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Regular bipolar fuzzy graphs

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ABSTRACT

In this paper, we studied three new operations on bipolar fuzzy graph; namely direct product, semi strong product and strong product. Also, we give sufficient condition for each one of them to be complete. Regularity on some bipolar fuzzy graphs whose understanding crisp graphs are path on 2m vertices, a cycle C_n are studied with some membership functions.

Keywords

Fuzzy Relation, Degree, Symmetric, bipolar fuzzy graph, Total Degree,Totally regular bipolar fuzzy graph, Induced sub graph, Complement, Crisp Graph

PRELIMINARIES OF REGULAR BIPOLAR FUZZY GRAPHS

In this section, we first review some definitions of undirected graphs that are necessary for this paper

Definition 1.1

By graph, we mean a pair G* = (V, E), where V is the set and E is a relation on V. The elements of V are vertices of G* and the elements of E are edges of G*. We write x y ∈ E to mean {x y} ∈ E; and if e = x y ∈ E; we say x and y are adjacent. Formally, given a graph G* = (V, E), two vertices x, y ∈ V are said to be neighbors, or adjacent nodes, if x y ∈ E: The neighborhood of a vertex v in a graph G* is the induced sub graph of G* consisting of all vertices adjacent to v and all edges connecting two such vertices. The neighborhood is often denoted N(v).

Definition 1.2

The degree deg (v) of vertex v is the number of edges incident on v or equivalently, deg (v) = |N(v)|. The set of neighbors, called a (open) neighborhood N(v) for a vertex v in a graph G*, consists of all vertices adjacent to v but not including v, that is N(v) = {u ∈ V | vu ∈ E}: When v is also included, it is called a closed neighborhood N[v], that is, N[v] = N(v) ∪ {v}. A regular graph is a graph where each vertex has the same number of neighbors, i.e., all the vertices have the same open neighborhood degree.

Definition 1.3

A complete graph is a simple graph in which every pair of distinct vertices is connected by an edge. An isomorphism of graphs G_1^* and G_2^* is a bijection between the vertex sets of G_1^* and G_2^* such that any two vertices v_1 and v_2 of G_1^* are adjacent in G_1^* if and only if
f (v_1) and $f(v_2)$ are adjacent in G_2^*. Isomorphic graphs are denoted by $G_1^* \simeq G_2^*$.

Definition 1.4

A fuzzy set A on a set X is characterized by a mapping m : X →[0,1], called the membership function. A fuzzy set is denoted as A = (X, m) . A fuzzy graph ξ = (V, σ, μ) is
a non-empty set V together with a pair of functions σ :V →[0,1] and μ :V ×V →[0,1] such that for all u, v ∈ V, μ (u, v) ≤ σ (u) ∧ σ (v) (here x ∧ y denotes the minimum of x and y). Partial fuzzy sub graph ξ ′ = (V, τ , ν) of ξ is such that τ (v) ≤ σ (v) for all v ∈ V and μ (u ,v) ≤ ν (u ,v) for all u, v ∈ V. Fuzzy sub graph ξ ″ = (P, σ ′, μ′) of ξ is such that P ⊆ V , σ(u) = σ (u) for all
u ∈ P, μ ′(u, v) = μ (u, v) for all u, v ∈ P .

Definition 1.5

A fuzzy graph is complete if μ (u, v) =σ (u) ∧ σ (v) for all u, v ∈ V. The degree of vertex u is d (u) = $\sum_{(u,v)\in\xi} \mu(u,v)$. The minimum degree of ξ is δ (ξ) = ∧ {d (u) | u∈V}. The maximum degree of

ξ is $\Delta(\xi) = \vee \{d(u) \mid u \in V\}$. The total degree of a vertex $u \in V$ is td (u) =
d(u) + σ(u) .

Definition 1.6

A fuzzy graph $\xi = (V, \sigma, \mu)$ is said to be regular if d(v) = k, a positive real number, for all v$\in$V . If each vertex of ξ has same total degree k, then ξ is said to be a totally regular fuzzy graph.

