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Reliability Evaluation of Stochastic Transportation Networks with Random Transmission Times

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ABSTRACT: Many important systems such as transportation systems and logistics/distribution systems that play important roles in our daily lives can be regarded as stochastic networks whose edges have independent, nonnegative and multi-valued random transmission times. Such a network is a multistate system with multistate components and so its reliability for level d, i.e., the probability that the shortest transmission time from a specified source node to another specified sink node is less than or equal to d, can be computed in terms of minimal path vectors to level d (named d-MPs here). The main objective of this paper is to present a simple and efficient method to generate all d-MPs of such a system for each level d in terms of minimal pathsets. Two examples are given to illustrate how all d-MPs are generated by our algorithm and then the reliability of one example is computed in terms of them by further applying the state space decomposition method.

KEYWORDS: System reliability, Stochastic transportation network, d-MP

I. INTRODUCTION

Reliability analysis usually assumes that the system under study is represented by a stochastic graph in a two state model, and the system operates successfully if there exists at least one operative path from the source node to the sink node. In such a situation, reliability is considered as a matter of connectivity only and so it does not seem to be reasonable to reflect some real world systems. Many systems such as transportation systems and logistics/distribution systems that play important roles in our modem society may be regarded as stochastic networks whose transmission time of edges are independent, limited, and integer-valued random variables. For such a network, it is very practical and desirable to compute its reliability for level d, the probability that the shortest transmission time from the source node to the sink node is less than or equal to d.

Virtually, reliability computation can be carried out in terms of minimal pathsets (MPs) or minimal cutsets (MCs) in the two state model case and d-MPs (i.e., minimal path vectors to level d [3], lower boundary points of level d [12], or upper critical connection vector to level d [7]) or d-MCs (i.e., minimal cut vectors to level d [3], upper boundary points of level d [8], or lower critical connection vector to level d [3]) for each level d in the multistate model case. The stochastic transportation network with random transmission times here can be treated as a multistate system of multistate components and so the need of an efficient algorithmto search for all of its d-MPs arises. The main purpose of this paper is to present a simple and efficient algorithm to generate all d-MPs of such a network in terms of minimal pathsets. Several examples are given to illustrate how all d-MPs are generated by our method and the reliabilities of such systems are computed by further applying the state-space decomposition method [3].

II. BASIC ASSUMPTION

Let $G=(N,E,L,U)$ be a directed stochastic transportation network with the uniquesource s and the unique sink t, where N is the set of nodes, $E=\{e_i \mid 1 \le i \le n\}$ is theset of edges, $L=(l_1,l_2,\ldots,l_n)$ and $U=(u_1,u_2,\ldots,u_n)$,where l_i and u_i denote theminimum and maximum timesof each edge e_i for $i=1,2,\ldots,n.$ Such a stochastic network is

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assumed to further satisfy the following assumptions:
1. Each node is perfectly reliable. Otherwise, the network will be enlarged by treatingeach of such nodes as an edge.
2. The transmission time of each edge e_i is an integer-valued random variable that takes integervalues from l_i to u_i according to a given distribution.
3. The transmission times of different edges are statistically independent.

Assumption 3 is necessary when reliability evaluation is required.

Let $X=(x_1, x_2, \ldots, x_n)$ be a system-state vector (i.e., the current transmission time of eachedge e_i under X is x_i ,where x_i takes integer values from l_i to u_i ,and $V(X)$, the shortest transmission time from s to t under X. Such a function $V(\cdot)$ plays the role of structure function of a multistate system with $V(L)=h$ and $V(U)=k$. Under the system-state vector $X=(x_1, x_2, \ldots, x_n)$, the edge set E has the following three important subsets: $N_X=\{e_i \in E \mid x_i<u_i\}$, $B_X=\{e_i \in E \mid x_i=u_i\}$, and $S_X=\{e_i \in N_X \mid V(X+I_i)>V(X)\}$, where $I_i=(d_{i1}, d_{i2}, \ldots, d_{in})$, with $\delta_{ij}=1$ if $j=i$ and 0 if $j \neq i$. In fact, $E=S_X \cup (N_X \setminus S_X) \cup B_X$ is a disjoint union of E under X .

