

# The explanation of basic concepts of electric circuit theory based on electromagnetic field theory

Le Thi Huyen Linh

<sup>1</sup>Thai Nguyen University of Technology, Thai Nguyen, Viet Nam  
Corresponding Author: Le Thi Huyen Linh

Submitted: 01-12-2021

Revised: 11-12-2021

Accepted: 14-12-2021

**ABSTRACT:** The electromagnetic field is one of the essential fields because of its formation, influence, existence, and impact on objects. This paper aims to explain the Circuit Model and Field Model, Electromagnetic Field and Substance interaction system, the relationship of Electric field and Magnetic field, based on the Theoretical Element of Electromagnetic Field to explain the basic concepts of Circuit Theory by mathematical modelling that are usually only described in words. Maxwell's Electromagnetic Field Theory and Classical Physics are fundamental to analyze and explain these concepts by Continuity of circuits and Potential property of voltage. Kirchhoff's 1st law is only a special case of Maxwell's equation 1, when ignoring probe and displacement currents, the conduction current is the current in the branches of the circuit and has continuity. Kirchhoff's law 2 is just a special case of Maxwell's equation 2, the potential depends on the selection of the landmark, the voltage does not depend on the selection of the landmark but only on the potential position of the 2 points to find the voltage. Based on this paper, the author provides to the students of Electrical Engineering as well as teachers and readers a clarification of the typical peculiarities of the Model of Electrical Circuits - Model of Electromagnetic Field.

**KEYWORDS:** Theory of Electromagnetic field, Theory of Circuit, Continuity of circuits, Potential property of voltage, Maxwell's equation, Kirchhoff's law

## I. INTRODUCTION

In order to control an object - an exact object, the first thing we need to do is assess which objective factors are affected or affected by other thing. To perceive standard objects and reflect correctly the properties of things, we usually refer

to establish the mathematical models for those entities. According to the characteristics of mathematics and physics, in practice we can temporarily divide the model of classes of physical phenomena into two types: Circuit Model and Field Model [1 - 4].

In reality, most electrical, electronic and telecommunication equipment can be described in terms of circuit model. Even devices of other disciplines such as heat transfer, sound transmission, etc., can also be described by circuit models and the same equations of Circuits - time-varying parameters:  $x(t)$ ,  $y(t)$ ,  $z(t)$ . Therefore, circuit theory has practical, popular and is the fundamental theories for all branches of electricity as well as many other disciplines. To investigate electrical equipment, we need to find out the laws of electromagnetic phenomena and processes occurring in the device; therefore, we are going to determine state parameters and behavior parameters (characteristics), as well as describe the rules of these processes by equations showing the relationship of those parameters [1], [5 - 8].

However, to analyze the electrical equipment, in many cases we have to use field theory. That is when it is necessary to consider the spatial distribution of the processes that affect the electromagnetic field on electrical equipment such as considering the distribution of state parameters: electric field strength vector, magnetic induction vector, electromagnetic wave distribution in space, or in waveguides. It is critical that we investigate in the spatial domain (in the coordinate system) and if these processes vary with time, the dependence on time must also be considered. We already know that physical quantities are determined purely by numbers such as: mass  $m$ , temperature  $T$ , length  $l$ , area  $S$ , resistance  $R$ , charge  $q$ ... or a mathematical function describe the change of some quantity in a

given domain such as  $\Phi$  - called scalar field; Other quantities such as velocity, force, and current density are described both in terms of value, and how the direction varies with position in spatial and time as a vector function - we call it the Vector Field [ 1 – 4], [9 – 12].

The major content of this paper focuses on a number of specific contents as follows:

1. Introduction to Circuit Modeling, Field Modeling.
2. Analysis of state parameters and basic characteristics of Model of Electrical Circuit and Model of Electromagnetic Field helps to understand about Circuit Model - Field Model, Electromagnetic Field - Environment interaction system, relationship of Electric field - Magnetic field.
3. Based on Maxwell's Electromagnetic Field Theory and classical physics as the foundation to analyze and explain basic concepts contained in the Theoretical Basics of Electric Circuits such as: continuity of current, potential properties of voltage – these concepts are often explained verbally or demonstrated experimentally.
4. Conclusions and further potential research.

