International Journal of Advances in Engineering and Management (IJAEM)
Volume 3, Issue 1 Jan-Feb 2021, pp: 123-129 www.ijaem.net ISSN: 2395-5252

Lower bound for span of radio k-distance labelling of Graphs

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Submitted: 10-01-2021 Revised: 23-01-2021 Accepted: 26-01-2021

ABSTRACT: The Channel Assignment Problem (CAP) is the problem of assigning channels (non-negative integers) to the transmitters in an optimal way such that interference is avoided. The problem, often modelled as a labelling problem on the graph where vertices represent transmitters and edges indicate closeness of the transmitters. A radio k-distance labelling of graphs is a variation of CAP. For a simple connected graph $G = (V(G), E(G))$ and a positive integer k, a radio k-distance labelling of G is a mapping $f: V(G) \to \{0,1,2, \dots\}$such that $|f(u) - fv \geq k+1-d(u,v)$ for each pair of distinct vertices u and v of G, where $d(u, v)$ is the distance between u and v in G. The span of a radio k -distance labelling f is the largest integer assigned to a vertex of G. The radio k -chromatic number of G is the minimum of spans of all possible radio k-labellings of G. In this article, we give a lowerbound for span of radio k-distance labelling of arbitrary graph G in terms some parameters related to metric closure of G.

KEYWORDS: Channel assignment; Metric closure Radio k-labelling; Radio k-chromatic number; Span.

I. INTRODUCTION

The Channel Assignment Problem (CAP) is the problem of assigning channels (non-negative integers) to the stations in an optimal way such that interference is avoided. In wireless communication, frequency reuse is limited by two kinds of radio interference, namely Co-channel interference and adjacent channel interference. Co-channel interference is caused by two simultaneous transmissions on the same channel. To avoid this, once a channel is assigned to a certain station, it should not be reused by another station in an area where it may cause significant interference. Adjacent channel interference is the result of signal energy from an adjacent channel spilling over into the current channel. Thus, CAP plays an important role in wireless network and a well-studied interesting problem. Many researchers have modelled CAP as an optimization problem as follows: Given a collection of transmitters to be assigned operating frequencies and a set of interference constraints on transmitter pairs, find an assignment that satisfies all the interference constraints and minimizes the value of a given objective function. In 1980, Hale [13] has modelled FAP as a Graph labelling problem (in particular as a generalized graph labelling problem) and is an active area of research now. Griggs and Yeh [12] concentrated on the fundamental case of L(1,2)-labellings. The $L(p, q)$-labelling problem $(p, q > 0)$ and its variants have been studied extensively (see e.g. [2, 3, 10, 11, 12, 13, 14, 16, 33, 34]). A major concern of this problem is to seek an assignment of labels (which are nonnegative integers) to the vertices of a graph such that the span (difference between the largest and smallest labels used) is minimized, subject to that adjacent vertices receive labels with separation at least p and vertices at distance two apart receive labels with separation at least q.

Motivated by FM channel assignments, a new model, namely the radio k-labelling problem was introduced in [4, 5] and studied further in [20, 21, 32]. For a simple connected graph $G = (V(G), E(G))$and a positive integer k with $1 \leq k \leq diam(G)$, a radio k-labelling of G is a mapping $f: V(G) \to \{0,1,2, \dots\}$ such that

$$|f(u) - f(v)| \geq k + 1 - d(u, v) \dots (1)$$

for each pair of distinct vertices uand vof G, where $diam(G)$ is the diameter of Gand $d(u, v)$is the distance between uand v in G. The span of a radio k -labelling f, denoted by $span(f)(G)$, is the largest integer assigned to a vertex of G. The radio k -chromatic number of G, denoted by $rc_k(G)$, is the minimum of spans of all possible radio k -labellings of G. A radio k -labelling fof Gis called minimal if $span_f(G) = rc_k(G)$.Without loss of generality, for a minimal radio labelling f we assume that $\min_{v \in V(G)} f(v)$, otherwise thespan of fcan be reduced further by subtracting the positive integer $\min_{v \in V(G)} f(v)$ from all the labels of the vertices of the graph. For some specific values of k there are specific names for radio k -labellings as well as the radio k -chromatic number in the literature, which are given in Table 1:

Table 1: Name of radio k -labelling for different Values of k

Values of k	Name of radio k - labelling	Name radio k -chromatic number
1	Vertex coloring	Chromatic number, $\chi(G)$
$diam(G)$	Radio labelling Radio number	Radio number, $rn(G)$
$diam(G) - 1$	Antipodal labelling	Antipodal number, $ac(G)$

The radio k-labelling problem can be viewed as an instance of the $L(p_1, \dots, p_m)$-labelling problem (see e.g. [12, 35]), where $m, p_1, \dots, p_m \geq 1$ are given integers, which aims at minimizing the span of a labelling $f: V(G) \to \{0,1,2,\dots\}$ subject to $|f(u) - f(v)| \geq p_i$ whenever $d(u,v) = i, 1 \leq i \leq m$.In the special casewhere $m = k$ and $p_i = \max\{k + 1 - i, 0\}$ for each i, the minimum span of such a labelling is exactly the radio k -chromatic number of G.

Determining the radio k -chromatic number of a graph is an interesting yet difficult combinatorial problemwith potential application to CAP. So far it has been explored for a few basic families of graphs and values of k near to diameter. The radio number of any hypercube was determined in [17] by using generalized binary Gray codes. Ortiz et al.[26] have studied the radio number of generalized prism graphs and have computed the exact value of radio number for some specific types of generalized prism graphs. For two positive integers $m \geq 3$ and $n \geq 3$, the Toroidal grids $T_{m,n}$ are the cartesian product of cycleC_m with cycle C_n. Morris et al. [25] have determined the radio number of $T_{n,n}$ and Saha et al.[28] have given exact valuefor radio number of $T_{m,n}$ when $mn \equiv 0 (mod 2)$. The radio numbers of the square of paths and cycles were studied in [22, 23]. For a cycle C_n, the radio number was determined by Liu and Zhu [21], and theantipodal number is known only for $n = 1,2,3 \pmod 4$ (see [6, 15]).

Surprisingly, even for paths finding the radio number was a challenging task. It is envisaged that in general determining the radio number would be difficult even for trees, despite a general lower bound for trees given in [20]. Till now, the radio number is known for very limited of families of trees. For paths P_n, complete m -ary trees the exact values of radio number were determined in [21, 24]. The results for paths were generalized [21] to spiders, leading to the exact value of the radio number in certain special cases. In [27], Reddy et al. give an upper bound for the radio number of some special type of trees. For a path n -vertex path P_n, the exact value of $rc_k(P_n)$ is known only for $k = n - 1$ [21], $n - 2$ [17], $n - 3$ [36], and $n - 4$ (n odd) [37].

In general, finding a good lower bound is comparatively difficult than finding an upper bound for the radio number, because every construction of a radio labelling of a graph leads to an upper bound of the radio number. Again, the radio k-coloring number of graphs for $k > diam(G)$ will be helpful to find radio k -chromatic number of graphs with bigger diameter likecartesian product of graphs. In this article, we givea lower bound for span of radio k -distance labelling mainly for higher values of k of arbitrary graph G interms some parameters related to metric closure of G.

From here to onwards by a graph G we mean that it is simple connected graph with vertex set $V(G)$and edge set $E(G)$.

II. PRELIMINARIES

Definition 2.1. (Metric Closure of a Graph) The metric closure of a connected graph G, denoted by G^c, is the complete weighted graph on $V(G)$ in which weight of an edge $\{u,v\}$ is the distance between $u$and v in G. Note that the edge weights in G^c satisfy the triangle inequality. The weight of a sub-graph H of G^c, denoted by $w(H)$, is the sum of weights of all edges in H.