For level $d=h, h+1, \ldots, k-2, k-1$, a system-state vector X is said to be a d-MP if and only if: (1) its system level is d (i.e., $V(X)=d$), and (2) each edge without maximum transmission time under X is sensitive (i.e., $N_X = S_X$). If level d is given, then $\Pr\{X \mid V(X) \le d\}$, i.e., the probability that the shortest transmission time from the source node to the sink node is less than or equal to d, is taken as the system reliability.

III. MODEL CONSTRUCTION

Suppose that $P^1, P^2, \ldots, P^m$ are total MPs of the system. For each P^i , the transmission time from the source node s to the sink node t is defined as the sum of the transmission time of all edges in it. Hence, we have $V(X)=\min_{1\le i\le m}\{\sum_j \{x_j \mid e_j \in P^i\}\}$ is the shortest transmission time from s to t under X . Because $V(X)$ is non-decreasing in each argument (edge length) under X , the stochastic transportation network with random transmission times can be viewed as a multistate monotone system with the structure function $V(\cdot)$.

A necessary condition for a system-state vector X to be a d-MP is stated in the following lemmas. Our algorithm relies mainly on such a result.

Lemma 1. If X is a d-MP, then
$S_X \subseteq \bigcap_i \{P^i \mid \sum_j \{x_j \mid e_j \in P^i\}=d\}$.

Lemma 2. If X is a d-MP. Then there exists at least one MP $P^r=\{e_{r_1}, e_{r_2}, \ldots, e_{rn_r}\}$ such that the following conditions are satisfied:

$x_{r_1}+x_{r_2}+\ldots+x_{rn_r}=d$ (3.1)

$l_i \le x_i \le u_i$ for all $e_i \in P^r$ (3.2)

$x_i=u_i$ for all $e_i \notin P^r$ (3.3)

Any system-state vector $X=(x_1, x_2, \ldots, x_n)$ that satisfies constraints (3.1) - (3.3) simultaneously will be taken as a d-MP candidate. A d-MP is obviously a d-MP candidate by Lemma 2. By definition, a d-MP candidate X is a d-MP if (a) $V(X)=d$ and (2) $N_X=S_X$.

Lemma 3. If the network is parallel-series, then each d-MP candidate is a d-MP.

IV. THE PROPOSED METHOD

Suppose that all MPs, $P^1, P^2, \ldots, P^m$, have been stipulated in advance [12, 15], the family of all d-MPs can then be derived by the following steps:

Step 1. For each $P^r=\{e_{r_1}, e_{r_2}, \ldots, e_{rn_r}\}$, find all integer valued solutions of the following constraintsby applying an implicit enumeration method:

(1) $x_{r_1}+x_{r_2}+\ldots+x_{rn_r}=d$

(2) $l_i \le x_i \le u_i$ for all $e_i \in P^r$

Step 2. Set $x_i=u_i$ for all $e_i \notin P^r$.

Step 3. Obtain the family of d-MP candidates $X=(x_1, x_2, \ldots, x_n)$ by steps 1 and 2.

Step 4. Check each d-MP candidate X whether it is a d-MP:

(A) If the network is parallel-series, then each candidate is a d-MP.

(B) If the network is non parallel-series, then check each candidate whether it is a d-MP as follows:

(4.1) If there exists an $i \neq r$ such that $\sum_j \{x_j \mid e_j \in P^i\}<d$, then X is not a d-MP and go to step (4.4).

(4.2) Let $I=\{i \mid \sum_j \{x_j \mid e_j \in P^i\}=d\}$.

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(4.3) If there exists an $e_j \in A \setminus \cup_{i \in I} P^i$ such that $x_j \neq u_j$, then X is not a d-MP.

(4.4) Next candidate.

V. NUMERICAL EXAMPLES

The following two examples are used to illustrate the proposed algorithm:

Example 1.

Consider the network in Figure 1.

