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Review Paper on Comparative Numerical Method to Solve Differential Equation

[1] Jatin yadav,[1] Mirza Ayad baig. [2] Vishal V.Mehtre

1.*Student , B.Tech, Department Of Electrical Engineering , Bharti Vidyapeeth (Deemed To Be University) College Of Engineering, Pune.*

2.*Assistant Professor, Department Of Electrical Engineering, Bharti Vidyapeeth (Deemed To Be University) College Of Engineering, Pune.*

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ABSTRACT

The main purpose of this article is to compare the work on Runge-Kutta Methods and Euler's method to study the Problems in Differential Equations. Runge–Kutta method can be used to construct high order accurate numerical method by functions self without needing the high order derivatives of functions. In order to achieve higher accuracy in the solution, the step size needs to be very small. In mathematics and computational science, the Euler method is a first-order numerical procedure for solving ordinary differential equation with a given initial value.It is the most basic explicit method of numerical integration of ordinary differential equation. The methods were compared and contrasted based on the results obtained. The comparison shows that Euler method gives accurate approximate result than Runge-Kutta method the comparison was done in regards to identify the formula with higher accuracy.

KEYWORDS - HIGHER ORDER DERIVATIVE, NUMERICAL INTEGRATION, DIFFERENTIAL EQUATION

I. INTRODUCTION

A differential equation is an equation that relates one or several functions and its derivatives. Differential equations help us to understand equations are the most important mathematical tools in generating models in Engineering[1]. An important type of problem that we must solve when we study ordinary differential equations is an initial value problem An "Initial Value Problem" is an ordinary differential equation together with the conditions imposed on the unknown function and the values of its derivatives in a single number is called an initial value problem[2]. At some point, the initial value problem is too complicated to solve exactly, and one of two approaches is taken to approximate the solution. The first approach is to simplify the differential equation to one that can be solved exactly and then use the solution of the simplified equation to approximate the solution to the original equation[3]. While studying literature, we came across the numerous works of numerical solution of Initial Value Problems applying the Runge-Kutta methods[4]. The author discussed accurate solutions of initial value problems for ordinary differential equations with fourth-order Runge kutta method.Numerical analysis naturally finds applications in all fields of engineering and the physical science[5], but in this 21^{st} century, the life science and even the arts have adopted elements of scientific computations. Euler's method is more preferable than Runge-Kutta method because it provides slightly better results[6]. Its major disadvantage is othe possibility of having several iterations that result from a round-error in a successive step[7].

DERIVATION

(1) Runge-kutta method

Runge-Kutta methods achieve the accuracy of a Taylor series approach without requiring the calculation of higher derivatives[

$$\phi = a_1k_1 + a_2k_2 + \cdots + a_nk_n$$

where $\emptyset(x_i,y_i,h)$ is called an increment function, which can be interpreted as a representative slope over the interval. The increment function is

$$y_{i+1} = y_i + \phi(x_1, y_i, h)h$$

Where, the **a**'s are constant and the **k**'s are

$$k_1 = f(x_1, y_1)$$

$$k_2 = f(x_1 + p_1 h, y_1 + q_{11} k_1 h)$$

$$k_3 = f(x_1 + p_2 h, y_1 + q_{21} k_1 h + q_{22} k_2 h)$$

$$k_n = f(x_1 + p_{n-1}h, y_1 + q_{n-1,1}k_1h + \cdots + q_{n-1,n-1}k_{n-1}h)$$

Fourth-order Runge-Kutta Methods

The classical fourth-order RK method

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$$y_{i+1} = y_i + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