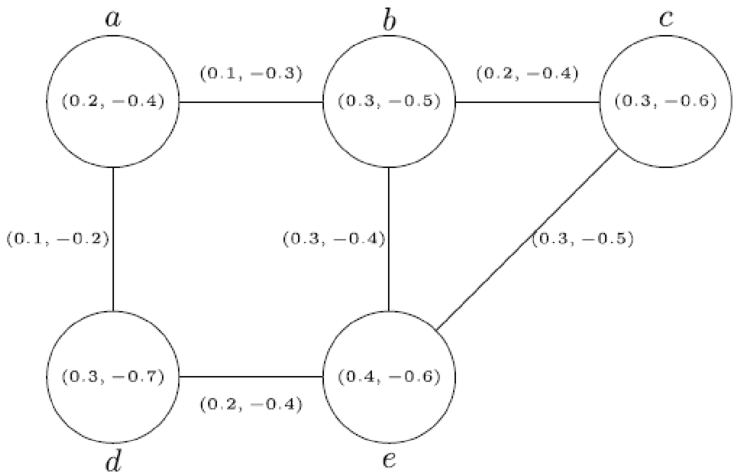
Regular Bipolar Fuzzy Line Graphs

Definition 2.1

A bipolar fuzzy graph, we mean a pair G = (A,B) where A = (μ_A^P, μ_A^N) is a bipolar fuzzy set in V and B = (μ_B^P, μ_B^N) is bipolar relation on V such that $\mu_B^P(\{x,y\}) \leq \min(\mu_A^P(x), \mu_A^P(y))$ and $\mu_B^N(\{x,y\}) \geq \max(\mu_A^N(x), \mu_A^N(y))$ for all {x, y} $\in$ E. We call A the bipolar fuzzy vertex set of V, B the bipolar fuzzy edge set of E, respectively.
Note that B is a symmetric bipolar fuzzy relation on A. We use the notation x y for an element of E. Thus, G = (A,B) is a bipolar graph of G* = (V,E) if $\mu_B^P(x,y) \leq \min(\mu_A^P(x), \mu_A^P(y))$ and $\mu_B^N(x,y) \geq \max(\mu_A^N(x), \mu_A^N(y))$ for all x y $\in$ E.

Example 2.2

Consider the bipolar fuzzy graph,



Definition 2.3

Let G = (A,B) be a bipolar fuzzy graph on G*. If all the vertices have the same open neighborhood degree n, then G is called an n-regular bipolar fuzzy graph. The open neighborhood degree of a vertex x in G is studied by deg(x) = $(deg^P(x), deg^N(x))$, where $deg^P(x) = \sum_{x\in V} \mu_A^P(x)$ and $deg^N(x) = \sum_{x\in V} \mu_A^N(x)$.

Definition 2.4

Let G = (A,B) be a regular bipolar fuzzy graph. The order of a regular bipolar fuzzy graph G is O(G) = $(\sum_{x\in V} \mu_A^P(x), \sum_{x\in V} \mu_A^N(x))$. The size of a regular bipolar fuzzy graph G is S(G) = $(\sum_{xy\in V} \mu_A^P(xy), \sum_{xy\in E} \mu_A^N(xy))$.

Definition 2.5

Let G = (A, B) be a bipolar fuzzy graph. If each vertex of G has same closed neighborhood degree m, then G is called a totally regular bipolar fuzzy graph. The closed neighborhood degree of a vertex x is studied by deg[x] = $(deg^P[x], deg^N[x])$, where
$deg^P[x] = deg^P(x) + \mu_A^P(x)$,
$deg^N[x] = deg^N(x) + \mu_A^N(x)$,
We show with the following examples that there is no relationship between n-regular bipolar fuzzy graph and m-totally regular bipolar fuzzy graph.

