

Study of Degree Based Topological Indices of Carbon Nanocones

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ABSTRACT: Let G be a simple and connected graph with n vertices and m edges. The degree d_u of a vertex $u \in V(G)$ is the number of vertices of G adjacent to u . The nanocones (CNCs) can be considered as the nanoscale conical carbon-based material. In carbon nanocones $CNC_k[n]$, the parameter k defines the length of inner cycle and n defines the number of layers of the graph. In this paper degree based topological indices for carbon nanocones $CNC_3[2]$, $CNC_4[2]$, $CNC_5[2]$, $CNC_6[2]$ and $CNC_7[2]$ and some inequalities are investigated.

KEYWORDS: Topological indices, carbon nanocones, inequality, one pentagonal nanocone, one heptagonal nanocone.

I. INTRODUCTION

A molecular graph is a simple graph such that its vertices correspond to the atoms and the edges to the bonds. A topological index is a number related to a molecular graph invariant automorphism of Graph. The ability of elemental carbon to form extended two dimensional sheet structures with extremely strong bond makes it a stable material to produce isolated objects. The sheets can be resealed notionally, to form a cone or horn [1].

Nanocones are discovered in 1994 [2]. Nanocones are carbon based structures formed by introducing 60° disinclination defects in two-dimensional graphenes sheets [3] (fig.1).

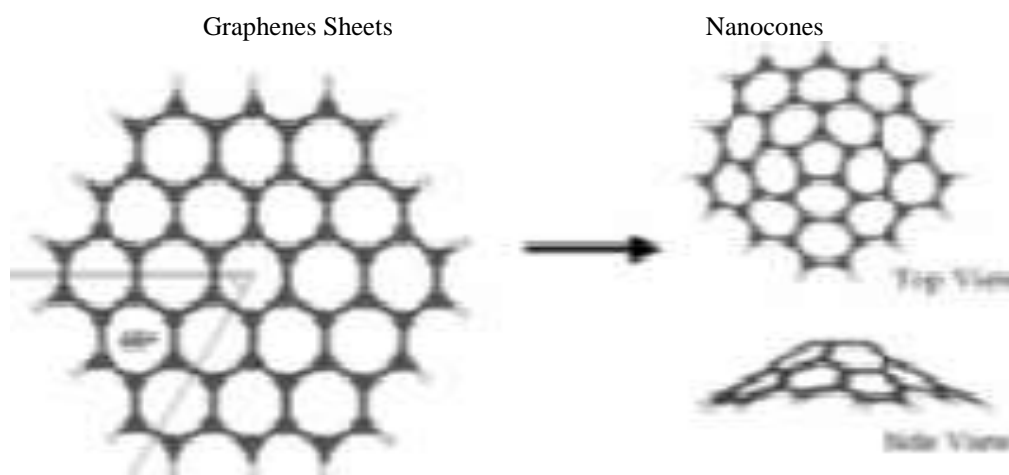


Figure (1): 2-Dimensional sheet of graphene.

Folding graphene sheets have five possible closed distinct CNC structures and the apex angles of a cone can be calculated as

Where θ is the disinclination angle in degrees can be taken as $60^\circ, 120^\circ, 180^\circ, 240^\circ$ and 360° [4]. In carbon nanocones $CNC_k[n]$, the parameter k defines the length of the inner cycle

and n defines the number of layers of the graph. The nanocones $CNC_3[2]$, $CNC_4[2]$, $CNC_5[2]$, consists of triangle, square and pentagon as its core surrounded by two layers of hexagons respectively fig.(2). The 2-dimensional of graphof $CNC_7[1]$ with heptagon as core is also shown in fig.2. The definitions for degree based topological indices are

taken from [5-10]. In this paper our notations is standard and mainly taken from standard books of graph theory [11-16]. In this paper vertex degree-based topological indices: Randic index $R(G)$, Reciprocal Randics index $RR(G)$, general Randic indices (with $\alpha = -3, -2, 2, 3$) where α is adjustable parameter, Zagreb indices (first $M_1(G)$, second $M_2(G)$ and third $M_3(G)$), modified second Zagreb index $M_2^*(G)$, second multiplicative Zagreb index, modified first Zagreb index Π_1^* , reduced second Zagreb index $R(M_2)$, Augmented Zagreb index $ABC(G)$, hyper Zagreb index $HM(G)$, Atom-bond connectivity index $ABC(G)$, Harmonic index $H(G)$, Sum-connectivity index $SCI(G)$, Sum-connectivity indices (with

$\alpha = -3, -2, 2, 3$ where α is adjustable parameter), Inverse indeg index $IN(G)$, Geometric-Arithmetic index $GA(G)$ and some inequalities are studied in $CNC_3[2]$, $CNC_4[2]$, $CNC_5[2]$, $CNC_6[2]$ and $CNC_7[2]$ nanocones.