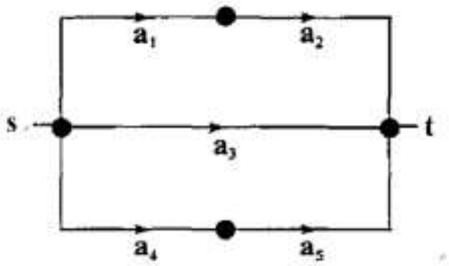

Figure 1: A series-parallel network

It is known that $L = (l_1, l_2, l_3, l_4, l_5) = (1,1,2,2,1)$ with $V(L) = 2$, $U = (u_1, u_2, u_3, u_4, u_5) = (2,5,4,5,3)$ with $V(U) = 4$, , and there exists three MPs; $P^1 = \{a_1, a_2\}$, $P^2 = \{a_3\}, P^3 = \{a_4, a_5\}$. Given d=3, the family of 3-MPs is derived as follows:

Step 1. For $P^1 = \{a_1, a_2\}$, find all integer-valued solutions of the following constraints by applying the enumeration method:

$$x_1 + x_2 = 3$$
$$1 \le x_1 \le 2$$
$$1 \le x_2 \le 5$$

Step 2. Set $x_3 = 4, x_4 = 5$, and $x_5 = 3$.

Step 3. Two 3-MP candidates $X = (1,2,4,5,3)$ and $X = (2,1,4,5,3)$ are obtained.

Step 4. Since the network is series-parallel, $X = (1,2,4,5,3)$ and $X = (2,1,4,5,3)$ are 3-MPs.

Step 1. For $P^2 = \{a_3\}$, find all integer-valued solutions of the following constraints by applying the enumeration method:

$$x_3 = 3$$
$$2 \le x_3 \le 4$$

Step 2. Set $x_1 = 2, x_2 = 5, x_4 = 5$, and $x_5 = 3$.

Step 3. One 3-MP candidate $X = (2,5,3,5,3)$ is obtained.

Step 4. Since the network is series-parallel, $X = (2,5,3,5,3)$ is a 3-MP.

Step 1. For $P^3 = \{a_4, a_5\}$, find all integer-valued solutions of the following constraints by applying the enumeration method:

$$x_4 + x_5 = 3$$
$$2 \le x_4 \le 5$$
$$1 \le x_5 \le 3$$

Step 2. Set $x_1 = 2, x_2 = 5$, and $x_3 = 4$.

Step 3. One 3-MP candidate $X = (2,5,4,2,1)$ is obtained.

Step 4. Since the network is series-parallel, $X = (2,5,4,2,1)$ is a 3-MP.

Example 2.

Consider the network in Figure 2. It is known that $L = (l_1, l_2, l_3, l_4, l_5, l_6) = (1,1,1,1,1,1)$ with $V(L) = 2$, $U = (u_1, u_2, u_3, u_4, u_5, u_6) = (3,2,2,2,2,3)$ with $V(U) = 5$,and there exists four MPs; $P^1 = \{a_1, a_2\}, P^2 = \{a_1, a_3, a_6\}$, $P^3 = \{a_2, a_4, a_5\}$, $P^4 = \{a_5, a_6\}$.

Given d=4, the family of 4-MPs is derived as follows:

Step 1. For $P^1 = \{a_1, a_2\}$, find all integer-valued solutions of the following constraints by applying the enumeration method:

$$x_1 + x_2 = 4$$
$$1 \le x_1 \le 3$$
$$1 \le x_2 \le 2$$

Step 2. Set $x_3 = 2, x_4 = 2$, and $x_5 = 3$.

Step 3. Two 4-MP candidates $X = (2,2,2,2,2,3)$ and $X = (3,1,2,2,2,3)$ are obtained.

Step 4. Check $X = (2,2,2,2,2,3)$ whether it is a 4-MP.

(4.1) $\sum_j \{x_j | e_j \in P^i\} > 4$, for each P^i with $i \neq 1$.

(4.2) $I = \{1\}$.

(4.3) $X = (2,2,2,2,2,3)$ is a 4-MP.

(4.4) Next candidate (i.e., check $X = (3,1,2,2,2,3)$ whether it is a 4-MP.)

(4.1) $\sum_j \{x_j | e_j \in P^i\} > 4$, for each P^i with $i \neq 1$.

(4.2) $I = \{1\}$.

(4.3) $X = (3,1,2,2,2,3)$ is a 4-MP.

