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RP-160: Solving some special standard quadratic congruence modulo an odd prime multiplied by eight

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ABSTRACT: In this paper, the author discussed the findings of the solutions of the special types of some standard quadratic congruence of composite modulus modulo an odd prime multiplied by eight. The formulation of solutions are well established, tested and verified using suitable numerical examples. The author selected the speciality of the positive integers appearing in thecongruence andalso on the odd prime p. A direct formulation of solutions is provided. Formulation of the congruence increased the interest of the readers and the students towards the study of the congruence. Thus, it can be said that the formulation of solutions is the merit of the paper.

KEYWORDS: Composite modulus, Formulation, Incongruent solutions, Quadratic residues.

I. INTRODUCTION

A standard quadratic congruence of composite modulus is a congruence of the type:
$x^2 \equiv a \pmod{m}$, a and m are composite positive integers.
For the solvability of the congruence, a must be quadratic residue of m.In this paper, the author imposes some speciality to a & p both, considering $m = 8p$, p being an odd prime.
Here the author classified the odd primes into four groups:

$$p \equiv 1 \pmod 8;\ p \equiv 3 \pmod 8;\ p \equiv 5 \pmod 8;\ p \equiv 7 \pmod 8.$$

Considering these four groups of p, the author also classified a into four groups as:

$$a = p, 3p, 5p, 7p.$$

These generates four types of standard quadratic congruence of composite modulus.
All of them have exactly four solutions each.

II. PROBLEM STAYEMENT

Here the problem is "To formulate the solutions of the congruence:

(1) $x^2 \equiv p \pmod{8p}$, p odd prime, $p \equiv 1 \pmod 8$.
(2) $x^2 \equiv 3p \pmod{8p}$, p odd prime, $p \equiv 3 \pmod 8$.
(3) $x^2 \equiv 5p \pmod{8p}$, p odd prime, $p \equiv 5 \pmod 8$.
(4) $x^2 \equiv 7p \pmod{8p}$, p odd prime, $p \equiv 7 \pmod 8$.

III. LITERATURE REVIEW

Now the aim of the author is to find the available literatureof the problem framed above.Burton [1], Zukerman [2], Koshy [3], remains silentabout the problem of this paper. Only Koshy had discussed a problem of the type $x^2 \equiv a \pmod{8p^2}$. Page $-$ 542, example $-$ 11.33;there he used Chinese Remainder Theorem[2] to find all the solutions. Nothing is relevant found in the literature of mathematics. But only the authors published papers related to the problem are found [4], [5], [6].

IV. ANALYSIS & RESULTS

Consider the congruence: $x^2 \equiv a \pmod{m}$.
Case-I: Let $a = p$, $m = 8p$ & $p \equiv 1 \pmod 8$.
Then congruence reduces to $x^2 \equiv p \pmod{8p}$.
As $p \equiv 1 \pmod 8$, hence $p - 1 = 8t$.
Now,$p^2 - p = p(p-1) = p.8t \equiv 0 \pmod{8p}$
Therefore,$p^2 \equiv p \pmod{8p}$.
This showed that$x \equiv \pm p \pmod{8p}$satisfied the congruence: $x^2 \equiv p \pmod{8p}$.
Hence, $x \equiv \pm p \pmod{8p}$ are the two solutions of the said congruence.
Also for $x \equiv 3p \pmod{8p}$,

$$x^2 - p \equiv (3p)^2 - p = 9p^2 - p = 8p^2 + p^2 - p \equiv p^2 - p \equiv 0 \pmod{8p}.$$

Hence, $x \equiv \pm 3p \pmod{8p}$ are also the solutions of the above congruence.
Therefore, $x \equiv \pm p, \pm 3p \pmod{8p}$

$$\equiv p, 8p - p, 3p, 8p - 3p \pmod{8p}$$

$\equiv p, 7p, 3p, 5p \pmod{8p}$ are the four solutions of the congruence.
Case-II: Let $a = 3p$, $m = 8p$ & $p \equiv 3 \pmod 8$.
Then congruence reduces to: $x^2 \equiv 3p \pmod{8p}$.
As $p \equiv 3 \pmod 8$, hence $p - 3 = 8t$.

So, $p^2 - 3p = p(p-3) = p.8t \equiv 0 \pmod{8p}$
Therefore,$p^2 \equiv 3p \pmod{8p}$ and hence $x \equiv \pm p \pmod{8p}$ are the solutions of the congruence: $x^2 \equiv 3p \pmod{8p}$.
Also for $x \equiv 3p \pmod{8p}$,

$$x^2 - 3p \equiv (3p)^2 - 3p = 9p^2 - 3p$$
$$= 8p^2 + p^2 - 3p \equiv p^2 - 3p$$
$$\equiv 0 \pmod{8p}.$$