## II. ELECTRIC CIRCUIT MODEL – ELECTROMAGNETIC FIELD MODEL

### ELECTRIC CIRCUIT MODEL

An electrical circuit is a model describing the localized distribution of energy processes, electromagnetic signals, it is coupled by a certain number of conductors in which the processes of the conversion, accumulation, transmission of the energy. The electromagnetic signal of electrical equipment is characterized by voltage  $u(t)$  and current  $i(t)$  distributed over time  $t$  [5 – 8].

The state parameters of the Electric Circuit depend only on time ( $t$ ):

Current is defined as the continuous variation of charge with time

$$i = \frac{dq}{dt} \quad (1)$$

Electric currents are described mathematically related to each other through Kirchhoff's law 1:

$$\sum_{k=1}^m i_k + \sum_{l=1}^p j_l = 0 \quad (2)$$

The algebraic sum of currents at a node is zero. Or the sum of currents entering the node is equal to the sum of currents leaving the node.

It is clearly confirm that whether the current is continuous or at a node without charge stagnation; In terms of geometrical meaning, this law confirms

the existence of node elements in electrical circuit structure.

Voltage is defined as the potential difference between any two points in an electric field

$$u(t) = u_{ab} = \varphi_a - \varphi_b \quad (3)$$

Voltages are described mathematically in relation to each other through Kirchhoff's 2 law

$$\sum_k u_k(t) = 0 \quad (4)$$

Follow any closed loop with arbitrary direction algebraic, sum of voltages in that loop equal to zero. Or algebraic sum of voltages on elements  $R$ ,  $L$ ,  $C$  is equal to algebraic sum of electromotive forces within that loop.

$$\sum_{k=1}^m \left( R_k i_k + L_k \frac{di_k}{dt} + \frac{1}{C_k} \int i_k dt \right) = \sum_k e_k \quad (5)$$

It confirms the physical meaning of the potential property of the circuit or in other words, in a closed loop, the voltage rise is zero; In terms of geometrical meaning, the law confirms the existence of loop elements in the electrical circuit structure.

The capacity to receive electromagnetic energy (electromagnetic power) is defined through a mathematical description by the relationship of the instantaneous voltage and current product:

$$p(t) = u(t).i(t) \quad (6)$$

The characteristic parameters (behavior) of the Electric Circuit are finite and defined elements:

The circuit is coupled by certain number of elements, so we can completely determine the number of loads and sources in the circuit through the characteristic parameters:  $R$  - resistance (characteristic of dissipation - conductor),  $L$  - inductance (characteristic for the phenomenon of accumulation of energy from the magnetic field – magnetic store),  $C$  – capacitance (characteristic for the phenomenon of accumulation of electric field energy – electricity storage),  $e(t)$  – source of electromotive force (voltage source),  $j(t)$  – current source.

The Electric Circuit structure is determined through geometric factors and finite state variables:

The electrical circuit is distributed centrally and has a specific structure through basic geometrical elements: the number of nodes, branches, loop, meshes determined with a pre-defined set of state variables  $u(t)$ ,  $i(t)$ ,  $p(t)$ .

### ELECTROMAGNETIC FIELD MODEL

Electromagnetic fields are a special form of material

Moving at a constant speed equal to the speed of light  $c = 299790.10$  m/s in all inertial

reference frames placed in a vacuum. It is a fundamental difference between the Electromagnetic Field and other entities, and is also the key to unlocking the miraculous relationship between space and time, between electric and magnetic fields. (In a vacuum, the propagation velocity of the Electromagnetic Field does not depend on the frame of reference, it means the usual addition velocity in the Galilean transform must be eliminated, so that for the Electromagnetic Field, space and time do not separate and stick together into a unity of space-time).

Electromagnetic field represents the existence and movement through interaction with charged particles or substance environment which is stationary or moving.

The electromagnetic field has two properties which are shown simultaneously by two sides: continuous and discontinuous

Continuity (as a wave structure), in a vacuum Electromagnetic field propagates electromagnetic waves with a constant velocity  $c$ , independent from the frequency of the field. The model of the interaction system is the Electromagnetic Field – Continuous material environment in space and time. In fact, electrical engineering equipment in the radio frequency range - frequencies lower than about 300 GHz (or equivalent to wavelengths longer than about 1 mm) operate according to local average value and independently exist in space and time.

Discontinuity (as a quantum-particle structure), each radiation quantum (also called photon) of the field carries an energy calculated according to Einstein's quantum theory. The model of the interaction system is the field-charged particle. Space and time coexist in a unity of space-time. But so far this has had no application to electrical engineering in the radio frequency band.

