

# Stability Analysis of the Covid-19 Spread Model Considering Physical Distancing and Self-Precaution

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**ABSTRACT:**The disease caused by SARS-CoV-2, is called Covid-19, and has become pandemic that must be considered in order to control its spread. Covid-19 can be transmitted between humans through mouth or nose when infected individuals sneezes, coughs, or talks. In this paper, we develop a mathematical model that introducing physical distancing and self-precaution in the spread of Covid-19. Dynamical model of Covid-19 spread in this paper consist of five variables: Susceptible, Traced, Quarantine, Infected, and Recovery. The results of its analysis obtained are two equilibrium points, called disease-free and endemic equilibrium points. Analysis of stability at the equilibrium points is determined by basic reproduction number. If the value of basic reproduction number is greater than one, then this dynamic model is asymptotically stable in endemic equilibrium point. Numerical simulation is provided to explain the behaviour of the system and to understand the impact of the physical distancing and self-precaution on the spread of Covid-19 in a population using real data of Covid-19 outbreak in Indonesia. Furthermore, if the level of physical distancing and self-precaution in the population is higher, then contact between individuals is also reduced. Thus, the transmission of Covid-19 infection can be controlled.

**KEYWORDS:**covid-19, physical distancing, self-precaution, dynamical model, stability

## I. INTRODUCTION

The world has been facing an outbreak by SARS-CoV-2 virus. This outbreak is called Covid-19. It has been proven to be an infectious disease that can transmitted between humans through mouth or nose once the contaminated people sneezes, coughs, or talks [1, 2, 3]. Based on World Health

Organization (WHO), the first identified case was a case in Wuhan, China on 31th December 2019. Then at the beginning

of 2020, Covid-19 was declared as pandemic and had 88.828.387 confirmed cases worldwide in January 2021 [4].

The existence of Covid-19 caused many negative effect in people's lives. Mathematical modelling is incredibly useful to analyse the behaviour of Covid-19 spread. As a result, the policies taken to beat the spread of Covid-19 are going to be a lot of effective [5, 6, 7]. There are many studies of Covid-19 modelling explains the dynamic of its spread in population. The general model that is commonly used to predict epidemics are SIR, SIS, and SEIR [8][9]. From several previous studies that discussed the model of Covid-19 outbreak that are SEIR model considering about vaccine and isolation as parameter [10], SEIR model considering hospitalization, isolation, and the impact of panic and anxiety [11, 12, 13], SPEIQRD (Susceptible, Insusceptible, Exposed, Infective, Quarantined, Recovered, Death) model based on case in Italy [14], SEI<sub>A</sub>I<sub>S</sub>RP (Susceptible, Exposed, Asymptomatic Infectious, Symptomatic Infectious, Recovered, Pathogen) model considering pathogens in the environment and social distancing [15, 16], STQIR model based on case in Central Java Province, Indonesia [17, 18], etc.

In this paper, we modified model that proposed by Fitriyani [17]. Model development is done by adding parameters related to crowd impact and the effectiveness of self-precaution by individual. In this model is introduced that virus can transmitted through the interaction between person in monitoring (ODP) and healthy individuals.

This paper is prepared as follows: in phase 2, we describe the formulation model in this paper. In phase 3, we discussed the positivity and boundedness of solutions. In phase 4, we analyse the global stability of the model. Then in phase 5 we present numerical simulation to illustrate the behaviour of this pandemic based on proposed model. Finally, in phase 6 we put on conclusions and discussion of this paper.

## II. DEVELOPMENT OF MODEL FORMULARIZATION

In this phase, we describe the development of mathematical model formulation using the proposed model by Fitriyani[17] i.e. the spread model of Covid-19 that divided the human population into five compartments namely susceptible (S) is healthy person who can be

infected with the disease, then person who is being monitored is individual who come into contact with the pathogen (T), while person under surveillance is individual who is infected the virus but not confirmed yet and do quarantine (Q), infected (I) is individual who have been confirmed to be infected and is capable to transmit the virus to others, and recovery is individual who was infected and is immune to the disease (R). In this paper, model is built by considering the crowd impact in populations during pandemic. It shows by the contact between person in monitoring (ODP) and healthy person, also not taking personal precautions like washing hands and wearing mask. Furthermore, we assumed that quarantine person and isolated infectious cannot transmit virus to healthy person. Figure 1 below shows a flow scheme based on above description.