Hence, $x \equiv \pm 3p \pmod{8p}$ are also the solutions of the congruence.
Therefore, $x \equiv \pm p, \pm 3p \pmod{8p}$

$$\equiv p, 8p-p, 3p, 8p-3p \ (mod\ 8p)$$

$\equiv p, 7p, 3p, 5p \ (mod\ 8p)$ are the four solutions.
These are the four solutions.
Case-III: Let $a = 5p$, $m = 8p$ & $p \equiv 5 \ (mod\ 8)$.
Then the congruence reduces to:$x^2 \equiv 5p \ (mod\ 8p)$.
As $p \equiv 5 \ (mod\ 8), hence\ p - 5 = 8t$.
So, $p^2 - 5p = p(p-5) = p.8t \equiv 0 \ (mod\ 8p)$
Therefore, $p^2 \equiv 5p \ (mod\ 8p)$ and hence $x \equiv \pm p (mod\ 8p)$ are the solutions of the congruence: $x^2 \equiv 5p \ (mod\ 8p)$.
Also for $x \equiv 5p \ (mod\ 8p)$.

$$x^2 - p \equiv (5p)^2 - 5p = 25p^2 - 5p$$
$$= 24p^2 + p^2 - 5p \equiv p^2 - 5p$$
$$\equiv 0 \ (mod\ 8p).$$

Hence, $x \equiv \pm 5p \ (mod\ 8p)$ are also the solutions of the congruence.
Therefore, $x \equiv \pm p, \pm 5p \ (mod\ 8p)$

$$\equiv p, 8p-p, 5p, 8p-5p \ (mod\ 8p)$$

$\equiv p, 7p, 5p, 3p \ (mod\ 8p)$ are the four solutions of the congruence.
Case-IV: Let$a = 7p$, $m = 8p$ & $p \equiv 7 \ (mod\ 8)$.
Then congruence reduces to: $x^2 \equiv 7p \ (mod\ 8p)$.
As $p \equiv 7 \ (mod\ 8), hence\ p - 7 = 8t$.
So, $p^2 - 7p = p(p-7) = p.8t \equiv 0 \ (mod\ 8p)$
Therefore, $p^2 \equiv 7p \ (mod\ 8p)$ and hence $x \equiv \pm p (mod\ 8p)$ are the solutions of the congruence: $x^2 \equiv 7p \ (mod\ 8p)$.
Also for $x \equiv 3p \ (mod\ 8p)$.

$$x^2 - 7p \equiv (3p)^2 - 7p = 9p^2 - 7p$$
$$= 8p^2 + p^2 - 7p \equiv p^2 - 7p$$
$$\equiv 0 \ (mod\ 8p).$$

Hence, $x \equiv \pm 3p \ (mod\ 8p)$ are also the solutions of the congruence.
Therefore, $x \equiv \pm p, \pm 3p \ (mod\ 8p)$

$$\equiv p, 8p-p, 3p, 8p-3p \ (mod\ 8p)$$

$\equiv p, 7p, 3p, 5p \ (mod\ 8p)$ are the solutions of the congruence.

V. ILLUSTRATIONS

Example-1: Consider the congruence $x^2 \equiv 17 \ (mod\ 136)$
It can be written as $x^2 \equiv 17 \ (mod\ 8.17)$
T is of the type $x^2 \equiv p \ (mod\ 8p)\ with\ p = 17 \equiv 1 \ (mod\ 8)$
It has exactly four solutions given by $x \equiv p, 3p, 5p, 7p \ (mod\ 8p)$

$$\equiv 17, 51, 85, 119 \ (mod\ 136).$$

Example-2: Consider the congruence $x^2 \equiv 57 \ (mod\ 152)$
It can be written as $x^2 \equiv 3.19 \ (mod\ 8.19)$
T is of the type $x^2 \equiv p \ (mod\ 8p)$ with $p = 19 \equiv 3 \pmod 8$
It has exactly four solutions given by $x \equiv p, 3p, 5p, 7p \pmod{8p}$

$$\equiv 19, 57, 95, 133 \pmod{152}.$$

Example-3: Consider the congruence $x^2 \equiv 65 \pmod{104}$
It can be written as $x^2 \equiv 5.13 (\text{mod } 8.13)$
T is of the type $x^2 \equiv 5p \pmod{8p}$ with $p = 13 \equiv 5 \pmod 8$
It has exactly four solutions given by $x \equiv p, 3p, 5p, 7p \pmod{8p}$

$$\equiv 13, 39, 65, 91 \pmod{104}.$$

Example-4: Consider the congruence $x^2 \equiv 91 \pmod{104}$
It can be written as $x^2 \equiv 7.13 (\text{mod } 8.13)$
T is of the type $x^2 \equiv 7p \pmod{8p}$ with $p = 23 \equiv 7 \pmod 8$
It has exactly four solutions given by $x \equiv p, 3p, 5p, 7p \pmod{8p}$

$$\equiv 23, 69, 115, 161 \pmod{184}.$$

VI. CONCLUSION

Therefore, it can be concluded that the special standard quadratic congruence modulo an odd prime multiplied by eight is correctly formulated. It has exactly four incongruent solutions in each case. The solutions are given by
$x \equiv p, 3p, 5p, 7p \pmod{8p}$.
The formulations are elaborated using numerical examples.

MERIT OF THE PAPER

The problemsof the said quadratic congruence is formulated for solutions. Formulation enabled finding the solutions orally. This is the merit of the paper.

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