Electromagnetic field can be expressed in two forms, electric field and magnetic field

Indeed, electric field and magnetic field are two different aspects in nature but closely related to each other. A changing electric field produces a magnetic field and conversely a changing magnetic field produces an electric field. Therefore, consider the Electromagnetic Field as a unified, indivisible entity, that is, a fundamental entity. However, Electric and Magnetic fields are both relative because they depend on the frame of reference we choose. Electric field is characterized by state parameters  $\vec{E}, \vec{D}, \varphi, \dots$ , Magnetic field is characterized by state parameters  $\vec{B}, \vec{H}, \vec{A}, \dots$

Electric field force  $\vec{F}_E$  depends only on the position of the object, not on the object's velocity, the

magnetic force  $\vec{F}_M$  acts only when the object is in motion with a non-zero velocity, so it depends on the object's velocity [1 - 4].

Parameters and distribution of the electromagnetic field

The parameters of the Model of Electromagnetic Field depend on both space and time according to Decac coordinate systems ( $x, y, z, t$ ), cylindrical coordinate systems ( $r, \alpha, z, t$ ), spherical coordinate system ( $R, \theta, \alpha, t$ ).

The Model of Electromagnetic Field is an infinite space, uncountable set of state variables that may or may not be time dependent such as:

$$\vec{E}_{x,y,z}(t), \vec{B}_{x,y,z}(t) \dots$$

The Model of Electromagnetic Field is investigated in three basic environments through the characteristic parameters of the substance medium: dielectric  $\epsilon$ , magnetic  $\mu$ , conductor  $\gamma$  – also known as conductivity of the conductive medium, based on the method related mathematical description of Electromagnetic Field - Substance medium in the simplest case:

$$\vec{D} = \epsilon \vec{E}, \vec{B} = \mu \vec{H}, \vec{J}_d = \gamma \vec{E} \quad (7)$$

The Model of Electromagnetic Field is distributed unfocused, uneven and scattered in space in the form of particles, but is distributed centrally according to the structure of the conductor.

Electromagnetic field has three forms: Static field, Standing field, Variable field.

So through the Model of the Electric Circuit and the Problem of the Electromagnetic Field, we find the following:

- The parameters of the problem Electric circuits depend only on time; while the parameters of the electromagnetic field problem depend on both space and time.

- The parameters of the electric circuit problem is finite and is determined through the characteristic parameters which are the elements: R (dissipation), L (magnetic storage), C (power storage), voltage source, current source; and the parameters of the problem Electromagnetic field is an infinite set in space, uncountable investigated in three substances: dielectric  $\epsilon$ , magnetic  $\mu$ , conductor  $\gamma$ .

- The electrical circuit is distributed centrally and has a specific structure; while the electromagnetic field is not concentrated, uneven and scattered in space in the form of particles, but is distributed centrally according to the structure of the conductor.

- DC circuit corresponds to Standing Field (time independent and has constant current density), AC circuit corresponds to Variable Field

(time dependent and has a non-zero current density) . Static fields are fields of statically charged particles (magnets).

Therefore, the problem of Circuits is only a special case of the problem of Electromagnetic fields.

### III. EXPLANATION OF SOME BASIC CONCEPTS OF ELECTRIC CIRCUIT THEORY ON THE BASIS OF ELECTROMAGNETIC FIELD THEORY

Continuity of electric current

Based on Maxwell's equation [1 - 4]:

$$\text{Rot}\vec{H} = \vec{J}_d + \frac{\partial \vec{D}}{\partial t} = \vec{J}_d + \vec{J}_{cd} = \vec{J}_a \quad (8)$$

Using the equality constants found in vector analysis: , we take Div (differentiation of variables by space) on both sides of (8):

$$\text{Div}\text{Rot}\vec{H} = \text{Div}\vec{J}_a = 0 \quad (9)$$

Apply Ostrogradsky - Gauss theorem to (9):

$$\oint_V \text{Div}\vec{J}_a \cdot dV = \oint_S \vec{J}_a \cdot d\vec{S} = 0 \quad (10)$$

According to the circuit conditions, when ignoring leakage current and displacement current, we get Kirchhoff's law 1 for a node or closed surface:

$$\sum id = 0 \text{ or } \oint_S \vec{J}_d \cdot d\vec{S} = \sum i_d = 0, \text{Div}\vec{J}_\Sigma = 0 \quad (11)$$

Therefore, it can be seen that Kirchhoff's law 1 is only a special case of Maxwell's equation 1 and is similar to the continuum principle of magnetic flux, we get:

$$\oint_S \vec{B} \cdot d\vec{S} = 0 \text{ hay } \text{Div}\vec{B} = 0 \quad (12)$$