II. RESULTS AND DISCUSSION

Degree-based Topological indices

In carbon nanocones $CNC_k[n]$, the parameter k defines the length of inner cycle and n defines the number of layers of the graph. The 2-dimensional graphs for $CNC_3[2]$, $CNC_4[2]$ and $CNC_5[2]$ and $CNC_7[1]$ are shown in figure (2).

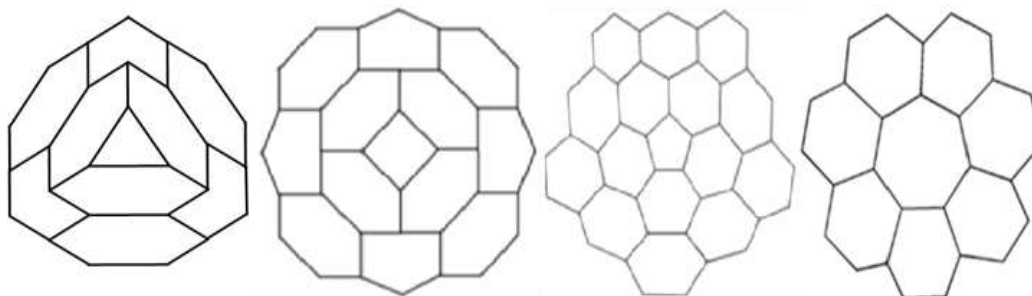


Figure (2): 2-Dimensional graphs of $CNC_3[2]$, $CNC_4[2]$, $CNC_5[2]$ and $CNC_7[1]$ nanocones

From 2-dimensional graphs of These nanocones the edges of degree $(3, 3)$, $(2, 2)$ and $(3, 2)$ are counted and given in table (1).

Table no (1): The number of edges $(d_u = 3, d_v = 3)$, $(d_u = 2, d_v = 2)$ and $(d_u = 3, d_v = 2)$ for nanocones $CNC_k[2]$.

Carbon Nanocones	The number of edges of type $(d_u = 3, d_v = 3, E_1)$	The number of edges of type $(d_u = 2, d_v = 2, E_2)$	The number of edges of type $(d_u = 3, d_v = 2, E_3)$
$CNC_3[2]$	21	3	12
$CNC_4[2]$	29	4	16
$CNC_5[2]$	35	5	20
$CNC_6[2]$	42	6	24
$CNC_7[2]$	49	7	28

The definitions of degree-based topological indices [5 – 10] :Randic index $R(G)$,Reciprocal Randic index $RR(G)$,Randic index (with $h = -3,-2,2,3$) where h is adjustable parameter, Zagreb index first $M_1(G)$, second $M_2(G)$, third $M_3(G)$,modified Zagreb index $M_2^*(G)$,second multiplicative Zagreb index Π_2 ,modified first Zagreb index Π_1^* , reduced second Zagreb index $R(M_2)$, Augmented Zagreb index $ABC(G)$,hyper

The Randic index

$$R(G) = \sum_{u,v \in E(G)} 1/\sqrt{d_u d_v}$$

$$= |E_1|/\sqrt{9} + |E_2|/\sqrt{4} + |E_3|/\sqrt{6}$$

Nanocone $CNC_3[2]$

$$R(G) = \sum_{u,v \in E(G)} 1/\sqrt{d_u d_v} = |E_1|/\sqrt{9} + |E_2|/\sqrt{4} + |E_3|/\sqrt{6} = 21/\sqrt{9} + 3/\sqrt{4} + 12/\sqrt{6}$$

$$= 13.399$$

$CNC_4[2]$

$$R(G) = \sum_{u,v \in E(G)} 1/\sqrt{d_u d_v} = 29/\sqrt{9} + 4/\sqrt{4} + 16/\sqrt{6} = 18.2 \text{ } CNC_5[2]$$

$$R(G) = \sum_{u,v \in E(G)} 1/\sqrt{d_u d_v} = 35/\sqrt{9} + 5/\sqrt{4} + 20/\sqrt{6} = 22.34 \text{ } CNC_6[2]$$

$$R(G) = \sum_{u,v \in E(G)} 1/\sqrt{d_u d_v} = 42/\sqrt{9} + 6/\sqrt{4} + 24/\sqrt{6} = 26.79 \text{ } CNC_7[2]$$

$$R(G) = \sum_{u,v \in E(G)} 1/\sqrt{d_u d_v} = 49/\sqrt{9} + 7/\sqrt{4} + 28/\sqrt{6} = 31.27$$

Using the number of edges of type $(d_u=3,d_v=3),(d_u=2,d_v=2),(d_u=3,d_v=2)$ and the definitions of degree-based indices [5-10],the TIs are computed and tabled in table (2) for $CNC_3[2], CNC_4[2],CNC_5[2], CNC_6[2]$ and $CNC_7[2]$ nanocones.

