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The New Exponential-Gamma Distribution with Application to Rainfall Data

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ABSTRACT: In this study, we aim to examine the performance of the new Exponential-Gamma distribution with the traditionally existing probability distribution that will perform better in fitting the data on rainfall in Ondo State from 2005-2019. The method of maximum likelihood was used to estimate the parameters of the distributions. We compared the distributions using the criteria for selecting the best model fit such as log-likelihood function, Akaike information criterion (AIC), and Bayesian information criterion (BIC). From the results obtained, it was observed that the Exponential-Gamma distribution performed better than the traditionally exiting distribution Exponential and Gamma respectively in terms of model fit; this indicated that the new Exponential distribution is more tractable and flexible in modeling and analyzing the rainfall data.

KEYWORDS:AIC, BIC, maximum likelihood estimate, Survival function, Exponential-Gamma distribution.

I. INTRODUCTION

Rainfall is a significant part of the hydrological cycle and changes in its pattern. Rain is a major factor in the water cycle and is responsible for depositing most of the fresh water on the Earth. Rainfall is considered the main source of domestic water for living as well as for agriculture. The analysis of the rainfall series of the State would improve the running of water resources as well as optimize its uses. One of the principal difficult tasks with rainfall data is to deal with inferring records of rainfall events in terms of future probabilities of existence. Hence, knowing the rainfall distribution that causes floods might play a crucial role in the ecological development and preservation of the natural resources of the state. Assessing a statistical distribution that provides a better fit for yearly rainfall has long been a concerning issue for hydrologists, meteorologists, and other water resource personnel. Knowing the rainfall distribution is essential for stochastic modeling, rainfall frequency analysis, and rainfall trend analysis. Rainfall is deliberated as one of the key natural resources which play a fundamental role in sustaining human life be it in agriculture, industry, or domestic, thus, scarcity or extreme rainfall might be very detrimental as there will be a food shortage, water pollution, flooding and telecommunication difficulties, etc. All of this can lead to economical beating in a region. Studies in many areas of life have made use of probability distributions in modeling rainfall data while many studies and research are continuously in progress and efforts are being made to model and predict future drifts of rainfall to improve planning such as [1],[2],[3],[4],[5],[6],[7],[8],[9],[10],[11],and[12]. Therefore, in this study, we aimed to model the new Exponential-Gamma distribution on rainfall data in Ondo State from 2005- 2019 using the model selection criteria like the Akaike information criterion (AIC), Bayesian information criterion (BIC), and the log-likelihood function (l)

II. METHODOLOGY

Several studies have emphasized how to study rainfall patterns throughout the universe and one of the major concerns in the analysis of rainfall data is with making a valid inference about the past data in terms of future probabilities of occurrences. A good number of probability distribution methods have been used in estimating rainfall data and they have been proven to be useful for rainfall studies and surveys.

The analysis of most rainfall data largely depends on its distribution pattern. It has now long been an emphasis of interest in the field of hydrology in establishing a probability distribution that provides a good fit to most rainfall data. In this study, we, therefore, aim to fit the new Exponential-Gamma distribution to rainfall data in Ondo State from 2005- 2019. The method of maximum likelihood will be used to estimate their parameters, while the log-likelihood, Bayesian information criterion (BIC) and Akaike information criterion (AIC) goodness of fit test will be employed to determine their goodness of fit and R-software would be used for data analysis.

The new Exponential-Gamma distribution was developed by Ogunwaleet al (2019) and its pdf is defined as

$$f(x;\alpha,\lambda)=\frac{\lambda^{\alpha+1}x^{\alpha-1}e^{-2\lambda x}}{\Gamma(\alpha)},x,\lambda,\alpha>0 \quad (1)$$

They defined its mean and variance as;

$$\mu=\frac{\alpha}{2^{\alpha+1}} \quad (2)$$

and
$$V(x)=\frac{\alpha\left(\alpha 2^{\alpha}-\lambda\alpha+2^{\alpha}\right)}{\lambda\left(2^{2(\alpha+1)}\right)} \quad (3)$$

The cumulative distribution function is defined as

$$F(x)=\frac{\lambda\gamma(\alpha,x)}{2^{\alpha}\Gamma(\alpha)} \quad (4)$$

The survival function for the distribution defined by $S(x)=1-F(x)$ was obtained as;

$$S(x)=1-\frac{\lambda\gamma(\alpha,x)}{2^{\alpha}\Gamma(\alpha)} \quad (5)$$

While the corresponding hazard function defined by $h(x)=\frac{f(x)}{S(x)}$ was obtained as;

$$h(x)=\frac{\lambda^{\alpha+1}x^{\alpha-1}e^{-2\lambda x}2^{\alpha}}{2^{\alpha}\Gamma(\alpha)-\lambda\gamma(\alpha,x)} \quad (6)$$

The cumulative hazard function for distribution is defined by

$$H(x)=W(F(x))=-\log(1-F(x))\equiv\int_0^x h(x)dx$$

and was obtained as;

$$H(x)=\frac{\lambda\gamma(\alpha,x)}{2^{\alpha}\Gamma(\alpha)-\lambda\gamma(\alpha,x)} \quad (7)$$

Maximum Likelihood Estimator (MLE)

