_j \xi - A_j \eta \prod_{j=1}^{q_1} \Gamma 1 - b_j - \beta_j \xi + B_j \eta \right] \qquad \dots(1.8)$$

Where the $\psi_2(\xi)$ and $\psi_3(\eta)$ are defined as (1.6) and for conditions of existence etc. of the $H(x,y)$ we refer to srivastava et al [2]

II. MAIN RESULT

Result I $L\left\{ t^{\alpha} H^{M,N}_{P,Q}\left[at^{\alpha} \,/ \begin{matrix} (a_j,\alpha_j)_{1,P} \\ (b_j,\beta_j)_{1,Q} \end{matrix} \right], P \right\}$

$$= P^{-1-\alpha} H^{M,N+1}_{P+1,Q}\left[at^{-\lambda} \,/ \begin{matrix} (-\alpha,\lambda)\,(a_j,\alpha_j)_{1,P} \\ (b_j,\beta_j)_{1,Q} \end{matrix} \right]$$

Provided Re(p) >0 1≥λ>a and Re(1+∝)>0

Result II

$$\int_0^1 \left\{ x^{\alpha-1}(1-x)^{\beta-1} H^{M_1,N_1}_{P_1,Q_1}\left[z_1 x^{\lambda} \,/ \begin{matrix} (a_j,\alpha_j)_{1,P_1} \\ (b_j,\beta_j)_{1,Q_1} \end{matrix} \right] H^{M_2,N_2}_{P_2,Q_2}\left[z_2 (1-x)^{\mu} \,/ \begin{matrix} (c_j,\gamma_j)_{1,P_2} \\ (d_j,\delta_j)_{1,Q_2} \end{matrix} \right] \right\} dx$$

$$= H^{0,\; N+2\; M_1,N_1,M_2,N_2}_{P+2,\; Q+1\; P_1,Q_1,P_2,Q_2}\left[\begin{matrix} z_1 \\ z_2 \end{matrix} \,/ \begin{matrix} (1-\alpha,\lambda)\,(1-\beta,\mu)\;(a_j,\alpha_j)_{P_1}\;(c_j,\gamma_j)_{P_2} \\ (1-b-\beta,\lambda;\mu)\;(b_j,\beta_j)_{Q_1}\;(d_j,\delta_j)_{Q_2} \end{matrix} \right]$$

Provided Re (α) >0 Re (β) >0 λ, μ >0

$$\mathrm{Re}\left(\alpha + \lambda \frac{b_j}{\beta_j}\right) > 0 \quad \mathrm{Re}\left(\beta + \mu \frac{d_j}{\delta_j}\right) > 0 \qquad j = 1, 2 \dots m \quad k = 1, 2 \dots g$$

$$|\arg z_1| < \tfrac{1}{2}\pi\Delta_1 \qquad |\arg z_2| < \tfrac{1}{2}\pi\Delta_2 \quad \Delta_1\ \Delta_2 > 0$$

$$\Delta_1 = \sum_{1}^{M_1} b_j - \sum_{M_1+1}^{Q_1} b_j + \sum_{1}^{N_1} a_j - \sum_{N_1+1}^{P_1} a_j \qquad \Delta_2 = \sum_{1}^{M_2} d_j - \sum_{M_2+1}^{Q_2} d_j + \sum_{1}^{N_2} c_j - \sum_{N_2+1}^{P_2} c_j$$

Result III

$$L\left\{ e^{-nt} t^h H_{P,Q}^{M,N}\left[zt^k \,/ \begin{matrix} (a_j,\alpha_j)_{1,P} \\ (b_j,\beta_j)_{1,Q} \end{matrix} \right], P \right\}$$

$$= (P+a)^{-1-h} H_{P+1,Q}^{M,N+1}\left[z(p+a)^{-k} \,/ \begin{matrix} (-h,k)\,(a_j,\alpha_j)_{1,P} \\ (b_j,\beta_j)_{1,Q} \end{matrix} \right]$$

Provided Re(p) >0 1≥λ>a and Re(1+∝)>0

Proof I First Taking by mellin barnes type contour integral for H- function for one variable and then convolution of laplace transform for H-function and get required result.

Proof II Taking by mellin barnes type contour integral for H- function for two variables and then using beta function we get required result.

Proof III same as proof I

III. CONCLUSION

From this Paper we get some many solution of integral equation of convolution from having H – Function of one or more veriables

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