

Momentum Analysis on Neutron Thermalization by a Proton with Head-on Collision

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ABSTRACT

Neutrons can be thermalized by some rich H-atom polymer materials such as polyethyleneterephthalate, PETE fibers because of mathematical demonstration have proved it clearly. Vector diagram analysis, law of momentum conservation and law of mechanical energy conservation were also applied in this mathematical physics proving. According to the result, the study found that all energy from neutrons can be transferred out of a neutrons into proton particles.

KEYWORDS: Neutron Thermalization, Polymer Materials, Mathematical Demonstration

I. INTRODUCTION

In the future, researcher will use polyethyleneterephthalate, PETE fibers for neutron attenuation in order to produce shielding concrete materials that will be applied in the nuclear power plants or in the hospitals. Because of neutron particles are normally dangerous to human body such as they will affect blood system or tissue system as be cancer (Cember, 1992). This polymer

material (PETE) was selected to analysis because it has high ratio of H-atom per molecule (Odiان, 2004). The study expected that this polymer will be applied for neutron attenuation effectively. In the similarity procedures of (Lamarsh, 1975) and (Jewett/Serway, 2008), Vector diagram analysis, law of momentum conservation and law of mechanical energy conservation were also applied in the mathematical expressions as physiochemical hypothesis.

II. MATHEMATICAL ANALYSIS

A simply situation can be considered below; first, a neutron mass m has an initial energy and linear momentum E_1 and \vec{P}_1 respectively. And then, it moves to collide with a rest proton mass M which has no an initial energy and linear momentum. After collision, the neutron scattered from the central line with an angle θ_1 which leads to E_2 and \vec{P}_2 remaining, whereas the proton mass M remains E_A and \vec{P}_A which has θ_2 as a scattered angle.

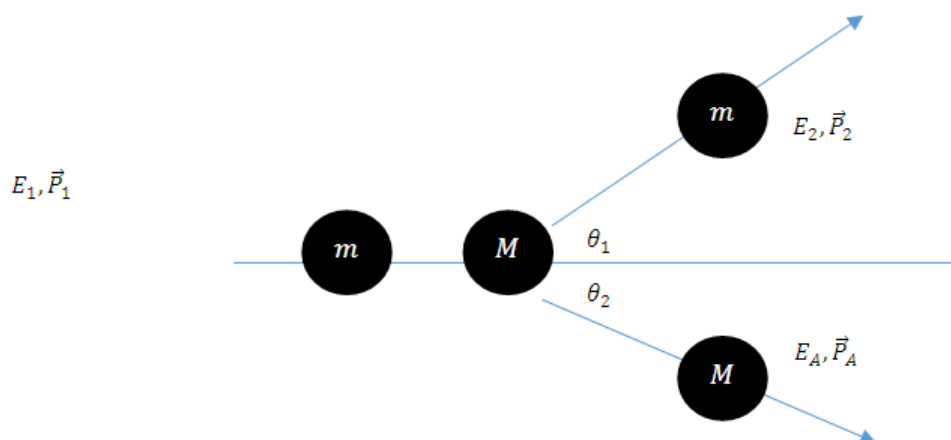


Figure 1: a proton is collided by a neutron

According to the law of energy conservation; $E_1 = E_2 + E_A$ and According to the law of linear momentum conservation; $\vec{P}_1 = \vec{P}_2 + \vec{P}_A$ Then, it can be represented figure 1 to the figure 2 as a vector illustration.

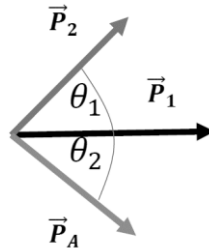


Figure 2: momentum expression as vector diagram

Forasmuch, the cosine's law can be applied to from the new mathematical relation;

$$P_A^2 = P_1^2 + P_2^2 - 2P_1P_2 \cos \theta_1 \text{----- (1)}$$

According to the classical mechanics, the quantity of momentum can be written as

$$P = mv$$

Thus $P_A = Mv_A$

$$P_A^2 = (Mv_A)^2$$

$$P_A^2 = M^2 v_A^2 \text{----- (2)}$$

Since

$$E_A = \frac{1}{2} M v_A^2$$

So

$$v_A^2 = \frac{2E_A}{M} \text{----- (3)}$$

Substitute (3) into (2)

$$P_A^2 = M^2 \left(\frac{2E_A}{M} \right)$$

$$P_A^2 = 2ME_A \text{----- (4)}$$

In the same way, using the previous procedure; now it can be derived as

$$P_1^2 = 2mE_1 \text{----- (5)}$$

And

$$P_2^2 = 2mE_2 \text{----- (6)}$$

Substitute (4), (5) and (6) into (1);

$$2ME_A = 2mE_1 + 2mE_2 - 2\sqrt{2mE_1}\sqrt{2mE_2}\cos\theta_1$$

$$2ME_A = 2mE_1 + 2mE_2 - 2(2)m\sqrt{E_1E_2}\cos\theta_1$$

Multiply by $\frac{1}{2}$ both sides

$$\text{So, } ME_A = mE_1 + mE_2 - 2m\sqrt{E_1E_2}\cos\theta_1 \text{----- (7)}$$

It decisively that $\frac{M}{m} \equiv A$

Where M is the target atomic mass

m is the neutron mass (1 a.m.u.)

