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The New Integral Transform "Soham Transform" For System of Differntial Equations

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ABSTRACT
In this paper we use Soham transform to solve the system of ordinary differential equations of first order and first degree keywords ; system of differential equations , Integral transforms , Soham transform .

I. INTRODUCTION

Recently , Integral transforms are one of the mostly used simple mathematical technique to obtain the solutions of advance problems of space , science , technology , engineering , commerce and economics . The important feature of these integral transform is to provide exact solution of of problem without lengthy calculations.

Due to this important feature of the integral transforms many researchers are attracted to this field and are engaged in introducing various integral transforms. Recently , in September 2021 , Kushare and Patil [1] introduced Kushare transform for facilitating the process of solving differential equations in time domain . Further in October 2021 Khakale and Patil [2] introduced Soham transform . As researchers are introducing the new integral transforms at the same time they are also interested in applying the transforms to various fields , various equations in different domain. In January 2022 , Sanap and Patil [3] used Kushare transform to solve the problems on Newton's law of Cooloing . In April 2022 D.P. Patil , etal [4] used Kushare transform for solving the problems on growth and decay . In October 2021 , D .P. Patil [5] used Sawi transform in Bessel functions . Further , Patil [6] used Sawi transform of error functions to evaluate improper integrals. Laplace transform and Shehu transforms are used to Patil [7] in chemical sciences . Patil [8] solved wave equation by using Sawi transform and its convolution theorem using Mahgoub transform , parabolic boundary value problems are solved by D .P. Patil [9]. Solution of wave equation is obtained by using double Laplace and double Sumudu transforms by D .P. Patil [10]. Dr. Patil [11] also obtained dualities between double integral transforms . Laplace , Elzaki and Mahgoub transforms are compared and used for solving system of first order and first degree by Kushare and Patil [12] . D.P.Patil [13] used Aboodh and Mahgoub transform for solving boundary value problems of the system of ordinary differential equations . Double Mahgoub transformed is used by Patil [14] to solve parabolic boundary value problems .

In 2018 , D .P. Patil [15] study comparatively Laplace , Sumudu , Aboodh , Elzaki and Mahagoub transform and used it for solving boundary value problems. Recently in March 2022 Dipali Kaklij [16] introduced double new general integra transform . Patil etal [17] used soham transform for solving Volterra Integral equations of first kind . Futher Patil with Tile and Shinde [18] used transform for solving volterra integral equations for first kind .

In this paper, we use soham transform to obtain the solution of the system of first order and first degree differential equations.

II. PRELIMINARIES:

In this section we state some basic requirements. Now we state some required definitions.

2.1 Soham transform: Soham Transform denoted by the operator S(.) is defined by the integral equation

$$S[f(t)] = P(v) = \ = \frac{1}{v}\int_0^\infty f(t)\, e^{-v^\alpha t}\, dt,$$

α is non zero real numbers $t \geq 0$, $k_1 \leq v \leq k_2$

2.2 Inverse Soham Transform: Inverse Soham transform is denoted as follows:
If Soham transform of f(t) is P(v) then inverse Soham transform is defined as

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$$S^{-1}[P(v)] = f(t)$$

2.3 SOHAM TRANSFORM OF THE ELEMENTARY FUNCTIONS:

For any function f(t), we assume that the integral equation exist. The sufficient conditions for the existence of Soham transform are that for t≥0 the function f(t) be piecewise continuous and of exponential order, otherwise Soham transform may or may not exist,

In this section we state Soham transform of elementary functions.

Sr.No.	f(t)	S[f(t)]= P(v)
1.	1	$\frac{1}{v^{\alpha+1}}$
2.	t	$\frac{1}{v^{2\alpha+1}}$
3.	t^n	$\frac{\Gamma(n+1)}{v^{\alpha n+\alpha+1}}$
4.	e^{at}	$\frac{1}{v(v^{\alpha}-a)}$
5.	e^{-at}	$\frac{1}{v(v^{\sigma}+a)}$
6.	sinat	$\frac{a}{v(v^{2\alpha}+a^2)}$
7.	cosat	$\frac{v^{\alpha}}{v(v^{2\alpha}+a^2)}$
8.	sinhat	$\frac{a}{v(v^{2\alpha}-a^2)}$
9.	coshat	$\frac{v^{\alpha}}{v(v^{2\alpha}-a^2)}$

2.4 Properties Of Soham Transform :

In this section we state some properties of soham transform.