Where,

$$k_1 = f(x_i, y_i)$$

$$k_2 = f\left(x_i + \frac{1}{2}h, y_i + \frac{1}{2}k_1 h\right)$$

$$k_3 = f\left(x_i + \frac{1}{2}h, y_i + \frac{1}{2}k_2 h\right)$$

$$k_4 = f(x_i + h, y_i + k_3 h)$$

RUNGA KUTTA METHOD IN MATLAB

```
%Runga Kutta Method
% Roll no. 22
function a=runga_kutta(df)
%asking initial condtions
x0=input('0.1');
y0=input('1');
x1=input('0.2');
tol=input('1');
n=ceil((x1-x0)/tol);
h=(x1-x0)/n;
for i=1:n
   X(0,1)=x0;Y(0,1)=y1;
   k1=h*feval(df,X(1,i),Y(1,i));
   k2=h*feval(df,X(1,i)+h/2,Y(1,i)+k1*h/2);
   k3=h*feval(df,X(1,i)+h/2,Y(1,i)+k2*h/2);
   k4=h*feval(df,X(1,i)+h,Y(1,i)+k3*h);
   k=1/6*(k1+2*k2+2*k3+k4);
   X(1,i+1)=X(1,i)+h;
   Y(1,i+1)=Y(1,i)+k;
end
%diplaying results
fprintf('forx=%g\ny=%g\n',x1,Y(1,n+1))
%diplaying graph
x=1:n+1;
y=Y(1,n+1)*ones(1,n+1)-Y(1,:);
plot(x,y,'r')
title[XvsY]
```

output of program

```
>>df
 (x-y/2)
Enter the initial value of x:
0
Enter the initial value of y:
1
Enter desired level of accuracy in the result:
0.01
y= 0.91451
```

(2) Euler's method

Consider the differential

$$\frac{dy}{dx} = f(x,y)$$

Let y=(x) be the solution of the above equation. Let's say that there are x_i, y_i, i=0,1,2,…….n+1 equispaced points on the curve y=ϕ(x). i.e.,

$$y_i = \phi(x_i)$$

For i=n+1, $y_{n+1} = \phi(x_{n+1})$

When the points are equally spaced, then we can write,

$$x_{n+1} - x_n = h$$

$$x_{n+1} = x_n + h$$

Putting this value of x_{n+1} in equation

$$y_{n+1} = \phi(x_n + h)$$

Let's expand this equation around x_n,

$$y_{n+1} = \phi(x_n) + h\phi'(x_n) + \frac{h^2}{2!}\phi''(x_n) + \cdots$$
$$= \phi(x_n) + h\phi'(x_n)$$

(Neglecting second and higher order terms)

$$y_{n+1} = y_n + hf(x_n, y_n)$$

EULER'S METHOD IN MATLAB

```
% Euler's Method
% Initial conditions and setup
h = (0.1);  % step size
x = (0):h:(enter the ending value of x here);  % the range of x
y = zeros(size(x));  % allocate the result y
y(1) = (enter the starting value of y here);  % the initial y value
n = numel(y);  % the number of y values
% The loop to solve the DE
for i=1:n-1
   f = the expression for y' in your DE
   y(i+1) = y(i) + h * f;
end
```

output of program

```
 Enter the function f(x,y):
inline('x-y')
|
 Enter initial value of x i.e. x0:
0

 Enter initial value of y i.e. y0:
1

 Enter the final value of x:
0.6

 Enter the step length h:
0.2

   x       y
 0.000   1.000
 0.200   0.800
 0.400   0.680
 0.600   0.624
 The value of y at x=0.60 is y=0.624
>>
```

EXAMPLES

1.Runge-kutta method

Question:- Find y(0.2) for $y'=\frac{x-y}{2}$, y(0) =1, with step length 0.1 using Runge-Kutta 4 method

Solution:-

Given $y'=\frac{x-y}{2}$, $y(0)=1, h=0.1, y(0.2)=?$

Forth order R-K method

$k_1=hf(x_0,y_0)=(0.1)f(0,1)=(0.1)\cdot(-0.5)=-0.05$

$k_2=hf(x0+\frac{h}{2},y_0+\frac{k_1}{2})=(0.1)f(0.05,0.975)=(0.1)\cdot(-0.4625)=-0.04625$

$k_3=hf(x_0+\frac{h}{2},y_0+\frac{k_2}{2})=(0.1)f(0.05,0.97688)=(0.1)\cdot(-0.46344)=-0.04634$