From (11), (12), we clearly see the continuity of the current when ignoring the leakage current and the displacement current as well as the continuity principle of the magnetic flux, which is always flowing continuously or in other words the property continuity of current is shown by Kirchhoff's law 1- at a node there is no charge stagnation or the algebraic sum of currents at a node is 0. The current referred here is the conduction current or  $i_d$  - is the current flowing in the branches of the circuit, this current has a continuous nature and has a dissipation phenomenon because it is the flow in a conductor

of free charges:  $\oint_S \vec{J}_d \cdot d\vec{S} = -\frac{\partial q_{ld}}{\partial t}$  (law of

conduction).

$\vec{J}_{cd}$  is the displacement current component

$$\vec{J}_{cd} = \frac{\partial \vec{D}}{\partial t} = \epsilon_0 \frac{\partial \vec{E}}{\partial t} + \frac{\partial \vec{P}}{\partial t} = \vec{J}_E + \vec{J}_P \quad (13)$$

The displacement current exists in a dielectric medium that is the medium of bound charges that exist between the electric dipoles (or between the plates of a capacitor). The displacement current has two components,  $\vec{J}_E$  and  $\vec{J}_P$  .

$$\text{Current density vector: } \vec{J}_E = \epsilon_0 \frac{\partial \vec{E}}{\partial t} \quad (14)$$

Equation (14) shows that  $\vec{J}_E$  does not represent the motion of any charged particles, but only corresponding to a changing electric field . However, in practice, it is found that it has completely similar properties to the conduction current density.

$$\text{Current density vector: } \vec{J}_P = \frac{\partial \vec{P}}{\partial t} = \frac{\partial N\vec{p}}{\partial t} \quad \text{or}$$

$$\vec{J}_P = \frac{\partial \vec{P}}{\partial t} = \frac{\partial Nq\vec{l}}{\partial t} = \frac{Nq}{\partial t} \vec{l} = Nq\vec{v} \quad (15)$$

Equation (15) shows that this current density component  $\vec{J}_P$  is the movement of charges around the equilibrium position with velocity  $\vec{v}$  . So the fundamental is the same as  $\vec{J}_d$  , however is the movement of the bound charges of the dipole while that of  $\vec{J}_d$  is the free charges. So the displacement current  $\vec{J}_{cd}$  is exactly the same as  $\vec{J}_d$  and is a part of the total current density  $\vec{J}_a$  .

### POTENTIAL PROPERTY OF VOLTAGE

Based on Maxwell's equation 2 [1-4]:

$$\text{Rot}\vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad (16)$$

Integrating both sides along a surface S bounded by a closed loop at rest in the frame of reference, S is not deformed in time - in other words, space and time are independent according to the interaction model of the Electromagnetic Field - The substance environment is continuous in space and time, so we have:

$$\int_S \text{Rot}\vec{E} \cdot d\vec{S} = -\int_S \frac{\partial \vec{B}}{\partial t} \cdot d\vec{S} = -\frac{\partial}{\partial t} \int_S \vec{B} \cdot d\vec{S} \quad (17)$$

Which follows the definite expression of the

$$\phi = \int_S \vec{B} \cdot d\vec{S}$$

Electromagnetic flux: where  $\phi$  is the flux flowing through the surface S. Hence:

$$\int_S \text{Rot}\vec{E} \cdot d\vec{S} = -\int_S \frac{\partial \vec{B}}{\partial t} \cdot d\vec{S} = -\frac{\partial}{\partial t} \int_S \vec{B} \cdot d\vec{S} = \frac{\partial \phi}{\partial t} \quad (18)$$

According to Lenz - Faraday law, it is the electromotive force induced on the closed loop L covering the surface S. Applying Green - Stokes theorem to the left side (18) we have:

$$\int_S \text{Rot} \vec{E} d\vec{S} = \oint_L \vec{E} d\vec{l} \text{ or } \oint_L \vec{E} d\vec{l} + \frac{\partial \phi}{\partial t} = 0 \quad (19)$$

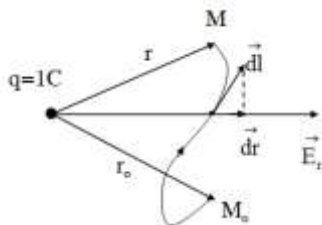
The terms in (19) are the same voltages including the induced emfs taken in the closed loop L. So we get Kirchhoff 2's law for the closed loop:  $\sum u = 0$ . Or the algebraic sum of the voltages in a closed loop is zero. Thus, it can be seen that Kirchhoff's law 2 is only a special case of Maxwell's equation 2.