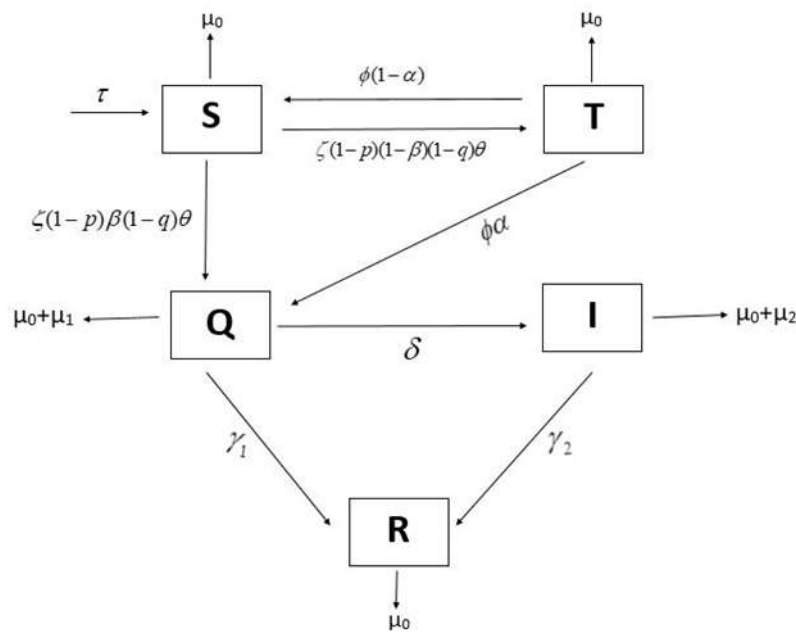


Figure 1: Scheme of Covid-19 spread model

Susceptible individuals are recruited from natural birth rate  $\tau$  and ODP person with the constant rate  $\phi(1-\alpha)$ . They are removed from their class with the infection rate  $\zeta(1-p)\beta(1-q)\theta$  and natural death rate  $\mu_0$ . It causes by the physical contact  $(1-p)$  between the susceptible individuals and ODP person, where  $p$  is the efficacy of physical distancing intervention, and there is no self-precautions  $(1-q)$  of the individuals, where  $q$  is the effectiveness of self-precaution by individuals. Average number of contacts between individuals and the proportion of monitored individuals who are able to transmit the infection to healthy person

respectively  $\zeta$  and  $\theta$ . The probability of successful infection of healthy individuals from infectious person is  $\beta$ . ODP person will be quarantine and become PDP person with the constant rate  $\phi\alpha$  and die at the rate  $\mu_0$ . PDP person is confirmed become positive Covid-19 infectious individual at the rate  $\delta$ , die at the rate  $\mu_0 + \mu_1$ , where  $\mu_1$  is the death rate of PDP individuals due to infected Covid-19, and recovery due to quarantine at the rate  $\gamma_1$ . The infectious individuals recovery due to treatment in hospital at the rate  $\gamma_2$  and die at the rate  $\mu_0 + \mu_2$ , where  $\mu_2$  is the death rate of positive Covid-19

individuals. Recovery individuals die with the rate  $\mu_0$ .

According to above assumptions and descriptions, the development mathematical model equations are

$$\left. \begin{aligned} \frac{dS}{dt} &= \tau - S\zeta(1-p)\beta(1-q)\theta T + (1-\alpha)\phi T - \mu_0 S \\ \frac{dT}{dt} &= S\zeta(1-p)(1-\beta)(1-q)\theta T - ((1-\alpha)\phi + \alpha\phi + \mu_0)T \\ \frac{dQ}{dt} &= S\zeta(1-p)\beta(1-q)\theta T + \phi\alpha T - (\mu_0 + \mu_1 + \delta + \gamma_1)Q \\ \frac{dI}{dt} &= \delta Q - (\mu_0 + \mu_2 + \gamma_2)I \\ \frac{dR}{dt} &= \gamma_1 Q + \gamma_2 I - \mu_0 R, \end{aligned} \right\} \quad (2.1)$$

given in (2.1) with the initial conditions are positive and the parameters interpretation are presented in Table 1.

with  $S(0) \geq 0, T(0) \geq 0, Q(0) \geq 0, I(0) \geq 0, R(0) \geq 0$  as the initial conditions.