The general Randic index with $h= -3$ has least values due the factor $\times = -3$ and second multiplicative Zagreb index has highest values as it

Inequalities for topological indices

- 1) $H(G) \leq GA(G)$, is obeyed for nanocones with $k=3,4,5,6,7$ and $n=2$.
- 2) $H(G) \leq X(G)$, is obeyed (where $X(G)$ -Sum-connectivity index) in $CNC_3[2]-CNC_7[2]$.

Zagreb index $HM(G)$,Atom-bond connectivity index $ABC(G)$,Harmonic index $H(G)$,Sum connectivity index $SCI(G)$,Sum-connectivity indices with $h = -3,-2,2,3$ where h is adjustable parameter, Inverse indeg index $IN(G)$, Geometric-Arithmetic index $GA(G)$ are used for computing TIs of $CNC_3[2], CNC_4[2],CNC_5[2], CNC_6[2]$ and $CNC_7[2]$ nanocones.

is defined in multiplication form, among the topological indices studied. By knowing number of degrees $(d_u,d_v):(3,3),(2,2),(3,2)$ the first ,second Zagreb and Harmonic polynomials can be computed for $CNC_3[2], CNC_4[2],CNC_5[2], CNC_6[2]$ and $CNC_7[2]$ nanocones[9,19]. The values of TIs increase from $CNC_3[2]$ to $CNC_7[2]$ nanocones due to increase in number of edges E_1,E_2 and E_3 in $CNC_3[2]-CNC_7[2]$.

- 3) The general inequality for Zagreb first and second indices with n vertices and m edges , $M_1(G)/n \leq M_2(G)/m$ is satisfied in $CNC_4[2]$, as is the case for all graphs.

Table number (2): Topological indices of $CNC_3[2]-CNC_7[2]$ carbon nanocones

Topological indices	$CNC_3[2]$	$CNC_4[2]$	$CNC_5[2]$	$CNC_6[2]$	$CNC_7[2]$
$R(G)$	13.399	18.2	22.34	26.79	31.27
$RR(G)$	98.39	134.18	163.98	196.77	229.57
$R(G)\lambda = -3$	0.13123	0.176349	0.21873	0.2625	0.3063
$R(G)\lambda = -2$	0.78008	1.052532	1.3005	1.5602	1.8202
$R(G)\lambda = 2$	2181	2989	3635	4362	5089
$R(G)\lambda = 3$	18093	24853	30155	36186	42217
M_1	198	250	330	396	462
M_2	273	373	455	546	637
M_3	12	16	20	24	28
M_2^*	5.084	6.889	8.473	10.167	11.862
Π_2	163296	400896	756000	1306368	2074464

Π_1^*	1512	2784	4200	6048	8232
$R(M_2)$	111	152	185	222	259
$AZI(G)$	359.21	490.33	598.69	718.4	834.15
$HM(G)$	1104	1508	1840	2208	2576
$ABC(G)$	24.61	34.18626	41.011	49.2132	57.4154
$H(G)$	13.3	18.07	22.17	26.6	31.34
$SCI(G)$	15.441	24.9975	30.7365	36.8338	43.0311
$SCI(\lambda = -3)$	0.2401	0.324759	0.4002	0.48019	0.5603
$SCI(\lambda = -2)$	1.25067	1.69562	2.0848	2.50176	2.91872
$SCI(\lambda = 2)$	1104	1508	1840	2208	2576
$SCI(\lambda = 3)$	6228	8520	10380	12456	14532
$IN(G)$	48.9	66.7	81.5	85.2	114.1
$GA(G)$	35.7578	48.6769	59.5962	71.5155	76.4347

III. CONCLUSION

The vertex degree-based topological indices are computed on 2-dimensional graphs of $CNC_3[2]$, $CNC_4[2]$, $CNC_5[2]$, $CNC_6[2]$ and $CNC_7[2]$ nanocones. By knowing number of degrees as (3,3), (2,2) and (3,2) the first, second Zagreb polynomial and Harmonic polynomial can be computed for $CNC_3[2]$ - $CNC_7[2]$ carbon nanocones. The values of vertex degree-based topological indices increase with $CNC_3[2]$ - $CNC_7[2]$ carbon nanocones.

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