Step 1. For $P^2 = \{a_1, a_3, a_6\}$, find all integer-valued solutions of the following constraints by applying the enumeration method:

$$x_1 + x_3 + x_6 = 4$$
$$1 \le x_1 \le 3$$
$$1 \le x_3 \le 2$$
$$1 \le x_6 \le 3$$

Step 2. Set $x_2 = 2, x_4 = 2,$ and $x_5 = 2.$

Step 3. Three 4-MP candidate $X = (1,2,1,2,2,2)$, $X = (1,2,2,2,2,1)$, and $X = (2,2,1,2,2,1)$ are obtained.

Step 4. The result is listed in Table 2.

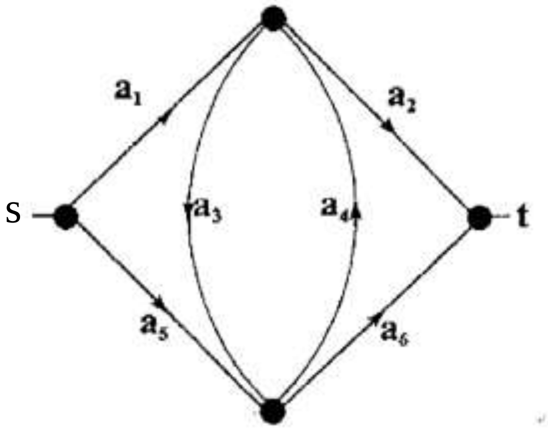

Figure 2:The 6-arcs Bridge Network

Table 1:Probability Distributions of Edge Transmission Time in Example 2.

Edge	Capacity	Probability	Edge	Capacity	Probability
a_1	3	0.60	a_4	1	0.90
	2	0.25		0	0.10
	1	0.10	a_5	2	0.80
	0	0.05		1	0.15
a_2	2	0.70		0	0.05
	1	0.20	a_6	3	0.65
	0	0.10		2	0.20
a_3	1	0.90		1	0.10
	0	0.10		0	0.05

Table 2: List of All 4-MPs in Example 2.

P^i	4-MP candidate	4-MP?	P^i	4-MP candidate	4-MP?
P^1	(2,2,2,2,2,3)	Yes	P^3	(3,1,2,1,2,3)	No
	(3,1,2,2,2,3)	Yes		(3,1,2,2,1,3)	No
P^2	(1,2,1,2,2,2)	No		(3,2,2,1,1,3)	No
	(1,2,2,2,2,1)	No	P^4	(3,2,2,2,1,3)	Yes
	(2,2,1,2,2,1)	No		(2,2,2,2,2,2)	Yes

VI. RELIABILITY EVALUATION

If $Y^1, Y^2, ..., Y^{m_d}$ are the collection of all d-MPs, then the system reliability for leveld is defined as $R_d = \Pr\{\cup_{i=1}^{m_d}\{X \mid X \le Y^i\}\}$. To compute it, several methods such as inclusion-exclusion [7, 12], disjoint subset [13], and state-space decomposition [3] are available. Here we apply the state-space decomposition method [3] to Example 2 and obtain that $R_4 = \Pr\{\cup_{i=1}^{m_4}\{X \mid X \le Y^i\}\} = 0.88796$ for demand level $d = 4$. Similarly, we have $R_2 = \Pr\{X \mid V(X) \le 2\} = 0.1876$, $R_3 = \Pr\{X \mid V(X) \le 3\} = 0.6278$, and $R_5 = \Pr\{X \mid V(X) \le 5\} = 1$. If the prior distribution of level d of example is $p_2 = 0.1$, $p_3 = 0.2$, $p_4 = 0.5$, and $p_5 = 0.2$, then the system reliability is $R = \sum_{d=2}^{5} R_d p_d = 0.861692$.

VII. SUMMARY AND CONCLUSIONS

Given all MPs that are stipulated in advance, the proposed method can generate all d-MPs of a stochastic transportation network with random edge transmission times for each level d. The system reliability, i.e., the probability that the shortest transmission time from a specified source node to another specified sink node t is less than or equal to d, can then be computed in terms of them by further applying the state space decomposition method.

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