Example 2.6

Consider a graph G* such that V = { a, b, c, d }, E = { ab, bc, cd, ad }. Let A be a bipolar fuzzy subset of V and let B be a bipolar fuzzy subset of E studied by

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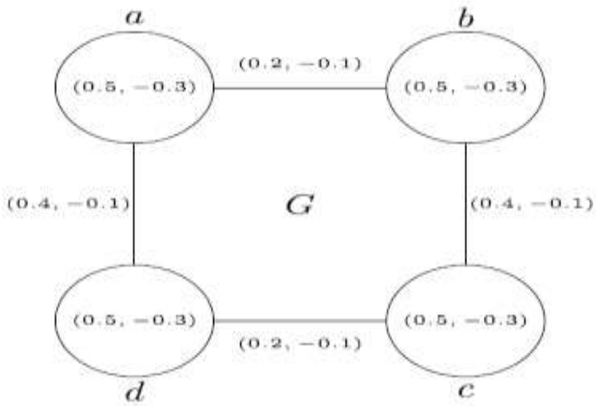
	a	b	c	d
μ_A^P	0.5	0.5	0.5	0.5
μ_A^N	−0.3	−0.3	−0.3	−0.3

	ab	bc	cd	da
μ_B^P	0.2	0.4	0.2	0.4
μ_B^N	−0.1	−0.1	−0.1	−0.1



Routine computations show that a bipolar fuzzy graph G is both regular and totally regular.

Definition 2.7

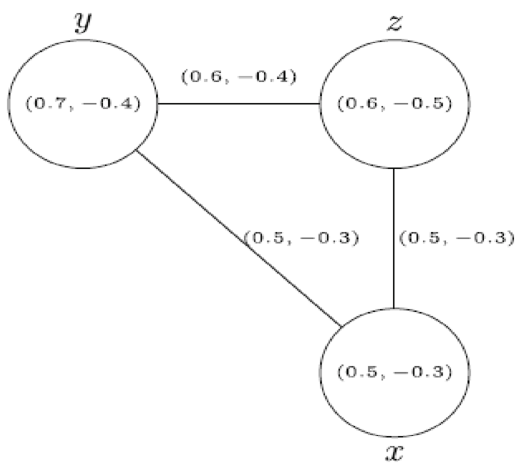
A bipolar fuzzy graph G = (A, B) is called complete if

μ_B^P (x y) = min (μ_A^P (x), μ_A^P (y) and μ_B^N (xy)
= max (μ_A^N(x), μ_A^N(y)) for all x, y ∈ V.

Example 2.8

Consider a graph G^* such that V = {x, y, z}, E = {xy, yz, zx}. Let A be a bipolar fuzzy subset of V and let B be a bipolar fuzzy subset of E studied by

μ_A^P (x) = 0.5, μ_A^P (y) = 0.7, μ_A^P (z) = 0.6,
μ_A^N (x) = -0.3, μ_A^N (y) = -0.4, μ_A^N (z) = -0.5,
μ_A^P (xy) = 0.5, μ_A^P (yz) = 0.6, μ_A^P (zx) = 0.5,
μ_A^N (xy) = -0.3, μ_A^N (yz) = -0.4, μ_A^N (zx) = -0.3,



G

Routine computations show that G is a both complete and totally regular bipolar fuzzy graph, but G is not regular since deg (x) ≠ deg (z) ≠ deg (y).

Theorem 2.9

Let G = (A, B) be a bipolar fuzzy graph of a graph G*. Then A = (μ_A^P, μ_A^N) is a constant function if and only if the following are equivalent:
(a) G is a regular bipolar fuzzy graph,
(b) G is a totally regular bipolar fuzzy graph.

