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`Study of Superimposition of Edges of Spanning Tree in PDN Using PDS of $\delta^2+\delta+1$ nodes

Rakesh Kumar Katare, Neha Singh, Manisha Singh, Charvi katare, Manish Bhardwaj
Department of Computer Science, A.P.S.University, Rewa (M.P.)

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ABSTRACT: We have shown in this paper the traversal of nodes as per the algorithms shown in PDS. The positive sign shows forward traversal and negative sign shows reverse traversal between nodes both "+ "and "–" sign for each node give the same value .We found that the traversal is in the form a subset of nodes which is a spanning tree.

Keywords: PDN, Spanning tree, Superimposition, PDS, Binary Relation.

I. INTRODUCTION:

Perfect Differences Sets were first discussed in 1938 by J. Singer. The formulation was in terms of points and lines in a finite projective plane, therefore it was not at all considered so much important until it was incorporated into perfect difference network [2]. The Perfect difference sets were considered a really good prospect for being developed into a network mainly through the works of Dr. Behrooz Parhami and Dr. Mikhail A. Rakov. In their paper[1] they have discussed low-diameter networks ,beginning with D=2 ,the next best value to that of the complete network and then proceeding to somewhat larger values leading to more economical networks. In their yet another paper[2,3] they have proposed an asymptotically optimal method for connecting a set of nodes into a perfect difference network with diameter 2 so that any node is reachable from any other node in one or two hops. A more exhaustive comparative study of hypercube and perfect difference network was done by katare et al. 2007[5], based on topological properties. Topological properties of perfect difference network compared with the corresponding properties of hypercube by katare et.al.2009 [6].In this scheme sparse linear system was implemented. It was proved that access function or routing function to map data on hypercube contains topological properties .The study of circuits based on the architecture of PDN is further taken forward by katare et.al 2013[8] in their research work on study of link utilization of PDN and Hypercube .they have shown that the circuits formed in PDN are a combination of odd and even length.

2. Study of node relation with others processors in PDN.

Perfect Difference Set: If the set S of $\delta+1$ distinct integers $S_0,S_1,S_2,\ldots,S\delta$ has the property that the $\delta 2+\delta$ differencesS_i-S_j($0\leq i,j\leq\delta, i\neq j$) are distinct modulo$\delta^2+\delta+1$, S is called a perfect difference set mod $\delta^2+\delta+1$.

The existence of perfect difference sets seems intuitively improbable, at any rate for large δ ,but in 1938 J.Singer proved that ,whenever δ is a prime or power of prime ,say $\delta=p^n$,a perfect difference set mod $p^{2n}+p^n+1$ exists .From now we on let δ denote pn and we write that n= $p^{2n}+p^n+1=\delta^2+\delta+1$[10].

In PDN each processor is connected with other processor by relation which is defined below.

$s_i-s_j = 0,1,\ldots,(\delta^2+\delta)\mathrm{mod}(\delta^2+\delta+1)$

We found from the perfect difference set that the mapping of nodes of a PDN is as follows

i) The direct nodes are connected is $i\pm1$, where i is the node representation.

ii) Cordial ring pattern node is represented by $i\pm s_j$ (mod n) for ($2\leq j\leq\delta$) i.e. j lies between 2 & δ .The reserve link is also exits. The network is drawn as undirected graph.

That is if we take $\delta=2$ then the number of nodes in PDN is$\delta^2+\delta+1$ i.e. 7 nodes of PDN is created

0, 1, 2,3,4,5 and 6. If we derive the appropriate relation then they can defined as

Positive node	Negative node / reflected node
For node 0 0-0 = (0) mod 7 0 = 0 mode 7 0 = 0	
For node 1 1-0 = 1 mod 7 1 = 1 mod 7 1=1	For node 6 0-1= -1 mod 7 -1 = -1 mod 7 -1= -1 1=1
For node 2 3-1= 2 mod 7 2 = 2 mod 7 2=2	For node 5 1-3= -2 mod 7 -2 = -2 mod 7 -2= -2 2=2
For node 3 3-0= 3 mod 7 3 = 3 mod 7 3=3	For node 4 0-3= -3 mod 7 -3 = -3 mod 7 -3= -3 3=3

In this way we see that the positive sign shows forward traversal and negative sign shows reverse traversal between nodes both "+ "and "–"sign for each node give the same value.