[1] If $f_1(t)$ and $f_2(t)$ be two functions of t and c_1 and c_2 be any two constants then

$S\{c_1 f_1(t) + c_2 f_2(t)\} = c_1 S\{f_1(t)\} + c_2 S\{f_2(t)\}$

[2]Let P(v) Soham transform of [S[f(t)] = P(v)] then :

(i) $S[f'(t)] = v^{\alpha}P(v) - \frac{1}{v}f(o)$

3. Application of Soham Transform in system of differential equations :

Example: (1) Consider the system of equations

$\frac{dx}{dt} - 2y = \cos 2t$ ……………. (1)

$\frac{dy}{dt} + 2x = \sin 2t$ ……………. (2)

with initial conditionsx(0) = 1 and y(0) = 0

Solution:

Applying the Soham transform to both sides of equations (1) & (2)

$$S\left(\frac{dx}{dt}\right) - 2S(y) = S(\cos 2t)$$

$$S\left(\frac{dy}{dt}\right) + 2S(x) = S\,(\sin 2t)$$

Let $S[x(t)] = P_1(v)$ and $S[y(t)] = P_2(v)$

$v^{\alpha}P_1(v) - \frac{1}{v}x(o) - 2P_2(v) = \frac{v^{\alpha}}{v(v^{2\alpha}+4)}$

$v^{\alpha}P_1(v) - 2P_2(v) = \frac{v^{\alpha}}{v(v^{2\alpha}+4)} + \frac{1}{v}$ ……….. (3)

Also from (2)

$v^{\alpha}P_2(v) - \frac{1}{v}y(o) + 2P_1(v) = \frac{2}{v(v^{2\alpha}+4)}$

$v^{\alpha}P_2(v) + 2P_1(v) = \frac{2}{v(v^{2\alpha}+4)}$ ………… (4)

Multiplying equation (3) by v^{α}& (4) by 2 and adding the obtained equations, we get,

$(v^{2\alpha}+4)\,P_1(v) = \frac{v^{2\alpha}}{v(v^{2\alpha}+4)} + \frac{v^{\alpha}}{v} + \frac{4}{v(v^{2\alpha}+4)}$

$(v^{2\alpha}+4)\,P_1(v) = \frac{v^{2\alpha}+4}{v(v^{2\alpha}+4)} + \frac{v^{\alpha}}{v}$

$(v^{2\alpha}+4)\,P_1(v) = \frac{1}{v} + \frac{v^{\alpha}}{v}$

$P_1(v) = \frac{1}{v(v^{2\alpha}+4)} + \frac{v^{\alpha}}{v(v^{2\alpha}+4)}$

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$P_1(\mathrm{v}) = \frac{1+v^{\alpha}}{v(v^{2\alpha}+4)}$ ………… (5)

Substituting value of $P_1(\mathrm{v})$ in equation (4), we get

$v^{\alpha} P_2(\mathrm{v}) + 2\left[\frac{1+v^{\alpha}}{v(v^{2\alpha}+4)}\right] = \frac{2}{v(v^{2\alpha}+4)}$

$v^{\alpha} P_2(\mathrm{v}) = \frac{2}{v(v^{2\alpha}+4)} - 2\left[\frac{1+v^{\alpha}}{v(v^{2\alpha}+4)}\right]$

$= -\frac{2v^{\alpha}}{v(v^{2\alpha}+4)}$

$P_2(v) = -\frac{2}{v(v^{2\alpha}+4)}$ ………… (6)

Applying inverse Soham transform to equations (5) & (6)

$S^{-1} P_1(\mathrm{v}) = S^{-1}\left[\frac{1+v^{\alpha}}{v(v^{2\alpha}+4)}\right]$

$= S^{-1}\left\{\left[\frac{1}{v(v^{2\alpha}+4)}\right] + \left[\frac{v^{\alpha}}{v(v^{2\alpha}+4)}\right]\right\}$