**Table 1: Interpretation of parameters**

Notation	Interpretation	Notation	Interpretation
$\tau$	Recruitment rate of S class	$\phi$	the proportion of ODP individuals but they are exposed and others become healthy
$\zeta$	The average of contacts number of each individuals	$\mu_0$	Natural death rate
$\beta$	The probability of successful infection of healthy individuals from infectious person	$\mu_1$	the death rate of PDP individuals due to infected Covid-19
$\alpha$	The probability of successful infection of ODP individuals from infectious person	$\mu_2$	the death rate of positive Covid-19 individuals
$p$	the efficacy of physical distancing intervention	$\delta$	the rate of PDP individuals become positive Covid-19 infection
$q$	the effectiveness of self-precaution by individuals	$\gamma_1$	the recovery rate of PDP individual due treatment in quarantine
$\theta$	the proportion of individuals in monitoring that can transmit the infection to healthy person	$\gamma_2$	the recovery rate of infectious persons due to treatment in hospital

### III. BASIC REPRODUCTION NUMBER

Before finding the basic reproduction number and analyzing the model, we need to verify that all variables are bounded and positive for all

$t \geq 0$ .

Let  $N(t) = S(t) + T(t) + Q(t) + I(t) + R(t)$  and for all initial conditions are positive. Then we have,

$$\begin{aligned} \frac{dN}{dt} &= \frac{dS}{dt} + \frac{dT}{dt} + \frac{dQ}{dt} + \frac{dI}{dt} + \frac{dR}{dt} \\ &= \tau + \zeta(1-p)(1-\beta)(1-q)\theta ST - \mu_0(S+T+Q+I+R) - \\ &\quad \mu_1 Q - \mu_2 I \\ &= \tau + \zeta(1-p)(1-\beta)(1-q)\theta ST - \mu_0 N - \mu_1 Q - \mu_2 I \leq \tau - \mu_0 N \end{aligned}$$

So that we have,

$$\frac{dN}{dt} \leq \tau - \mu_0 N \quad (2.2)$$

Next, by integrating the inequality (2.2), we obtain the solution

$$N(t) \leq \frac{\tau}{\mu_0} + N(0)e^{-\mu_0 t}$$

Letting  $t$  tends to infinity, we get  $0 \leq N(t) \leq \frac{\tau}{\mu_0}$ . Therefore, for system (2.1) is bounded.

Further, using integral factor we prove that system (2.1) is positive for all solutions.

We have,

$$\frac{dS}{dt} = \tau - \zeta(1-p)\beta(1-q)\theta ST + \phi(1-\alpha)T - \mu_0 S$$

For  $t \geq 0$  and let  $j = \zeta(1-p)\beta(1-q)\theta T + \mu_0$ , we find the solution

$$\begin{aligned} S(t) &= e^{-\int j(t)dt} \left( \int (\tau + \phi(1-\alpha)T) e^{\int j(t)dt} dt + C \right) \\ S(t) &= C e^{-\int j(t)dt} + e^{-\int j(t)dt} \int (\tau + \phi(1-\alpha)T) e^{\int j(t)dt} dt \\ S(t) &= S(0) e^{-\int_0^t j(u)du} + e^{-\int_0^t j(u)du} \int_0^t (\tau + \phi(1-\alpha)T) e^{\int_0^u j(u)du} dt \geq 0, \forall t \geq 0 \end{aligned}$$

with the same method, we can find that  $T(t)$ ,  $Q(t)$ ,  $I(t)$ ,  $R(t)$  are positive.

Furthermore, we will derive a basic reproduction number. The basic reproduction number is a parameter that used to determine the extent of Covid-19 infection, which determine whether the disease will disappears or persist within the population and is stated by  $\mathcal{R}_0$ . Afterwards, we apply the next generation matrix approach to determine  $\mathcal{R}_0$ . If  $\mathcal{R}_0 > 1$ ,

it means that one primary infection can cause multiple secondary infections so the disease-free equilibrium is unstable and it causes the outbreak. On the other hand, if  $\mathcal{R}_0 < 1$  the disease-free equilibrium is asymptotically steady. It means the infection is under control. Now based on explanation on the phase 2, we know that classes T, and I are the only classes directly involved in the spread of Covid-19 infection. Consequently we have,

$$\left. \begin{aligned} \frac{dT}{dt} &= \zeta(1-p)(1-\beta)(1-q)\theta ST - (\phi(1-\alpha) + \alpha\phi + \mu_0)T \\ \frac{dI}{dt} &= \delta Q - (\mu_0 + \mu_2 + \gamma_2)I \end{aligned} \right\} (2.3)$$