In an electromagnetic field that varies without vortex, without loss of generality, we can also determine the scalar potential function  $\phi$  as work A moving a charge from point M0 (reference point) to any point M:

$$\phi_{(M)} = A = \int_L -\vec{F}_E d\vec{l} = -q \int_L \vec{E} d\vec{l} = -q \int_{M_0}^M \vec{E} d\vec{l} \quad (20)$$

Assuming there is a point charge  $q = 1C$ , its density  $\rho$  is a Dirac distribution of volume  $\rho = \delta(v)$  for a total of 1C. We have the potential at any point M relative to the reference point M0 is determined:

$$\phi_M = - \int_{M_0}^M \vec{E} d\vec{l} \quad (21)$$



**Figure1.** The potential formation at point M relative to the reference point M0 is caused by a charge  $q = 1C$

Figure 1 shows the potential formation at any point M. For simplicity, we put the origin of the reference system at the position of the charge q, paying attention to the radial symmetry with the tangent of the curve L from M0 to any point M in space,  $dr$  is the perpendicular projection of  $d\vec{l}$  to  $\vec{E}_r$ . We have:

$$\begin{aligned} \phi_M &= - \int_{M_0}^M \vec{E} d\vec{l} = - \int_{M_0}^M \vec{E}_r d\vec{l} = - \int_{M_0}^M E_r dl \cos \theta \\ &= - \int_{r_0}^r E_r dr = - \frac{1}{4\pi\epsilon_0} \int_{r_0}^r \frac{1}{r^2} dr \in M_0 \end{aligned} \quad (22)$$

Where:  $\theta$  is the clamping angle between  $\vec{E}_r$  and  $d\vec{l}$

$$\text{So: } \phi_M = \frac{1}{4\pi\epsilon_0} \left( \frac{1}{r} - \frac{1}{r_0} \right) \quad (23)$$

The potential  $\phi_M$  is understood as the potential value at point M, this value is determined depending on how the reference point M0 is selected. Because we can choose the reference point, so we choose the reference point M0 at infinity, then the

potential function corresponding to a unit charge will have a very simple form (simplified Green function):  $\phi_M = \frac{1}{4\pi\epsilon_0 r}$  (24)

In order to avoid losing generality, we apply the choice of reference point potential  $\phi_{M_0} = 0$  to the problem of electric circuit by the method of node potential. We arbitrarily choose the potential at a most convenient node with a potential value of zero, then set up a system of equations to determine the potential for the remaining nodes. For each reference point that we are allowed to optionally determine, the voltage value at the nodal points of the circuit with different false values because the potential always depends on the reference point M0 - or in other words depends on Choosing the reference point M0 leads to countless different potential values. However, choosing a reference point at any node of the circuit does not affect the resulting voltage value - in other words, the potential difference between any two points M1, M2 is always determined without any problems and does not depend on the location of the reference point M0. We call that difference the voltage value U12 from point M1 to point M2. Indeed we have:

$$U_{12} = \phi_{M_1} - \phi_{M_2} = - \int_{M_0}^{M_1} \vec{E} d\vec{l} - \left( - \int_{M_0}^{M_2} \vec{E} d\vec{l} \right)$$

Or we have:

$$U_{12} = \int_{M_1}^{M_0} \vec{E} d\vec{l} + \int_{M_0}^{M_2} \vec{E} d\vec{l} = \int_{M_1}^{M_2} \vec{E} d\vec{l} \in M_1, M_2 \notin M_0 \quad (25)$$

From expression (25) it is clear that the voltage U12 depends only on the position - or the value of the initial voltage M1 and the end point M2 but no longer depends on the selection of the reference point M0. Therefore, with the node potential method, we can freely choose any reference point at any node of the circuit, the final result of calculating the voltage and current in the circuit is unchanged.

In addition, to confirm the potential property of the circuit that follows any closed loop with an arbitrary direction, the potential increase is zero, we can rely on the theory of Electromagnetic Fields and Physics to have: The work of moving a charge from one point to another is definite because it depends only on the position of the starting and ending points and not on the path, inferring the work of moving a charge along a closed loop  $L$  is equal to 0.