Definition 2.2. (Triameter of a Graph) Let G be a simple connected graph with at least 3 vertices. The triameter of G, denoted by $tr(G)$, of the graph G is defined as the smallest positiveinteger M such that $d(u,v) + d(v,w) + d(w,u) \leq M$ for every triplet u, v and w in $V(G)$. In another way we can say that the triameter $tr(G)$ is the maximum weight of triangle in metric closure G^c.

From the definition, it follows that $tr(G)$ is always greater than or equal to 3. Now, we investigate other bounds on $tr(G)$.

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Lemma 2.1. For a graph G, $tr(G) = 3$ if and only if G is complete graph.

Theorem 2.1. For any connected graph G, $2\ diam(G) \leq tr(G) \leq 3\ diam(G)$ and the bounds are tight.

Proof. Let q be the diameter of G. Since $max\{d(u,v) + d(v,w) + d(w,u) :$ $:for\ all\ u,v,w \in VG \leq 3q$ and trG is the smallest integer such that $d(u,v) + d(v,w) + d(w,u) \leq M$ for all $u, v, w \in V(G)$, we have $tr(G) \leq 3q$. If the vertices u and v are chosen in such a way that $d(u,v) = q$, then from the triangle inequality $d(v,w) + d(w,u) > d(u,v) = q$. Therefore $d(u,v) + d(v,w) + d(w,u) > 2q$. Since $tr(G)$ is thesmallest positive integer M such that $d(u,v) + d(v,w) + d(w,u) \leq M$ for every triplet$u$, v and w in $V(G)$, we have $tr(G) > 2\ diam(G)$.

III. LOWER BOUND FOR RADIO K-CHROMATIC NUMBERS OF TREES

Definition 3.1. Let T be any tree. The measure of separability of a vertex $v \in V(T)$, denoted by$\beta_T(v)$, isthe size of maximum component of $T - \{v\}$. A vertex is called centroid if it has minimal separability overall vertices in T.

Let T be a tree with centroid S. Define the level of $u \in V(T)$ (with respect to S) by $L(u) = d(S,u)$. Avertex u of T is in level l if $L(u) = l$. For distinct $u, v \in V(T)$, define $\varphi(u,v) :=$length of the common part of the paths of T from S to u and v.

Lemma 3.1. Let T be a tree rooted at r. Then for distinct $u, v \in V(T)$ the following (a) − (b) hold.
(a) $d(u,v) = L(u) + L(v) - 2\varphi(u,v)$
(b) $\varphi(u,v) = 0$ if and only if $r \in \{u,v\}$ or u and v belongs to the different branches.

Lemma 3.2. For an n-vertex tree T , the following (a) − (c) are hold.
(a) If a vertex v is centroid, then $\beta_T(v) \leq \left\lfloor \frac{n}{2} \right\rfloor$
(b) A tree with odd number of vertices has exactly one centroid.
(c) A tree T with even number of vertices has two centroids $S_1$and S_2which are neighbours and

$$\sum_{u \epsilon V(T)} d(S_1, u) = \sum_{u \epsilon V(T)} d(S_2, u)$$

Lemma 3.3. Let S be a centroid of an n-vertex tree T. Then there exist a sequence u_0, u_1, … , u_{n-1} of vertices of T such that no two consecutivevertices are in same branch of $T - S$.

Definition 3.2. For a given tree T, a maximum weight Hamiltonian path is a Hamiltonian path in $T^c$of maximum weight. For $u, v \in V(T)$, the uv-maximal Hamiltonian path in T^c is a maximum weight Hamiltonian path in T^c whose end vertices are u and v. Let us denote $w^*_{uv}(T^c)$ and $w^*(T^c)$ be the weight of uv-maximal weight Hamiltonian path and maximum weight Hamiltonian path in T^c respectively.

Finding the weight of a maximum Hamiltonian path in G^c is NP-hard. Lemma 3.4 gives the weight of a maximum Hamiltonian path in T^c for any tree T. This lemma also describes the weight of a uv-maximal weight Hamiltonian path in T^c.