Let $X_1, X_2 \dots X_n$ be a random sample of size n drawn from a probability density function $f(x;\theta)$, the likelihood function is defined as;

$$f(x_1,x_2...x_n;\theta)=\prod_{i=1}^{n} f(x_i;\theta) \quad (8)$$

Now, we estimate the parameters of both the exponential-gamma distributions using maximum likelihood estimation as follows,

If $X_1, X_2, \dots, X_n$ be a random sample of size n from Exponential-Gamma distribution. Then the likelihood function is given by;

$$L(\alpha,\lambda;x)=\left(\frac{\lambda^{\alpha+1}}{\Gamma(\alpha)}\right)^n\prod_{i=1}^{n}x_i^{\alpha-1}\exp\left(-2\lambda\sum_{i=1}^{n}x_i\right) \quad (9)$$

by taking the logarithm of (9), we find the log-likelihood function as;

$$\log(L)=\alpha n\log\lambda+n\log\lambda-n\log\Gamma(\alpha)+(\alpha-1)\sum_{i=1}^{n}\log x_i-2\lambda\sum_{i=1}^{n}x_i \quad (10)$$

Therefore, the MLE which maximizes (10) must satisfy the following normal equations;

$$\frac{\partial\log L}{\partial\alpha}=n\log\lambda-\frac{n\Gamma'(\alpha)}{\Gamma(\alpha)}+\sum_{i=1}^{n}\log x_i \quad (11)$$

$$\frac{\partial\log L}{\partial\lambda}=\frac{\alpha n}{\lambda}+\frac{n}{\lambda}-2\sum_{i=1}^{n}x_i \quad (12)$$

The solution of the non-linear system of equations is obtained by differentiating (10) with respect to (α,λ) gives the maximum likelihood estimates of the model parameters. The estimates of the parameters can be obtained by solving (11) and (12) numerically as it cannot be done analytically. The numerical solution can also be obtained directly by using python software using the data sets.

III. RESULTS AND DISCUSSION

This work used secondary data collected based on the monthly amount of rainfall data in Ondo State from 2005- 2019. The Exponential-

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Gamma probability distribution was applied to the data to determine the best fit distribution for the data. The data were analyzed using the Python software package. The measures of goodness were determined by using selection criteria such as the Akaike information criterion (AIC), Bayesian information criterion (BIC), and the log-likelihood function (l). The model with the least Akaike information criterion (AIC), Bayesian information criterion (BIC), or the highest log-likelihood function (l) value is considered the best model, and the results are obtained as follows;

TABLE 1: SUMMARY OF THE DATA

Statistic	Value
n	168
Mean	184.29
Var.	30837.07
SD	175.60
Skewness	1.42
Kurtosis	2.31

Interpretation

The results from table 1 above indicated that the distribution of the data is skewed to the right with skewness of 1.42. Also, it was observed that the kurtosis is 2.31 which is lesser than 3. This implies that the distribution of the data has a shorter and lighter tail with a light peakedness when compared to that of the Normal distribution.

TABLE 2: THE MAXIMUM LIKELIHOOD ESTIMATES FOR THE DISTRIBUTIONS

Distribution	Exponential-Gamma		Exponential	Gamma	
Parameters	**Shape(α)**	**Scale(λ)**	**rate(λ)**	**Shape (α)**	**Scale(λ)**
Values	1.87849	1.9748	2.7418	2.1450	118.8754
Standard Error	0.0879	0.0985	0.2587	0.2687	90.6587

TABLE 3: THE LOG-LIKELIHOOD VALUE FOR THE DISTRIBUTIONS

Goodness of Fit	Exponential-Gamma	Exponential	Gamma
Log-Likelihood	**-12.0597**	-28.0897	-23.8974

Interpretation

The log-likelihood value for the data is presented in Table 3. It was observed that the new Exponential-Gamma distribution provides a better fit as compared to Exponential and Gamma distributions respectively since it has the highest value of log-likelihood (l). Hence, the Exponential distribution performed better than the Exponential and Gamma distributions.

TABLE 4: THE AIC AND BICVALUE FOR THE DISTRIBUTIONS

Goodness of Fit	Exponential-Gamma	Exponential	Gamma
AIC	**52.3014**	172.0681	151.7893
BIC	**58.25578**	178.70716	153.067

Interpretation

The AIC value for the data is presented in Table 4. It was observed that the Exponential-Gamma distribution provides a better fit as compared to the existing Exponential and Gamma distributions since it has a smaller value of AIC while the BIC value for the data also presented in Table 4 showed that the Exponential-Gamma distribution provides a better fit as compared to the existing Exponential and Gamma distributions since it has a smaller value of BIC. Hence, the Exponential-Gamma distribution performed better than the exiting Exponential and Gamma distributions.

IV. CONCLUSION

The probability distribution in which the AIC and BIC value is the lowest and the log-likelihood is the highest respectively was selected; it was discovered that the Rayleigh distribution is noticeably low and high values concerning the AIC, BIC, and log-likelihood values respectively. Therefore, the superiority of fit of the Exponential-Gamma distributions was established without distinguishing superiority. Equally, the suitability of the Exponential-Gamma distribution for life data analysis has been shown in advanced studies. Therefore, for higher precision in rainfall data in Ondo State, the use of Exponential-Gamma distribution is highly recommended and also purposefully useful in various fields where analysis of such data is crucial.

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