Then system (2.3) can be transformed into the form  $\frac{dx}{dt} = (F - V)(x)$ , where  $x = [T, I]$

$$\begin{aligned} F(x) &= \begin{bmatrix} F_1 \\ F_2 \end{bmatrix} = \begin{bmatrix} \zeta(1-p)(1-\beta)(1-q)\theta ST \\ 0 \end{bmatrix} \\ V(x) &= \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} (\phi(1-\alpha) + \alpha\phi + \mu_0)T \\ \delta Q - (\mu_0 + \mu_2 + \gamma_2)I \end{bmatrix} \end{aligned}$$

Since the model has DFE

$E^0 = (S^0, T^0, Q^0, I^0, R^0) = \left( \frac{\tau}{\mu_0}, 0, 0, 0, 0 \right)$ , so we get Jacobian matrix of  $\mathbf{F}(\mathbf{x})$  dan  $\mathbf{V}(\mathbf{x})$  at the DFE and

compute  $\mathbf{FV}^{-1}$ , where

$$F = \begin{pmatrix} \zeta(1-p)(1-\beta)(1-q)\frac{\tau}{\mu_0} & 0 \\ 0 & 0 \end{pmatrix}$$

$$V^{-1} = \begin{pmatrix} -\frac{1}{\phi + \mu_0} & 0 \\ 0 & \frac{1}{\mu_0 + \mu_1 + \gamma_2} \end{pmatrix}$$

Thus, we obtain the solution as follows:

$$\mathcal{R}_0 = \frac{\zeta(1-p)(1-\beta)(1-q)\theta\tau}{\mu_0(\phi + \mu_0)} = \frac{\eta\tau}{\mu_0(\phi + \mu_0)},$$

with  $\eta = \zeta(1-p)(1-\beta)(1-q)\theta$ .

### 3.1 Equilibrium Point

System of this model has two equilibriums. The first one is disease free equilibrium point (DFE)

$E^0 = (S^0, T^0, Q^0, I^0, R^0) = \left( \frac{\tau}{\mu_0}, 0, 0, 0, 0 \right)$ . The other one is endemic equilibrium point (EE)

$(E^*) = (S^*, T^*, Q^*, I^*, R^*)$ , where

$$S^* = \frac{\phi + \mu_0}{\eta}; \quad T^* = \frac{\mu_0(\phi + \mu_0)(\mathcal{R}_0 - 1)}{\chi(\phi + \mu_0) - \phi(1 - \alpha)\eta};$$

$$Q^* = \left( \chi \left( \frac{\phi + \mu_0}{\eta} \right) + \frac{\eta\phi\alpha}{\eta} \right) \frac{1}{(\mu_0 + \mu_1 + \delta + \gamma_1)} T^*;$$

$$I^* = \frac{\delta Q^*}{(\mu_0 + \mu_2 + \gamma_2)}; \quad R^* = \frac{\gamma_1 Q^* + \gamma_2 I^*}{\mu_0}$$

According to the expression of EE above,  $E^*$  is feasible if  $\mathcal{R}_0 > 1$ .

### 3.2 Stability Analysis of the Equilibrium Point

In this phase we verify the locally and globally asymptotic stability criteria with the following theorems.

**Theorem 3.1** Let  $\mathcal{R}_0 = \frac{\eta\tau}{\mu_0(\phi + \mu_0)}$ . DFE point of the system is locally asymptotically steady if  $\mathcal{R}_0 < 1$  and unsteady if  $\mathcal{R}_0 > 1$ .