Lemma 3.4. Let T be an n -vertex tree with centroid S. Then the following are hold :

(a)

$$w^*_{uv}(T^c) = 2 \sum_{u \in V(T)} d(S,u) - d(u,S) - d(v,S)$$

(b)

$$w^*(T^c) = 2 \sum_{u \in V(T)} d(S,u) - 1.$$

Proof. (a) For any Hamiltonian path $P: u_1 \ldots u_{n-1} = v$ in T^c, the weight of the path is

$$w(P) = \sum_{i=0}^{n-2} d(u_{i,}u_{i+1})$$

$$\leq \sum_{i=0}^{n-2} [d(S,u_i) + (S,u_{i+1})] \ldots (2)$$

$$= 2\sum_{i=0}^{n-2} d(S,u_i) - d(S,u_0) - d(S,u_{n-1}) \ldots (3)$$

$$= 2\sum_{i=0}^{n-2} d(S,u_i) - d(S,u) - d(S,v) \quad \ldots (4).$$

Equality occurs in (2) if $u_i$and u_{i+1} are in different branches of $T - S$ and such types ofchoices of $u_i^{'}$ sis arepossible due to the Lemma 3.3. hence the part (a) of this lemma is proved.

(b) The maximum value of $w^*_{uv}(T^c)$ in (b) will occur if we take $u = S$ and v is a vertex adjacent to S.Thus, the weight of a maximum weight Hamiltonian path in T^c is equals to

$$2\sum_{u\in V(T)} d(S,u)-1.$$

Notation 3.1. For an n -vertex tree T and acentroid S of T , we call the number $\sum_{u\in V(T)} d(S,u)$ is the weightof T and denoted it by $w(T)$.

Theorem 3.1. Let f be a radio k- labeling of an n -vertex tree T with first and last-colored vertices u and$v$. Then

$$span(T)\geq (n-1)(k+1)-2w(T)+f(u) + d(S,u)+d(S,v)$$

where $w(T)$ denotes the weight of T and S denotes a centroid of T. Moreover, $k=diam(T)$ if the equality holds if and only if there exist a centroid S and a radio labelling f with
$f(u_0)=0<f(u_1)<\cdots<f(u_{n-1})$,
where all the following hold (for all $0\leq i\leq n-2$):

(a) $u_0,u_1,\dots,u_{n-1}$is a maximum weight Hamiltonian path in T^c

(b) $f(u_{i+1})=f(u_i)+diam(G)+1-L_S(u_i)-L_S(u_{i+1})$

Proof : Since f is radio k-labelling of T , f induces a linear order $u_0,u_1,\dots,u_{n-1}$ of the vertices of Tsuch that
$u=f(u_0)\leq f(u_1)\leq f(u_2)\dots\leq f(u_{n-1})=v$.
Then

$$span(T)=f(v)$$
$$=\sum_{i=0}^{n-2}[f(u_{i+1})-f(u_i)]+f(u)$$
$$\geq\sum_{i=0}^{n-2}[k+1-d(u_i,u_{i+1})]+f(u)$$
$$\geq (n-1)(k+1)-\textstyle\sum_{i=0}^{n-2} d(u_i,u_{i+1})+f(u)$$
$$\geq (n-1)(k+1)-\sum_{i=0}^{n-2}[d(S,u_i)+d(S,u_{i+1})]+f(u)$$
$$=(n-1)(k+1)-2w(T)+f(u)+d(S,u)+d(S,v).$$

Remark 3.1. To compute this lower bound first we have to find a centroid S and then the distancesof other vertices from this centroid. One can give an algorithm to find a centroid S and compute $w(T)=\sum_{u\in V(T)} d(S,u)$ with time complexity of order $|V(T)|$. Therefore, an algorithm with worst case time complexity$|V(T)|$can be presented to compute the above lower bound.

Corollary 3.1. For an n-vertex tree T,

$$rc_k(T)\geq (n-1)(k+1)-2w(T)+1,$$

where $w(T)$ denotes the weight of T.