Proof:

Suppose that A = (μ_A^P, μ_A^N) is constant function. Let μ_A^P (x) = c_1 and $\mu_A^N(x) = c_2$ for all x ∈ V. (a)⇒(b): assume that G is n-regular bipolar fuzzy graph, then deg^P(x) = n_1 and deg^N(x) = n_2 for all x ∈ V:
So $deg^P[x] = deg^P(x) + \mu_A^P(x)$,
$deg^N\ [x] = deg^N(x) + \mu_A^N(x)$. for all x ∈ V ;
Thus deg^P[x] = $n_1 + c_1$, deg^N(x) = $n_2 + c_2$ for all x ∈ V . Hence, G is a totally regular bipolar fuzzy graph. (b) ⇒ (a): suppose that G is a totally regular bipolar fuzzy graph, then
deg^P[x] = k_1, deg^N[x] = k_2 for all x ∈ V
or $deg^P(x) + \mu_A^P(x) = k_1$, $deg^N(x) + \mu_A^N(x) = k_2$ for all x ∈ V
or $deg^P(x) + c_1 = k_1$, $deg^N(x) + c_2 = k_2$ for all x ∈ V
or $deg^P(x) = k_1 - c_1$, $deg^N(x) = k_2 - c_2$ for all x ∈ V .
Thus, G is a regular bipolar fuzzy graph. Hence (a) and (b) are equivalent. The converse part is obvious.

Proposition 2.10

If a bipolar fuzzy graph G is both regular and totally regular, then A = (μ_A^P, μ_A^N) is constant function.

Proof:

Let G be a regular and totally regular bipolar fuzzy graph, then deg^P(x) = n_1, deg^N(x) = n_2 for all x ∈ V, deg^P[x] = k_1, deg^N[x] = k_2 for all x ∈ V, Now deg^P[x] = k_1, ⇔ $deg^P(x) + \mu_A^P(x) = k_1$, ⇔ $n_1 + \mu_A^P(x) = k_1$, ⇔ $\mu_A^P(x) = k_1 - n_1$ for all x ∈ V, Likewise, $\mu_A^N(x) = k_2 - n_2$ for all x ∈ V, Hence A = (μ_A^P, μ_A^N) is constant function.
Remark The converse may not be true, in general. We state the following Theorem without its proof.

Bipolar fuzzy line graphs

Definition 3.1

Let P(S) = (S,T) be an intersection graph of a simple graph G* = (V, E). Let G = (A_1, B_1) be a bipolar fuzzy graph of G*. We studied a bipolar fuzzy intersection graph
P(G) = (A_2, B_2) of P(S) as follows:
(1) A_2, B_2 are bipolar fuzzy sets of S and T, respectively,
(2) $\mu_{A2}^P(S_i) = \mu_{B1}^P(v_i)$, $\mu_{A2}^N(S_i) = \mu_{B1}^N(v_i)$,
(3) $\mu_{B2}^P(S_i, S_j) = \mu_{B1}^P(v_i, v_j)$, $\mu_{B2}^N(S_i, S_j) = \mu_{B1}^N(v_i, v_j)$,
for all Si,Sj ∈ S, S,Si,Sj ∈ T. That is, any bipolar fuzzy graph of P(S) is called a bipolar fuzzy intersection graph. The following Proposition is obvious.

Proposition 3.2

Let G = (A_1, B_1) be a bipolar fuzzy graph of G*, then
• P(G) = (A_2, B_2) is a bipolar fuzzy graph of P(S),
• G ≃ P(G).
This Proposition shows that any bipolar fuzzy graph is isomorphic to a bipolar fuzzy intersection graph.