**Proof.**

The Jacobian matrix of the system at DFE is

$$J(E^0) = \begin{bmatrix} -\mu_0 & -B \frac{\tau}{\mu_0} + \phi(1-\alpha) & 0 & 0 \\ 0 & A \frac{\tau}{\mu_0} - \phi(1-\alpha) - \phi\alpha - \mu_0 & 0 & 0 \\ 0 & B \frac{\tau}{\mu_0} + \phi\alpha & -(\mu_0 + \mu_1 + \delta + \gamma_1) & 0 \\ 0 & 0 & \delta & -(\mu_0 + \mu_2 + \gamma_2) \end{bmatrix}$$

The characteristic roots of the Jacobian at DFE are

$$(\lambda + \mu_0)(\lambda^3 a_0 + \lambda^2 a_1 + \lambda a_2 + a_3) = 0, \text{ where}$$

$$a_0 = 1 \quad a_1 = \gamma_1 + \gamma_2 + \delta + 2\mu_0 + \mu_1 + \mu_2 + (\mu_0 + \phi)(1 - \mathcal{R}_0)$$

$$a_2 = \mu_0^2 + \mu_0(\mu_1 + \delta + \gamma_2 + \gamma_1 + \mu_2) + \gamma_1\gamma_2 + \delta\gamma_2 + \mu_1\mu_2 + \delta\mu_2 + \mu_1\gamma_2 + \mu_2\gamma_1 + 2(\mu_0^2 + \mu_0\phi)(1 - \mathcal{R}_0) + (\mu_0 + \phi)(\mu_1 + \delta + \gamma_2) \quad a_3 = (\mu_0 + \mu_2 + \gamma_2)(\mu_1 + \delta + \gamma_1 + \mu_0)(\phi + \mu_0)(1 - \mathcal{R}_0) + \gamma_1 + \mu_2)(1 - \mathcal{R}_0)$$

Since we get  $\lambda = -\mu_0$  and  $a_1, a_2, a_3$  will be positive if  $\mathcal{R}_0 < 1$ . Based on Routh-Hurwitz properties, the DFE( $E_0$ ) is locally asymptotically steady if  $\mathcal{R}_0 < 1$  and unsteady if  $\mathcal{R}_0 > 1$ .

**Theorem 3.2** Let  $\mathcal{R}_0 = \frac{\eta\tau}{\mu_0(\phi + \mu_0)}$ . EE point of the system is locally as well as globally asymptotically steady if  $\mathcal{R}_0 > 1$  and unsteady if  $\mathcal{R}_0 < 1$ .

**Proof.**

Local stability of the EE point is verified by Manifold Center Theory. We assumed that DFE is lose its stability when  $\mathcal{R}_0 > 1$ . Therefore, we can state that bifurcation is occur around DFE in this system when  $\mathcal{R}_0 = 1$ . We choose  $\beta$  as bifurcation

parameter. So we have,  $\beta^* = 1 - \frac{\mu_0(\phi + \mu_0)}{\zeta(1-p)(1-q)\theta\tau}$ .

Then, from Jacobian matrix  $J(E^0, \beta^*)$ , there is eigen value  $\lambda_2 = 0$ .

Let  $W = [w_1 \ w_2 \ w_3 \ w_4]^t$  is the right eigen vector and  $V = [v_1 \ v_2 \ v_3 \ v_4]$  is the left eigen vector. From the  $J(E^0, \beta^*)W = 0$  and  $VJ(E^0, \beta^*) = 0$ , we have

$$\begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \end{bmatrix} = \begin{bmatrix} \frac{1}{\mu_0\delta(\mu_0^2 + \phi(1-\alpha)\mu_0 - \zeta(1-p)(1-q)\theta\tau)} \left( (\mu_0^2 + \phi(2-\alpha)\mu_0) - \zeta(1-p)(1-q)\theta\tau \right) \\ \frac{(\mu_0^2 + \mu_0(\mu_2 + \gamma_2 + \gamma_1 + \delta + \mu_1) + (\mu_2 + \gamma_2)(\gamma_1 + \delta + \mu_1))}{\delta(\mu_0^2 + \phi(1-\alpha)\mu_0 - \zeta(1-p)(1-q)\theta\tau)} \\ \frac{\mu_0(\mu_0^2 + (\gamma_1 + \mu_2 + \gamma_2 + \delta + \mu_1)\mu_0 + (\mu_2 + \gamma_2)(\gamma_1 + \delta + \mu_1))}{\delta(\mu_0^2 + \phi(1-\alpha)\mu_0 - \zeta(1-p)(1-q)\theta\tau)} \\ \frac{(\mu_0 + \gamma_2 + \mu_2)}{\delta} \\ 1 \end{bmatrix}$$