Example 3.1. For an n -vertex path P_n, a centroid is $\left\lfloor\frac{n}{2}\right\rfloor$and the length of a maximum weight Hamiltonian path P_n^c (which does not depend on the choice of a centroid) is given by

$$w^*(P_n^c)=2\sum_{u\in V(T)} d(S,u)-1 =\begin{cases}\dfrac{n^2-2}{2}, & if\ n\ is\ an\ even\ integer\\ \dfrac{n^2-3}{2}, & Otherwise.\end{cases}$$

By applying above theorem, the lower bound of $rc_k(P_n)$ is stated as

$$rc_k(P_n)\geq\begin{cases}(n-1)k-\frac{1}{2}n(n+2)+2, & if\ n\ is\ even\\ (n-1)k-\frac{1}{2}(n-1)^2+1, & if\ n\ is\ odd\end{cases}$$

Kchikech et al. [4] have given exact value of radio k-chromatic number of $P_n$for $k\geq n$ as below.

$$rc_k(P_n)\geq\begin{cases}(n-1)k-\frac{1}{2}n(n-2), & if\ n\ is\ even\\ (n-1)k-\frac{1}{2}(n-1)^2+1, & if\ n\ is\ odd\end{cases}$$

Thus, above lower bound is sharp for odd path $P_n$with $k\geq n$.

Example 3.2. Consequences of Theorem 3.1 include the radio number for complete m-ary tree $T_{l,m}\geq 3$ (which was settled in [24] by a different approach). Note that the root r is the centroid of $T_{l,m}$, $diam(T_{l,m})=2l$ and level l is the bottom level of $T_{l,m}$. Now the length of the maximum wight Hamiltonian path in $T_{l,m}^c$ is given by
$w^*(T_{l,m})=2w(T_{l,m})-1$, where

$$w(T_{l,m})=\sum_{u\epsilon V(T_{l,m})} d(r,u)=\sum_{i=1}^{l} m^i i$$

$$= \frac{lm^{l+2} - (l+1)m^{l+2} + m}{(m-1)^2}$$

Complete m-ary trees $T_{l,m}$ $(l \geq 2, m \geq 3)$ have radio numbers equal to the bound in Theorem 3.1, asone can find a radio labelling satisfying Theorem 3.1 (cf. [24]).

Definition 3.3. A subgraph H of a graph G is said to be maximal k-diameteral subgraph if diameter of $H$is k and it contains maximum number of vertices of G.

Definition 3.4. Let $f: E \to F$ be a mapping from a set E to a set F. For a set $A \subset E$, we call the mapping $f|_A: A \to F$ as the restriction of f on A.

Lemma 3.5. Let G be a graph with diameter d and H be a maximal k-diameteral subgraph of G with $k < d$. If $rc_k(G)$ and $rn(H)$ be the radio k-chromatic number of G and H, respectively, then $rc_k(G) > rn(H)$.

Proof: Let f be a radio k-labelling of G. Here the diameter of H is k with $k < d$. Thus $V(H) \subset V(G)$. Let $g = f|_{V(H)}$be the restriction of f on $V(H)$.Then $span_f(G) \geq span_f(H)$ and this is true for any radio$k$-labelling of G and its restriction $g = f|_{V(H)}$. Since the diameter of H is k, we obtain the required result.

The minimum value of span of a radio k-labelling for tress T given in Theorem 3.1 may give weak resultsfor small values of k. Thus, next we give another lower bound for the same in terms of spans of maximal$k$-diameteral subgraphs.

Theorem 3.2. Let T be an n-vertex tree and be the set of all maximal k-diameteral subgraphs of T. Then$rc_k \geq max_{H \in \Omega}\{rn(H)\}$.
Proof. From Lemma 3.5, $rc_k(G) > rn(H)$ for any maximal k-diameteral subgraphs H of T . Thus the result follows.
Finding maximal Hamiltonian path of any graph G is NP hard problem. In the next section we givelower bound for radio k-chromatic number for arbitrary graph G in terms of triameter of the graph.