And A is an atomic mass number

$$\text{So, } M = A \text{----- (8)}$$

Substitute (8) into (7);

$$AE_A = mE_1 + mE_2 - 2m\sqrt{E_1E_2}\cos\theta_1$$

$$AE_A = (1)E_1 + (1)E_2 - 2(1)\sqrt{E_1E_2}\cos\theta_1 ; m = 1 \text{ a. m. u.}$$

$$AE_A = E_1 + E_2 - 2\sqrt{E_1E_2}\cos\theta_1 \text{----- (9)}$$

Then, substitute $E_A = E_1 - E_2$ into (9)

$$A(E_1 - E_2) = E_1 + E_2 - 2\sqrt{E_1 E_2} \cos \theta_1$$

$$\begin{aligned} AE_1 - AE_2 &= E_1 + E_2 - 2\sqrt{E_1 E_2} \cos \theta_1 \\ -AE_2 - E_2 + 2\sqrt{E_1 E_2} \cos \theta_1 + AE_1 - E_1 &= 0 \\ AE_2 + E_2 - 2\sqrt{E_1 E_2} \cos \theta_1 - AE_1 + E_1 &= 0 \\ (A + 1)E_2 - 2\sqrt{E_1 E_2} \cos \theta_1 - (A - 1)E_1 &= 0 \\ (A + 1)E_2 - 2\sqrt{E_1} \sqrt{E_2} \cos \theta_1 - (A - 1)E_1 &= 0 \end{aligned}$$

$$(A + 1)\sqrt{E_2}^2 - 2\sqrt{E_1} \sqrt{E_2} \cos \theta_1 - (A - 1)E_1 = 0 \text{----- (10)}$$

Then, the following quadratic equation below can be used to find the roots of the former equation (10);

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$(A + 1)\sqrt{E_2}^2 - 2\sqrt{E_1} \sqrt{E_2} \cos \theta_1 - (A - 1)E_1 = 0$$

$$\sqrt{E_2} = \frac{2\sqrt{E_1} \cos \theta_1 \pm \sqrt{(2\sqrt{E_1} \cos \theta_1)^2 - 4(A + 1)(-(A - 1)E_1)}}{2(A + 1)}$$

$$\sqrt{E_2} = \frac{2\sqrt{E_1} \cos \theta_1 \pm \sqrt{4E_1 \cos^2 \theta_1 + 4(A + 1)((A - 1)E_1)}}{2(A + 1)}$$

$$\sqrt{E_2} = \frac{2\sqrt{E_1} \cos \theta_1 \pm \sqrt{4E_1 \cos^2 \theta_1 + 4(1+1)((1-1)E_1)}}{2(1+1)}$$

$$\sqrt{E_2} = \frac{2\sqrt{E_1} \cos \theta_1 \pm \sqrt{4E_1 \cos^2 \theta_1 + 4(2)(0)E_1}}{2(2)}$$

$$\sqrt{E_2} = \frac{2\sqrt{E_1} \cos \theta_1 \pm \sqrt{4E_1 \cos^2 \theta_1 + 0}}{2(2)}$$

$$\sqrt{E_2} = \frac{2\sqrt{E_1} \cos \theta_1 \pm \sqrt{4E_1 \cos^2 \theta_1}}{4}$$

$$\sqrt{E_2} = \frac{2\sqrt{E_1} \cos \theta_1 \pm 2\sqrt{E_1} \cos \theta_1}{4}$$

$$\sqrt{E_2} = \frac{\sqrt{E_1} \cos \theta_1 \pm \sqrt{E_1} \cos \theta_1}{2}$$

$$\sqrt{E_2} = \frac{1}{2} [\sqrt{E_1} \cos \theta_1 \pm \sqrt{E_1} \cos \theta_1]$$

$$E_2 = \frac{1}{4} [\sqrt{E_1} \cos \theta_1 \pm \sqrt{E_1} \cos \theta_1]^2$$

Since the addition operation in the previous equation cannot be operated according to the obtained E_2 will be greater than an initial energy E_1 , thus use

$$E_2 = \frac{1}{4} [\sqrt{E_1} (\cos \theta_1 - \sqrt{\cos^2 \theta_1})]^2$$

$$E_2 = \frac{1}{4}E_1(\cos\theta_1 - \sqrt{\cos^2\theta_1})^2; \text{ if } \theta_1 = 0$$
$$E_2 = 0 \text{ ----- (11)}$$

III. RESULT AND DISCUSSION

According to the previous mathematical solving and some of the given conditions, it shows that all energy from neutron will be transferred out of a neutron into proton particle as can be seen from equation (11). Because of polyethylene family is one of the rich H-atom polymers, so they probably can thermalize a neutron satisfactory which agree with (Cember, 1992) that used polyethylene for neutron shielding.

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