And

$$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} = \begin{bmatrix} 0 \\ \frac{\delta(\mu_0^2 + \phi(1-\alpha)\mu_0 - \zeta(1-p)(1-q)\theta\tau)}{\mu_0((\mu_0^2 + \mu_0(\mu_2 + \gamma_2 + \gamma_1 + \delta + \mu_1) + (\mu_2 + \gamma_2)(\gamma_1 + \delta + \mu_1)))} \\ 0 \\ 0 \end{bmatrix}$$

Then let  $S = x_1, T = x_2, Q = x_3, I = x_4$  and

$$\frac{dS}{dt} = f_1, \frac{dT}{dt} = f_2, \frac{dQ}{dt} = f_3, \frac{dI}{dt} = f_4, \text{ we get}$$

$$\begin{aligned}
 a &= \sum_{k,i,j=1}^n v_k w_i w_j \frac{\partial^2 f_k}{\partial x_i \partial x_j} \\
 &= v_2 w_1 w_2 \left[ \frac{\partial^2 f_2}{\partial x_1 \partial x_2} + \frac{\partial^2 f_2}{\partial x_2 \partial x_1} \right] \quad \text{and} \\
 &= - \frac{1}{\mu_0 \delta \left( \mu_0^2 + \phi(1-\alpha)\mu_0 - \mathcal{R}_0 \frac{\mu_0^2 + \mu_0 \phi}{(1-\beta)} \right)} 2(\mu_0^2 + (\gamma_1 + \mu_2 + \gamma_2 + \delta + \mu_1)\mu_0 + \\
 &\quad (\mu_2 + \gamma_2)(\gamma_1 + \delta + \mu_1)) \left( \mu_0^2 + \phi\mu_0 + (1-\alpha)\mu_0 - \mathcal{R}_0 \frac{\mu_0^2 + \mu_0 \phi}{(1-\beta)} \right) \zeta(1-p)(1-q)\theta \\
 \\
 b &= \sum_{k,i=1}^n v_k w_i \frac{\partial^2 f_k}{\partial x_i \partial \beta} \\
 &= v_2 w_2 \left[ \frac{\partial^2 f_2}{\partial x_2 \partial \beta} \right] \\
 &= - \frac{\zeta(1-p)(1-q)\theta\tau}{\mu_0}
 \end{aligned}$$

From the expressions above, we have  $a < 0$  and  $b < 0$ . According to Manifold Center Theory [19], when  $a < 0$  and  $b < 0$ , then we conclude that EE is locally asymptotically steady.

Afterwards, we verify the global stability of EE by constructing Lyapunov function as

$$\begin{aligned}
 V(t) &= \left( S - S^* - S^* \ln \frac{S}{S^*} \right) + \left( T - T^* - T^* \ln \frac{T}{T^*} \right) + \\
 &\quad \left( Q - Q^* - Q^* \ln \frac{Q}{Q^*} \right) + \left( I - I^* - I^* \ln \frac{I}{I^*} \right)
 \end{aligned}$$

- $V(t)$  consist of logarithm function, so it is clear that its function is continuous.
- $V(t)$  is definite positive. It proved for  $V(S, T, Q, I) = V(S^*, T^*, Q^*, I^*)$  and  $V(S, T, Q, I) \neq V(S^*, T^*, Q^*, I^*)$ .
- $\dot{V}(t)$  is definite negative.

Using arithmetical mean (AM) and geometrical mean (GM), where define the two interdependent sequences  $\{a_n\}$  and  $\{g_n\}$  by  $a_{n+1} = \frac{1}{2}(a_n + g_n)$  and  $g_{n+1} = \sqrt{a_n g_n}$ , and satisfy the inequality  $AM \geq GM$  [20]. Thus, we have

$$\frac{S^*}{S} + \frac{S}{S^*} \geq 2 \quad \text{and} \quad \frac{I^*}{I} + \frac{I}{I^*} \geq 2$$

As a result we get  $\dot{V}(t) < 0$  when  $\mathcal{R}_0 > 1$ . Therefore, we can conclude that EE point is globally asymptotically stable. Hence theorem is verified.

#### IV. NUMERICAL SIMULATION

We estimated the value of parameters using least square method [21, 22, 23] in table 2 for

numerical simulation. The computational in this paper using the real Covid-19 data in Indonesia from 11 July 2021 until 31 August 2021.