IV. LOWER BOUND FOR RADIO K-CHROMATIC NUMBER OF ARBITRARY GRAPHS

Theorem 4.1. For an -vertex simple connected graph G,

$$(a) rc_k(G) \geq \left\lceil \frac{3(k+1) - tr(G)}{2} \right\rceil \left(\frac{n-2}{2}\right) + max\{k+1 - diam(G), 0\} \ if\ n\ is\ even$$

$$(b) \left\lceil \frac{3(k+1) - tr(G)}{2} \right\rceil \left(\frac{n-1}{2}\right), \ if\ n\ is\ odd. \quad (5)$$

Proof. Let f be any radio k-coloring of G and $u_0, u_1, \dots, u_{n-1}$be an ordering of the vertices of G suchthat $0 = f(u_0) \leq f(u_1) \dots, \leq f(u_{n-1})$. Then the span of f is $f(u_{n-1})$. Since f is a radio k-coloring of G,

$$f(u_{i+1}) - f(u_i) \geq k + 1 - d(u_i, u_{i+1}) \quad (6)$$
$$f(u_{i+2}) - f(u_{i+1}) \geq k + 1 - d(u_{i+1}, u_{i+2}) (7)$$
$$f(u_{i+2}) - f(u_i) \geq k + 1 - d(u_{i+2}, u_i) \quad (8)$$

Adding $(6-8)$, we get

$$2(f(u_{i+2}) - f(u_i)) \geq 3(k+1) - d(u_i, u_{i+1}) - d(u_{i+1}, u_{i+2}) - d(u_{i+2}, u_i) \quad (9)$$

Now it is clear that
$tr(G) \geq d(u_i, u_{i+1}) + d(u_{i+1}, u_{i+2}) + d(u_{i+2}, u_i)$.
Then (9) reduces to
$$2\big(f(u_{i+2}) - f(u_i)\big) \geq 3(k+1) - tr(G)$$
for all i with $0 \leq i \leq n-3$ (10)

Since $f(u_{i+2}) - f(u_i)$ is a non-negative integer, the inequality (10) gives

$$\mathrm{f(u_{i+2}) - f(u_i)} \geq \left\lceil \frac{3(k+1) - tr(G)}{2} \right\rceil$$
$0 \leq i \leq n-3$ (11)

Case I : Here we take n as an even integer. Let $S \in \{0,2,\dots,n-4\}$. Since
$spanf = f(u_{n-1}) - f(u_0)$

$$= \sum_{i \in S} [\, f(u_{i+2}) - f(u_i)] + [f(u_{n-1}) - f(u_{n-2})]$$

and the inequality (11) holds for all $i \in S$,

$$span(f) \geq \left\lceil \frac{3(k+1) - tr(G)}{2} \right\rceil \left(\frac{n-2}{2}\right) + [\, f(u_{n-1}) - f(u_{n-2})]$$

Using the condition of radio k -labelling
$f(u_{n-1}) - f(u_{n-2}) > k + 1 - d(u_{n-2}, u_{n-1})$,

the above inequalityreduces to

$$\text{span}(f) \geq \left\lceil \frac{3(k+1) - \text{tr}(G)}{2} \right\rceil \left(\frac{n-2}{2}\right) + \max\{k + 1 - \text{diam}(G), 0\}$$

Case II : In this case, we take n an odd integer. Let $S = \{0,2,\dots,n-3\}$. Since

$$\text{span}(f) = f(u_{n-1}) - f(u_0) = \sum_{i\in S}[\, f(u_{i+2}) - f(u_i)]$$

and theinequality (11) holds for all $i \in S$,

$$\text{span}(f) \geq \left\lceil \frac{3(k+1) - \text{tr}(G)}{2} \right\rceil \left(\frac{n-2}{2}\right)$$

and this completes the proof.

Remark 4.1. The result presented in Theorem 4.1 is sharp for radio number of cycles, hypercube.

Acknowledgement: The author is thankful to the National Board for Higher Mathematics (NBHM), India for its financial support (Grant No. 2/48(22)/R & D II/4033).

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International Journal of Advances in Engineering and Management
ISSN: 2395-5252

IJAEM

Volume: 03 Issue: 01 DOI: 10.35629/5252

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