Definition 3.3

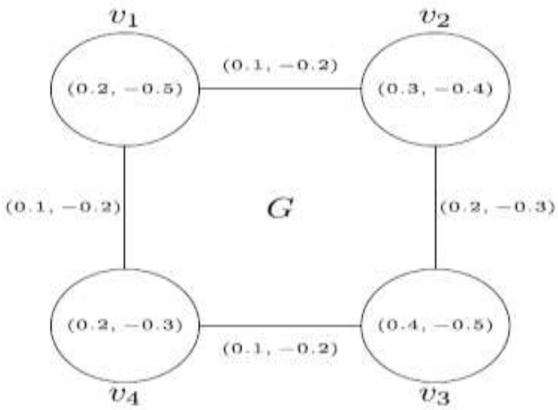
Let L(G*) = (Z,W) be a line graph of a simple graph G* = (V, E).
Let G = (A_1, B_1) be a bipolar fuzzy graph of G*. We studied a bipolar fuzzy line graph
L(G) = (A_2, B_2) of G as follows,
(1) A_2 *and* B_2 are bipolar fuzzy sets of Z and W, respectively,
(2) $\mu_{A2}^P(S_x) = \mu_{B1}^P(x) = \mu_{B1}^P(u_x, v_x)$
(3) $\mu_{A2}^N(S_x) = \mu_{B1}^N(x) = \mu_{B1}^N(u_x, v_x)$
(4) $\mu_{B2}^P(S_x S_y)$ = min (μ_{B1}^P(x),μ_{B1}^P(y))
(5) $\mu_{B2}^N(S_x S_y)$ = max (μ_{B1}^N(x) , μ_{B1}^N(y)), for all $S_x S_y$ ∈ Z, S_x, S_y ∈ W.

Example 3.4

Consider a graph G* = (V , E) such that V = { v_1 , v_2 , v_3 ,v_4 } and E = { $x_1 = v_1 v_2$, $x_2 = v_2 v_3$, $x_3 = v_3 v_4$, $x_4 = v_4 v_1$ }. Let A_1 be a bipolar fuzzy subset of V and let B_1 be a bipolar fuzzy subset of E studied by

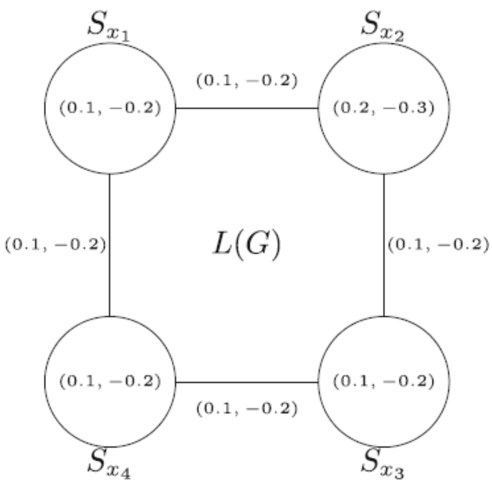
	v_1	v_2	v_3	v_4
$\mu^P_{A_1}$	0.2	0.3	0.4	0.2
$\mu^N_{A_1}$	−0.5	−0.4	−0.5	−0.3

	x_1	x_2	x_3	x_4
$\mu^P_{B_1}$	0.1	0.2	0.1	0.1
$\mu^N_{B_1}$	−0.2	−0.3	−0.2	−0.2



By routine computations, it is easy to see that G is a bipolar fuzzy graph.
Consider a line graph L (G*) = (z, w) such that Z = { $S_{x1}, S_{x2}, S_{x3}, S_{x4}$ } and W = { $S_{x1}\,S_{x2}$, $S_{x2}\,S_{x3}$, $S_{x3}\,S_{x4}$, $S_{x4}\,S_{x1}$ } Let $A_2 = (\mu^P_{A2}, \mu^N_{A2})$ and $B_2 = (\mu^P_{B2}, \mu^N_{B2})$ be bipolar fuzzy sets of Z and W , respectively. Then , by routine computations, we have

$\mu^P_{A2}(S_{x1}) = 0.1$, $\mu^P_{A2}(S_{x2}) = 0.2$, $\mu^P_{A2}(S_{x3}) = 0.1$, $\mu^P_{A2}(S_{x4}) = 0.1$,
$\mu^N_{A2}(S_{x1}) = -0.2$, $\mu^N_{A2}(S_{x2}) = -0.3$, $\mu^N_{A2}(S_{x3}) = -0.2$, $\mu^N_{A2}(S_{x4}) = -0.2$,
$\mu^P_{B2}(S_{x1}\,S_{x2}) = 0.1$, $\mu^P_{B2}(S_{x2}\,S_{x3}) = 0.1$, $\mu^P_{B2}(S_{x3}\,S_{x4}) = 0.1$, $\mu^P_{B2}(S_{x4}\,S_{x1}) = 0.1$,
$\mu^N_{B2}(S_{x1}\,S_{x2}) = -0.2$, $\mu^N_{B2}(S_{x2}\,S_{x3}) = -0.2$, $\mu^N_{B2}(S_{x3}\,S_{x4}) = -0.2$, $\mu^N_{B2}(S_{x4}\,S_{x1}) = -0.2$,



By routine computations, it is clear that L(G) is a bipolar fuzzy line graph. it is neither regular bipolar fuzzy line graph nor totally regular bipolar fuzzy line graph.