$k_4=hf(x_0+h,y_0+k_3)=(0.1)f(0.1,0.95366)=(0.1)\cdot(-0.42683)=-0.04268$

$y_1=y_0+\frac{1}{6}(k_1+2k_2+2k_3+k_4)$

$y_1=1+\frac{1}{6}[-0.05+2(-0.04625)+2(-0.04634)+(-0.04268)]$

$y_1=0.95369$

$\therefore y(0.1)=0.95369$

Again taking (x_1,y_1) in place of (x_0,y_0) repeat the process

$k_1=hf(x_1,y_1)=(0.1)f(0.1,0.95369)=(0.1)\cdot(-0.42684)=-0.04268$

$K_2=hf(x_1+\frac{h}{2},y_1+\frac{k_1}{2})=(0.1)f(0.15,0.93235)=(0.1)\cdot(-0.39117)=-0.03912$

$K_3=hf(x_1+\frac{h}{2},y_1+\frac{k_2}{2})=(0.1)f(0.15,0.93413)=(0.1)\cdot(-0.39206)=-0.03921$

$K_4=hf(x_1+h,y1+k_3)=(0.1)f(0.2,0.91448)=(0.1)\cdot(-0.35724)=-0.03572$

$y_2=y_1+\frac{1}{6}(k_1+2k_2+2k_3+k_4)=0.95369+16[-0.04268+2(-0.03912)+2(-0.03921)+(-0.03572)]$

$y_2=0.91451$

$\therefore y(0.2)=0.91451$

$\therefore y(0.2)=0.91451$

2.Euler's method

Question:- Find y(0.2) for $y'=\frac{x-y}{2}$, y(0) = 1, with step length 0.1 using Euler method

solution:

Given, $y'=\frac{x-y}{2}$, $y(0)=1, h=0.1, y(0.2)=?$

Euler method,

$y_1=y_0+hf(x_0,y_0)=1+(0.1)f(0,1)=1+(0.1)\cdot(-0.5)=1+(-0.05)=0.95$

$y_2=y_1+hf(x_1,y_1)=0.95+(0.1)f(0.1,0.95)=0.95+(0.1)\cdot(-0.425)=0.95+(-0.0425)=0.9075$

$\therefore y(0.2)=0.9075$

RUNGE KUTTA METHOD	EULER'S METHOD
1: The main advantages of Runge-Kutta methods are that they are easy to implement, they are very stable, and they are ``self-starting" (i.e., unlike muti-step methods,[8] we do not have to treat the first few steps taken by a single-step integration method as special cases).	Euler's method is simple and can be used directly for the non-linear IVPs
2: It is used for temporal discretization of ordinary differential equation.	Only need to calculate the given function.
3. Runge-Kutta method are generally unsuitable for the solution of stiff equations	euler's method is less accurate and numerically unstable.
4:It increase the number of iteration steps.	Approximation error is proportional to the step size h. Hence good approximation is obtained with a very small h[9]. This requires a large number of time discretization leading to a large computation time.

II. RESULT AND DISCUSSION

In this journal we have learned about runge-kutta and euler method to solve different types of differential equation during this journal we came to know that R-K method provides more accurate answer than the euler's method. Runge-Kutta of fourth-order method, The Runge-Kutta method attempts to overcome the problem of the Euler's method, as far as the choice of a sufficiently small step size is concerned, to reach a reasonable accuracy in the problem resolution.

STEP SIZE(h)	EULER'S ERROR	R-K METHODS
h= 0.5	0.298	$4.8*10^{-4}$
h=0.1	0.042	$6.72*10^{-7}$
h=0.05	0.0203	$4.14*10^{-8}$
h=0.01	$3.98*10^{-3}$	$6.54*10^{-11}$

Runge-kutta method gives the exact approximate values and euler's method gives the number of approximate values.

III. CONCLUSION

The Euler's method and runge-kutta method were analysis in order to demonstrate the efficiency of the method to other techniques. The effect of the step on the accuracy of the techniques were also explained.

The result obtained by solving some differential equation as compared in this paper analyzed based on the theory of comparison.

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