$= S^{-1}\left[\frac{1}{v(v^{2\alpha}+4)}\right] + S^{-1}\left[\frac{v^{\alpha}}{v(v^{2\alpha}+4)}\right]$

$\mathrm{x(t)} = \frac{1}{2}\sin 2t + \cos 2t$

and

$S^{-1} P_2(v) = S^{-1}\left[-\frac{2}{v(v^{2\alpha}+4)}\right]$

$\mathrm{y(t)} = -\sin 2t$

Thus, the required solution of given system of differential equations is $\mathrm{x(t)} = \frac{1}{2}\sin 2t + \cos 2t$ & $y(t) = -\sin 2t$

Example : (2) Consider the system of equations

$\frac{dx}{dt} + y = sint$ ………………. (7)

$\frac{dy}{dt} + x = cost$……………. (8)

with initial conditions x(0)= 0 and y(0)= 2

Solution :

Applying the Soham transform of both sides of equations (7) & (8)

$$S\left(\frac{dx}{dt}\right) + S(y) = S(sint)$$

$$S\left(\frac{dy}{dt}\right) + S(x) = S(cost)$$

$S[x(t)] = P_1(\mathrm{v})$ and $S[y(t)] = P_2(v)$

$v^{\alpha} P_1(\mathrm{v}) - \frac{1}{v}\mathrm{x(o)} + P_2(\mathrm{v}) = \frac{1}{v(v^{2\alpha}+1)}$

$v^{\alpha} P_1(\mathrm{v}) + P_2(\mathrm{v}) = \frac{1}{v(v^{2\alpha}+1)}$ ………….. (9)

Also from (8)

$v^{\alpha} P_2(\mathrm{v}) - \frac{1}{v}\mathrm{y(o)} + P_1(\mathrm{v}) = \frac{v^{\alpha}}{v(v^{2\alpha}+1)}$

$v^{\alpha} P_2(\mathrm{v}) + P_1(\mathrm{v}) = \frac{v^{\alpha}}{v(v^{2\alpha}+1)} + \frac{2}{v}$ …………….. (10)

Multiplying equation (9) by v^{α} & subtracting equation (10) from the obtained equations, we get,

$(v^{2\alpha} - 1)P_1(v) = \frac{1}{v(v^{2\alpha}+1)} - \left[\frac{v^{\alpha}}{v(v^{2\alpha}+1)} + \frac{2}{v}\right]$

$(v^{2\alpha} - 1)P_1(v) = -\frac{2}{v}$

$P_1(v) = -\frac{2}{v(v^{2\alpha}-1)}$ …………..... (11)

Substituting value of $P_1(v)$ in equation (9), we get,

$v^{\alpha}\left[-\frac{2}{v(v^{2\alpha}-1)}\right] + P_2(v) = \frac{1}{v(v^{2\alpha}+1)}$

$P_2(v) = \frac{1}{v(v^{2\alpha}+1)} + \left[\frac{2v^{\alpha}}{v(v^{2\alpha}-1)}\right]$ ………….. (12)

Applying inverse Soham transform to equations (11) & (12)

$S^{-1} P_1(v) = S^{-1}\left[-\frac{2}{v(v^{2\alpha}-1)}\right]$

$\mathrm{x(t)} = -2\sinh t$

and

$S^{-1} P_2(v) = S^{-1}\left\{\frac{1}{v(v^{2\alpha}+1)} + \left[\frac{2v^{\alpha}}{v(v^{2\alpha}-1)}\right]\right\}$

$= S^{-1}\left[\frac{1}{v(v^{2\alpha}+1)}\right] + S^{-1}\left[\frac{2v^{\alpha}}{v(v^{2\alpha}-1)}\right]$

$\mathrm{y(t)} = \sin t + 2\cosh t$

Thus, the required solution of given system of differential equationsis $\mathrm{x(t)} = -2\sinh t$ & $\mathrm{y(t)} = \sin t + 2\cosh t$

III. CONCLUSION:

We have applied Soham transform for obtaining the solution of the system of first order and first degree differential equations.

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