3. $\alpha\beta -$ Band Matrix representation:

In large number of applications sparse matrices are involved. $\alpha\beta -$ Band Matrix is a sparse matrix whose non-zero entries are confined to a diagonal band comprising the main diagonal and zero or more diagonals on either side. When we obtained adjacency matrices of architecture we got most of them are of $\alpha\beta -$ Band matrices type. So, the indexing formula of $\alpha\beta -$ Band Matrix can be used as routing function for those architectures which has $\alpha\beta -$ Band matrix pattern [8]. Considering the row-major ordering for the memory allocation the indexing formula is explained as below [9].

Case 1: $1 \le i \le \beta$

Address (a_{ij}) = Number of elements in first (i-1)th rows + Number of elements in i^{th} row up to j^{th} columns

$$= \alpha * (i-1) + \frac{(i-1)(i-2)}{2} + j$$

Case 2: $\beta < i \le n - \alpha + 1$

Address (a_{ij}) = Number of first β rows + Number of elements in between $(\beta + 1)$th row and (i-1) th row+ number of elements in ith row.

$$= \alpha\beta + \frac{\beta(\beta-1)}{2} + (\alpha\alpha + \beta\beta - 1i-\beta-1+j-i+\beta$$

Case 3: $n\text{-}\alpha + 1 < i$

Address (a_{ij}) = Number of elements in first $(n - \alpha + 1)$ rows + Number of elements after $(n - \alpha + 1)$th row and up to (i-1) th row+ number of elements in ith row.

$$= \alpha\beta + \frac{\beta(\beta-1)}{2} + (\alpha + \beta - 1)(n - \alpha-\beta+1+\alpha+\beta i-n+\alpha-1 \mathbf{-i-n+\alpha-1+(i-n+\alpha-2)2+1}$$

Here we create $\alpha\beta -$ Band Matrix for edge to edge connectivity of Perfect difference Network. Number of bits in row vector of an edge to edge will show the connectivity with other processor which also the density of connectivity of processor.

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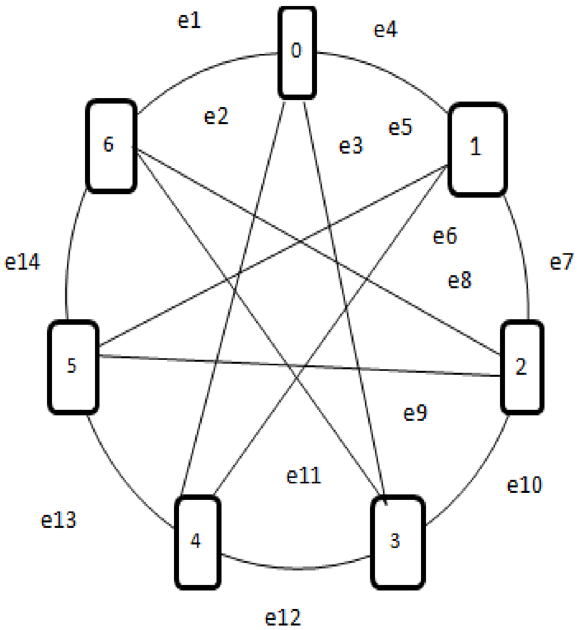


Fig3.1 PDN for $\delta = 2$.

	e1	e2	e3	e4	e5	e6	e7	e8	e9	e10	e11	e12	e13	e14
e1	1	1	1	1	0	0	0	1	0	0	1	0	0	1
e2	1	1	1	1	0	1	0	0	0	0	0	1	1	0
e3	1	1	1	1	0	0	0	0	0	1	1	1	0	0
e4	1	1	1	1	1	1	1	0	0	0	0	0	0	0
e5	0	0	0	1	1	1	1	0	1	0	0	0	1	1
e6	0	0	0	1	1	1	1	0	0	0	0	1	1	1
e7	0	0	0	1	1	1	1	1	1	1	0	0	0	0
e8	1	0	0	0	0	0	1	1	1	1	1	0	0	1
e9	0	0	0	0	1	0	1	1	1	1	0	0	1	1
e10	0	0	1	0	0	0	1	1	1	1	1	1	0	0
e11	1	0	1	0	0	0	0	1	0	1	1	1	0	1
e12	0	1	1	0	0	1	0	0	0	1	1	1	1	0
e13	0	1	0	0	1	1	0	0	0	1	0	0	1	1
e14	1	0	0	0	1	0	0	1	1	0	1	0	1	1