Proposition 3.5

If L(G) is a bipolar fuzzy line graph of bipolar fuzzy graph G. Then L(G*) is the line graph of G*.

Proof:

Since G = (A_1, B_1) is a bipolar fuzzy graph and L(G) is a bipolar fuzzy line graph,
$\mu^P_{A1}(S_x) = \mu^P_{B1}(x)$, $\mu^N_{A1}(S_x) = \mu^N_{B1}(x)$ for all x ∈ E.
And so $S_x \in Z \Leftrightarrow$ x ∈ E. also $\mu^P_{B2}(S_xS_y)$ = min (μ^P_{B1}(x), μ^P_{B1}(y)), $\mu^N_{B2}(S_xS_y)$ = max (μ^N_{B1}(x), μ^N_{B1}(y)) for all S_xS_y ∈ Z, and so W = { $S_xS_y \mid S_x \cap S_y \neq \emptyset$, x,y ∈ E, x≠ y}.
This completes the proof.

Proposition 3.6

L(G) is a bipolar fuzzy line graph of some bipolar fuzzy graph G if and only if $\mu^P_{B2}(S_xS_y)$ = min ($\mu^P_{A2}(S_x)$, $\mu^P_{A2}(S_y)$) for all S_xS_y ∈ W,
$\mu^N_{B2}(S_xS_y)$ = max ($\mu^N_{A2}(S_x)$, $\mu^N_{A2}(S_y)$) for all S_xS_y ∈ W

Proof:

Assume that $\mu^P_{B2}(S_xS_y)$ = min ($\mu^P_{A2}(S_x)$, $\mu^P_{A2}(S_y)$) for all S_xS_y ∈ W,
We studied $\mu^P_{A1}(x) = \mu^P_{A2}(S_x)$ for all x ∈ E, Then
$\mu^P_{B2}(S_xS_y)$ = min ($\mu^P_{A2}(S_x)$, $\mu^P_{A2}(S_y)$) = min ($\mu^P_{A1}(x)$, $\mu^P_{A1}(y)$),
$\mu^N_{B2}(S_xS_y)$ = max ($\mu^N_{A2}(S_x)$, $\mu^N_{A2}(S_y)$) = max (μ^N_{A1}(x), μ^N_{A1}(y)).
A bipolar fuzzy set A_1= (μ^P_{A1}, μ^N_{A1}) that yields that the property
$\mu^P_{B1}(xy) \leq \min(\mu^P_{A1}(x), \mu^P_{A1}(y))$,
$\mu^N_{B1}(xy) \geq \max(\mu^N_{A1}(x), \mu^N_{A1}(y))$
will suffice. The converse part is obvious.

REFERENCES:

[1]. Akram, M. and Dudek, W. A. (2012). Regular bipolar fuzzy graphs, Neural Computing & Applications, Vol. 21.

[2]. Deb, M. and De, P.K. (2015). Optimal solution of a fully fuzzy linear fractional programming problem by using graded mean integration representation method, Applications and Applied Mathematics, Vol. 10.

[3]. Dinesh, T. and Ramakrishnan, T. V. (2011). On generalised fuzzy graph structures, Applied Mathematical Sciences, Vol. 5.

[4]. Zadeh, L. A. (1965). Fuzzy sets, Information and Control, Vol. 8.

[5]. Zhang, W.-R. (1998). Bipolar fuzzy sets, Proc. of FUZZ-IEEE.