**Table 2: Parameters estimation for Covid-19 case in Indonesia**

Notation	Value	Source
$\tau$	3.99	Estimated

$\zeta$	$8.8408 \times 10^{-1}$	Estimated
$\beta$	0.99	Estimated
$\alpha$	$6.263 \times 10^{-1}$	Estimated
$p$	$5.9448 \times 10^{-3}$	Estimated
$q$	0.11	Estimated
$\phi$	$0.83051 \times 10^{-5}$	Estimated
$\mu_0$	0.1199	Estimated
$\mu_1$	0.134	Estimated
$\mu_2$	0.2065	Estimated
$\delta$	$9.529 \times 10^{-2}$	Estimated
$\gamma_1$	$8.99 \times 10^{-2}$	Estimated
$\gamma_2$	$6.097 \times 10^{-2}$	Estimated
$\theta$	0.89	Estimated

Numerical simulation of model (2.1) with parameter value in Table 2 and the initial value is set to be  $S(0) = 17010; T(0) = 1477051; Q(0) = 38079;$

$I(0) = 256763; R(0) = 211947$ . Simulations are used to verify the results of the analysis discussed in the previous section. We depicted the model of the transmission of Covid-19 by using MATLAB R2019b.

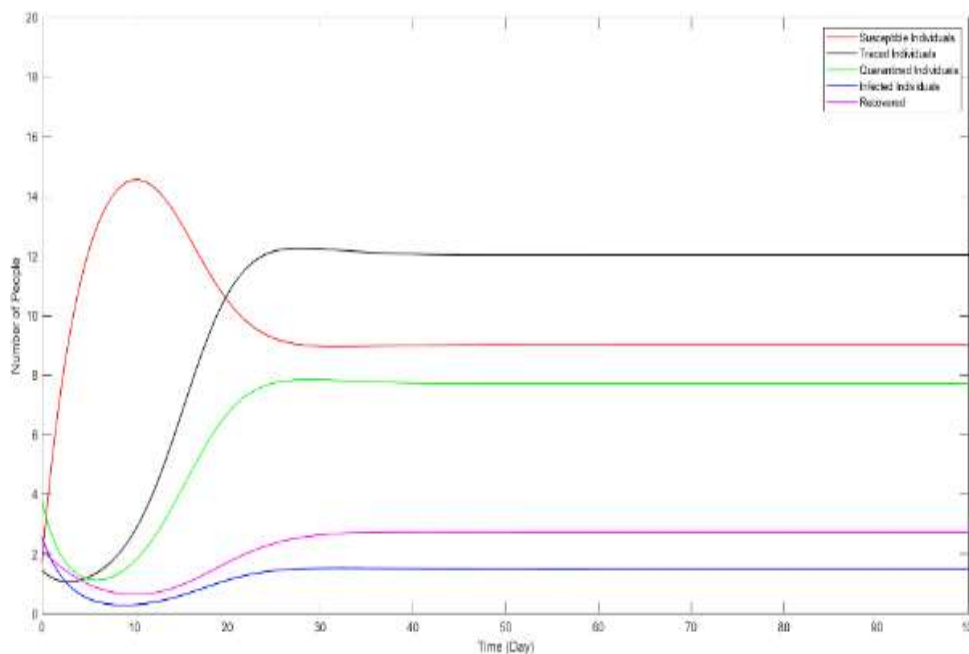


Figure 2: Model Simulation of Covid-19 Spread in Indonesia

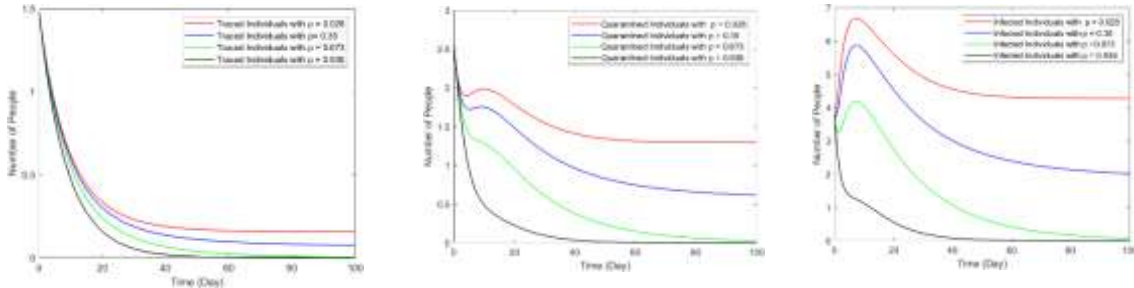
From Figure 2, it can be seen that the susceptible compartment is increasing at the beginning but then decreasing with time, because healthy individual had contact with person in monitoring (traced Individual) and there is crowd

in population. Then in quarantine class is increasing because healthy person had contact with ODP and the infection is successfully transmitted. Therefore, person in monitoring (traced Individual) is changed to, quarantined individual. It can be