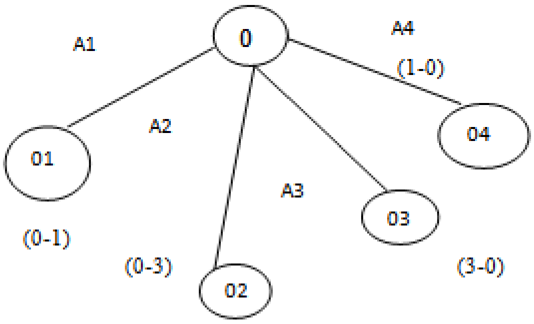
Table 3.1 $\alpha\beta$ – Band matrix for edge to edge connectivity of PDN

4. Superimposition of edges in PDN using Spanning Tree:

There are some common edges for spanning tree of each node which we will say as superimposition of edges. It is the placement of one thing over another typically both layers are still evident or both seen at once. In terms of perfect difference network when we use the spanning tree an image is layers with another image such that existence of both image retain.

Suppose if we built a spanning tree for each node is a mirror image & this value make a combination

from PDS. For example if we built PDN $\delta = 2$ then number of nodes $\delta^2+\delta+1$ i.e. 7 nodes which we built 7 spanning tree.
For example spanning tree for node 0, each node is connected to four nodes in PDN & values of each node is represented by combination of PDS value. Such as node 0 has from edges A1, A2, A3, and A4 according to PDS value of nodes
01=(0-1) , 04=(1-0)
02=(0-3), 03=(3-0) ,



Figre4.1 Spanning Tree for Node 0as per PDS

Similarly we can create spanning trees for all other nodes of PDN according to PDS value because of superimpose of edges in PDN using spanning these are the values of nodes.

0-0	0-0
1-0	0-1
3-1	1-3
3-0	0-3

Lemma 1: Spanning tree of PDN satisfy the properties of binary relation in the mathematical property of PDN.
Proof: Binary Relation- A binary relation R between pairs (A_i, A_j) may exist, in which case we write
A_i R A_j
And say that A_i has relation R to A_j.[4]
Reflexive –For some relation R it may happen that every element is in relation R to it self, such a relation R on set A that satisfies
A_i R A_i
, for every $A_i \in A$ is called a reflexive relation.[4]
Transitive- A relation R is said to be transitive if for any three elements A_i,A_j and A_k in the set
A_i R A_j and A_j R A_k
, the binary relation is transitive relation [4].
Symmetric- for some relation R it may happen that for all Ai and A_j if
A_i R A_j holds, and then
A_j R A_i also holds.
Such a relation is called a symmetric relation.[4]
We can create edge to edge, Vertex to vertex and Edge to vertex relation matrix of spanning (Fig2.1) tree in PDN.

Vertex to Vertex-

	00	01	02	03	04
00	1	1	1	1	1
01	1	1	0	0	0
02	1	0	1	0	0
03	1	0	0	1	0
04	1	0	0	0	1

Edge to Vertex-

	00	01	02	03	04
A1	1	1	0	0	0
A2	1	0	1	0	0
A3	1	0	0	1	0
A4	1	0	0	0	1

Edge to Edge-

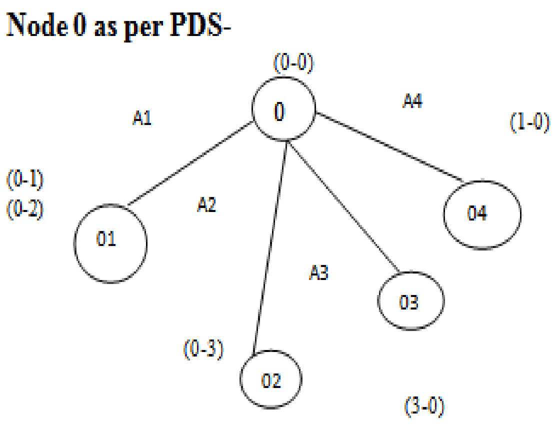
	A1	A2	A3	A4
A1	1	1	1	1
A2	1	1	1	1
A3	1	1	1	1
A4	1	1	1	1