Indeed, if the charge moves in a closed loop  $L$ , the work is calculated:

$$A = -q \oint_L \vec{E} d\vec{l} \quad (26)$$

Assume that charge  $q = 1C$  is chosen. According to the Green - Stokes theorem  $\oint_L \vec{E} d\vec{l} = \int_S \text{Rot} \vec{E} d\vec{S}$ :

with the attention that when the electric field is not eddy,  $\text{Rot} \vec{E} = 0$  then:  $A = \int_S \text{Rot} \vec{E} d\vec{S} = 0$

(27)

where  $S$  is the surface enclosed by a closed loop  $L$  or in other words follows a closed loop, the potential increase is zero.

#### IV. CONCLUSION

Based on the theoretical basis of the electromagnetic field, it helps us to better understand the nature and explain the basic concepts such as continuity of current, potential of voltage in electrical circuits. Through the article, the subject The Theoretical Foundation of Electromagnetic Fields has clarified as well as applied to the subject The Theoretical Fundamentals of Electric Circuits effectively; Especially, not only that, it helps us to understand the close relationship between Kirchhoff's laws and Maxwell's equations, between Electric and Magnetic fields, which is the fundamentals and basis for many subjects of the Electrical, Electronics and Telecommunications. Through the article, the author wishes to help readers get closer to the subject Theoretical foundations of electromagnetic fields and see the importance and necessity of this subject in University program in Electrical Engineering. The field of Electromagnetic Field Theory is also a research direction that the author will continue to learn and apply more in the future with practical applications when deeply analyzing Macwell's electromagnetic equations into Matlab.

#### ACKNOWLEDGEMENTS

This research was funded by Thai Nguyen

University of Technology, No. 666, 3/2 Street, Thai Nguyen, Viet Nam.

#### REFERENCES

- [1]. D. H. Dang, K. L. Lai, T. H. L. Le and T. T. H. Tran, "Textbooks Electromagnetic Field Theory," (in Vietnamese), TNU publishing company, 2017.
- [2]. D. K. Cheng, "Fundamentals of Engineering Electromagnetics," Pearson New International Edition, 2014.
- [3]. Matthew N. O. Sadiku, "Elements of Electromagnetics," Oxford University Press, 2018.
- [4]. F. Ulaby and U. Ravaioli, "Elements of Electromagnetics," Pearson New International Edition, 2020.
- [5]. S.A. Mitkowski, A.M. Dąbrowski, A. Porębska and E. Kurgan, "Electrical engineering education in the field of electric circuits theory at AGH University of Science and Technology in Kraków," in 1st World Conference on Technology and Engineering Education, Kraków, Poland, pp. 47–53, 2010.
- [6]. J. Ma, G. Zhang, T. Hayat and G. Ren, "Model electrical activity of neuron under electric field," Springer Nature B.V. 2018
- [7]. A. Gero, Y. Stav and N. Yamin, "Use of real world examples in engineering education: the case of the course Electric Circuit Theory," World Transactions on Engineering and Technology Education Vol.15, No.2, pp. 120 – 125, 2017.
- [8]. [8].J. Gulowski, T. P. Stefanski and D.Trofimowicz, "On Applications of Elements Modelled by Fractional Derivatives in Circuit Theory," Energies MDPI, 13, 5768; doi:10.3390/en13215768, 2020.
- [9]. M. T. Nguyen , C. V. Nguyen, L. H. Truong, A. M. Le, T. V. Quyen, A. Masaracchia and K. A. Teague, "Electromagnetic Field Based WPT Technologies for UAVs: A Comprehensive Survey," Electronics MDIP,9, 461, 2020.
- [10]. B. Salarieh, H. M. J. De Silva, A. M. Gole, A. Ametani and B. Kordi, "An Electromagnetic Model for the Calculation of Tower Surge Impedance Based on thin WireApproximation," IEEE Transactions on Power Delivery, 2020.
- [11]. T. D. Nguyen, T. Kim, J. Noh, H. Phung, H. R. Choi IEEE Fellow, and G. Kang, "Skin-type proximity sensor by using the change of electromagnetic field," IEEE Transactions



- on Industrial Electronics, DOI  
10.1109/TIE.2020.2975503, 2020.
- [12]. K. E. I. Elnail, X. Huang, C. Xiao, L. Tan  
and X. Haozhe, "Core Structure and  
Electromagnetic Field Evaluation in WPT  
Systems for Charging Electric Vehicles,"  
Energies MDPI, 11, 1734, 2018.