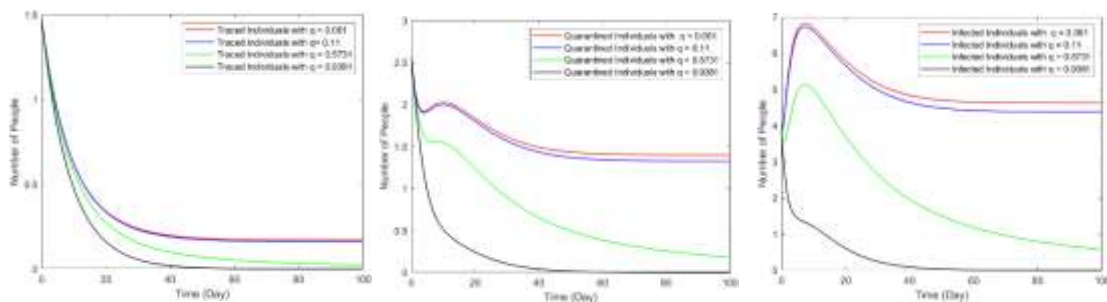


seen that the higher the crowd level, then the higher contact between individual happens. Consequence,

the outbreak occurs within population with the value of  $\mathcal{R}_0 = 1.9319 > 1$ .



**Figure 3: Effect of physical distancing intervention to the dynamics of Covid-19 on traced, quarantine, and infected individuals**



**Figure 4: The effectiveness of self-precaution by individuals to the dynamics of Covid-19 on traced, quarantine, and infected individuals**

Furthermore, we observed the effect of physical distancing intervention and self-precaution by individuals such a wearing mask and washing hands to the behavior dynamics model that has been proposed. Figure 3 represent the result of the efficacy of physical distancing intervention. This simulation is carried out with various values of  $p = 0.028, 0.35, 0.673, 0.936$  and the other parameters constant. According to Figure 3, it said that the greater  $p$  value, then it would reduce the number of traced Individual, quarantined individual, and infected individuals. If we look at the difference between the numbers of population in each value of  $p$ , we can state that physical distancing intervention is significant to overcome the spread of Covid-19 outbreak. It means, if the level of crowd in the population is lower, then contact between individuals are also reduced. Thus, the transmission of Covid-19 infection can be controlled. The second simulation, in Figure 4 represent the effect of the effectiveness of self-precaution by individuals. To run this simulation, we provide various value of

$q = 0.061, 0.11, 0.5731, 0.9381$  and keep the other parameters constant. As the result, it said that the greater value of  $q$  then traced Individual, quarantined individual, and infected individuals are reduced. The difference number of population between various  $q$  can confirm that self-precaution by individuals is significant to control the outbreak. This means that if the individuals in population take self-precaution obediently then the transmission of Covid-19 can be controlled.

## V. CONCLUSION OF RESULT

In this research, we developed a STQIR model for Covid-19 outbreaks based on case in Indonesia by considering the physical distancing and self-precaution intervention and proved its stability at equilibrium points. Theoretically, the stability of the system has been proven depends on the basic reproduction number value. We have shown that all properties necessary for epidemiological relevance is verified. We estimated parameters value using real data from Indonesia. Numerical simulation show that the epidemic occurs in the population. It can be seen

from basic reproduction number value, that is  $R_0 = 1.9319$ , which means that every one person can infect two persons.

Furthermore, based on the simulation with the various value of the efficacy physical distancing intervention and effectiveness of self-precaution by individuals parameters conclude that these two parameters are significant to reduce the Covid-19 transmission. The higher efficacy physical distancing intervention means that the crowd in the population is lower thus the level contact between individual is reduced. The higher effectiveness of self-precaution by individuals means that every individual in the population is taking seriously to wear mask and washing hand to keep hygiene. Finally we can conclude that Covid-19 outbreak can be controlled by complying with the policy of physical distancing and taking seriously to do self-precaution.

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