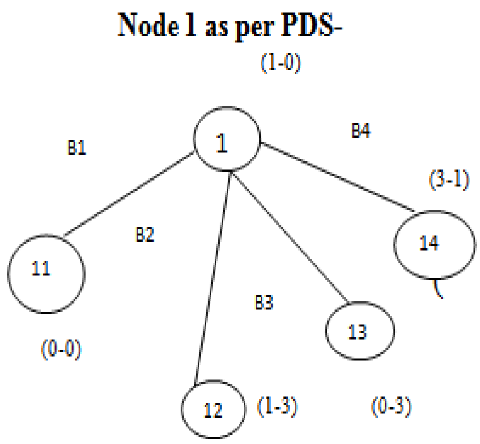
Table 4.1 Connectivity Matrix

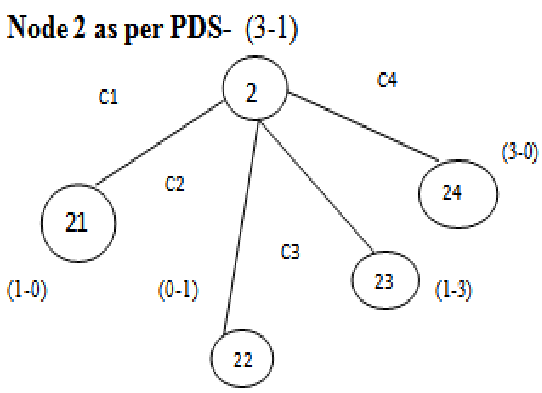
For node 0
Set of edges E = (A1, A2, A3, A4)
Set of Vertex V = (00, 01, 02, 03, 04)
V*V ={ <00,00>,<00,01>,<00,02>,<00,03>,<00,04>, <01,00> , <01,01> , <02,00> ,<02,02>, <03,00> , <03,03> ,<04,00> ,<04, 04>}
E*V = { <A1,00>,<A1,01> , <A2,00> , <A2 ,02> ,<A3,00>,<A3,A3>,<A4,00> ,<A4,04>}
E*E = {< A1, A1>, < A1, A2>,< A1,A3>,< A1,A4>,< A2,A1>,< A2,A2>,< A2,A3>, < A2,A4>, < A3,A1>, < A3,A2>, < A3,A3>, < A3,A4> , < A4,A1> , < A4,A2> ,< A4,A3> , < A4,A4>}
If we can apply some binary properties in above sets we have found that V*V is reflexive and symmetric, E*V is reflexive and E*E is reflexive, symmetric and transitive i.e. edge to edge relation should be equivalence relation. Now we have show that spanning tree of PDN satisfies the properties of binary relation in the mathematical property of PDN.

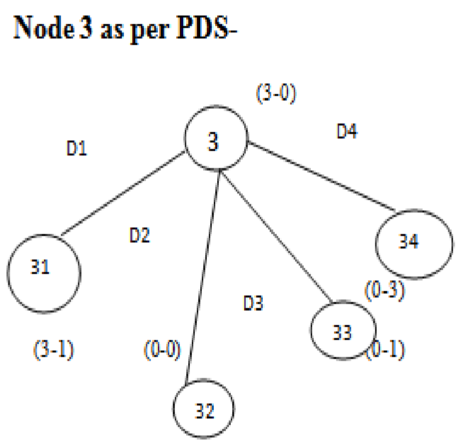
Lemma2: The Union of spanning tree of PDS is the PDN.

Proof: if we take $\delta = 2$ then the set of nodes = {0, 1, 2, 3, 4, 5, 6} as per the value apply PDS of $(0,\pm1,\pm3)$ .PDS give us the algorithm for traverse between nodes. If we combine the all the spanning tree based on PDS value then we get complete architecture of PDN i.e. if we have spanning tree of nodes 0, 1, 2,3,4,5 and 6 then
PDN= $\sum(\mathbf{0} \cup \mathbf{1} \cup \mathbf{2} \cup \mathbf{3} \cup \mathbf{4} \cup \mathbf{5} \cup \mathbf{6})$









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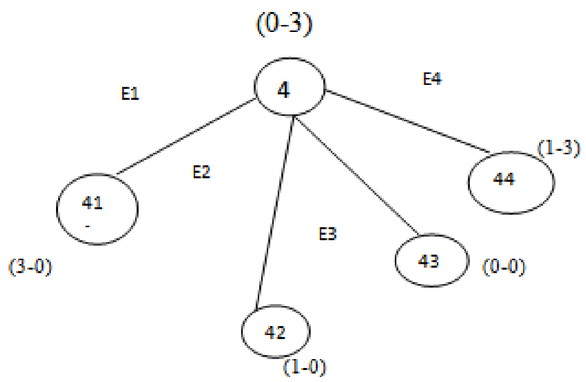
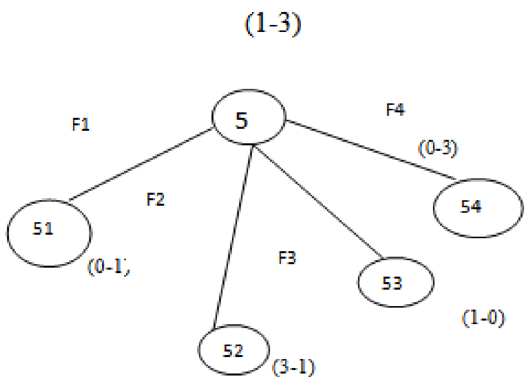
Node 4 as per PDS-



Node 5 as per PDS-



Node 6 as per PDS-

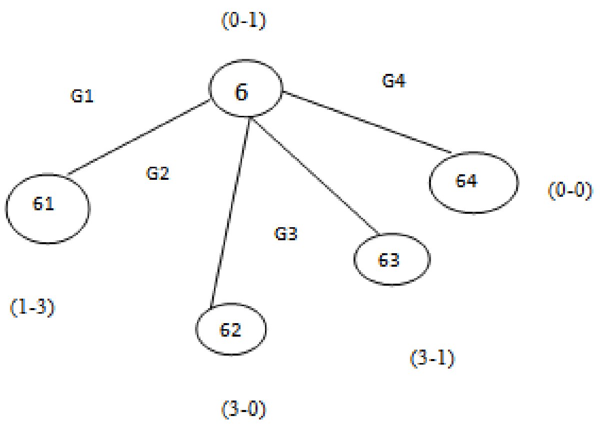


Fig 4.2 spanning trees as per PDS $(0, \pm 1, \pm 3)$

Lemma3- The connectivity matrix of spanning tress of PDS shows all possible connection between nodes of PDN.

Proof-The PDN is converted into spanning tree node by node .Here we are taken 7 node PND i.e. create 7 spanning tree .Spanning tree is converted into its equivalent edge to edge connectivity matrix. This relation matrix or connectivity matrix shows the all possible patterns of connectivity between nodes & edges. The bits of each vector show the connectivity of each processors/edges with other processors/edges.

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Node 0-	A1	A2	A3	A4
A1	1	1	1	1
No A2	1	1	1	1
	C1	C2	C3	C4
A3	–	–	–	–
C1	1	1	1	1
No				
C2	E1	E2	E3	E4
E1	1	1	1	1
N E2	1	1	1	1
E3	G1	G2	G3	G4
G1	1	1	1	1
G2	1	1	1	1
G3	1	1	1	1
G4	1	1	1	1

	B1	B2	B3	B4
B1	1	1	1	1
B2	1	1	1	1
	D1	D2	D3	D4
B3	1	1	1	1
D1	1	1	1	1
D2	F1	F2	F3	F4
F1	1	1	1	1
F2	1	1	1	1
F3	1	1	1	1
F4	1	1	1	1

Table 4.2 connectivity matrix of spanning tree

II. CONCLUSION:

We calculate the common edge for each node of all spanning tree. Now comparing the edge to edge relation matrix of node in a spanning tree with $\alpha - \beta$ band matrix of PDN are get all the common edges of given PDN by using spanning tree as per PDS$(0,\pm1,\pm3)$.

Spanning tree of Node 0	Spanning tree of Node 1	Spanning tree of Node2	Spanning tree of Node3	Spanning tree of Node 4	Spanning tree of Node 5	Spanning tree of Node 6	PDN(COMMON TO ALL edges)
A1A2A3A4	B1B2B3B4	C1C2C3C4	D1D2D3D4	E1E2E3E4	F1F2F3F4	G1G2G3G4	e1- e1e2e3e4 e2- e1e2e3e4
A1(1111)	B1(1111)	C1(1111)	D1(1111)	E1(1111)	F1(1111)	G1(1111)	e3-e1e2e3e4 e4-e1e2e3e4
A2(1111)	B2(1111)	C2(1111)	D2(1111)	E2(1111)	F2(1111)	G2(1111)	-e2e3e4e5 -e3e4e5e6
A3(1111)	B3(1111)	C3(1111)	D3(1111)	E3(1111)	F3(1111)	G3(1111)	-e4e5e6e7 e5-e4e5e6e7
A4(1111)	B4(1111)	C4(1111)	D4(1111)	E4(1111)	F4(1111)	G4(1111)	e6 –e4e5e6e7 e7-e4e5e6e7 -e5e6e7e8 -e6e7e8e9 -e7e8e9e10 e8-e7e8e9e10 -e8e9e10e11

							e9-e7e8e9e10 e10-e7e8e9e10 -e8e9e10e11 -e9e10e11e12 e12- e10e11e12e13

We have found that there are some common edges for spanning tree of each node which we will say as superimposition of edges and to increase the efficiency of Perfect Difference Network we can choose any one of the common edge to get optimal solution and

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