

The scale-free law of the human tourist city network

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ABSTRACT: Scale-free networks are common in nature and society, such as the world wide web, the human language web and the web of human sexual contacts. In this paper, we build a human tourist city network (HTCN), study its probability distribution and entropy, and find the network follows the scale-free laws, which makes us better understand the rules of tourism network and provide the reasonable preparation between the supply and the demand in order to improve the passenger satisfaction and the popularity of cities.

Key words. tourist city network ; scale-free law; entropy

I. INTRODUCTION

Complex systems, such as economic systems, companies in social and biological, show cooperative phenomena between constituents through diverse interactions and adaptations to the pattern they create(Watts, 2004; Ameraoui et al., 2016; Barabasi et al., 2004). In order to study the systems, Interactions may be described in terms of networks, consisting of vertices and edges, where vertices and edges represent the constituents and their interactions, respectively. In networks, the distance between the two vertices is denoted by the minimum number of connecting edges, the average distance of the network is defined as the average distance between every two vertices. The clustering coefficient of vertices in the network is the proportion of the number of connected edges between all the neighboring vertices to the maximum possible number of links. The average of all the clustering coefficients of the vertices is called the

clustering coefficient of the network. The first type of network that comes to

people's mind is the regular lattice, which plays a fundamental role in solids. They are characterized by invariance under translation in space by a lattice spacing along a lattice axis. In this case the vertices of a network could represent the atoms of a crystal. The edges could then

indicate the most important interactions. Afterwards, Erdos and Renyi(ER) (Erdős et al., 1960) proposed the ER model, in which the number of vertices is fixed, while edges connecting one vertex to another occur randomly with certain probability. It has been shown by research that a regular network has a higher clustering coefficient and average distance, while the random network has a relatively smaller clustering coefficient and average distance. It is clearly that the regular network exists many limitations for the complex systems and the ER model is too random to describe real complex systems.

Recently, Watts and Strogatz(WS) (Watts et al., 1998) introduced a small-world network, where a fraction of edges on a regular lattice is rewired with probability p to other vertices, which makes the network show a short average path length and high clustering. The popular "six degrees of separation" is a case of the small-world, which is revealed by the social psychologist Stanley Milgram in the 1960s. More recently, Barabasi and Albert(BA) (Ablert et al., 2002) introduced an evolving network where the number of vertices N increases rather than fixed, and a newly introduced vertex is connected to m already existing vertices, following the so-called preferential attachment(PA) rule. When the number of edges k incident upon a vertex is called the degree of the vertex, the PA rule means that the probability for the new vertex to connect to an already existing vertex is proportional to the degree k of the selected vertex. Then the degree distribution follows a power-law distribution for the BA model. Networks whose degree distribution follows a power-law are called scale-free(SF) network(Stegehuis et al., 2017; Yao et al., 2018; Gerlach et al., 2019; Voitalov et al., 2019; Holme et al., 2019; Broido et al., 2019; Corral et al., 2019), which are ubiquitous in real-world networks such as food webs (Camacho et al., 2002), scientific collaboration (Newman, 2001; Newman, 2001a) and sexual relations networks (Liljeros et al., 2001)

etc.

Tourism as a complex system, there have been many related studies (Paul et al., 2018; Birendra et al., 2019; Caldeira et al., 2020). In this paper, we build a human tourist city network and study its properties. The rest of the paper is organized as follows, we first recall the properties of the scale-free network and build the a new network model about the tourist number in section 2. Then theoretical analysis are given in section 3. Finally, the conclusions are given in section 4.

II. THE MODEL

Many "real-world" networks are clearly defined while most "social" networks are to some extent subjective. Indeed, the accuracy of empirically-determined social networks is a question of some concern because individuals may have distinct perceptions of what constitutes a social link. One unambiguous type of connection is the tourist contact. Next, we propose the model of tourist.

In the model of tourist, we see the tourist cities as the nodes, if there exists people from a city to another city, a link is connected. Many real-world networks typify the "small-world" phenomenon, which is so-called because of the surprisingly small average path lengths between nodes in the presence of a large degree of clustering. The network decided by the growth and preferential attachment is the scale-free network. small-world networks depend on their connectivity distribution, $p(k)$, where k is the number of links connected to a node. Scale-free networks, which are characterized by a power-law decay of the cumulative distribution $p(k) = k^{-\lambda}$, may be formed as a result of the preferential attachment of new links between highly connected nodes.

The above characters forming the scale-free networks is in accord with the development and evolution of the actual HTCEN. In the beginning, some cities has not attached importance to tourism, there are only some vertices and connections between some important nodes(hubs). With the gradually emphasis on tourism, the vertices increase, the development and evolution of HTCEN basically meet the preferential attachment conditions. the vertices with many connections are call hubs, which connections have may be as many as thousands, it can explain the HTCEN is a scale-free network. Furthermore, the preferential attachment is an important factor forming scale-free networks. Correspondingly, the property of preferential attachment, namely the rich being richer, also plays a key role in forming the HTCEN.

In the HTCEN, the original m_0 cities are connected arbitrarily or completely. With the development of tourism, we add a new node with $m(m \leq m_0)$ edges that link the new node to m different nodes already present in the system. According to the preferential attachment, when choosing the nodes to which the new nodes connects, we assume that the probability p that a new node will be connected to node i depends on the degree k of node i , i.e.

$$P(k_i) = \frac{k_i}{\sum_{j=1}^{N-1} k_j}$$

After t time steps, a network with $N = t + m_0$ nodes and $mt + L_0$ edges is formed.

In addition, the entropy is also used to measure the non-homogeneous nature of the network. Shannon (Shannon, 1948) first introduced the thermodynamic entropy into the information theory and see the entropy as the uncertainty of a random event and a measure of the amount of information. Specifically, if the value of the random variable X is $x_i, i = 1, 2, \dots, n$, and $x = \{x_i\}$ are pairwise incompatible, the probability of x_i is p_i ,

$$i = 1, 2, \dots, n, \quad \sum_{i=1}^n p_i = 1, \quad \text{Shannon proved}$$

$$H(X) = -c \sum_{i=1}^n p_i \log(p_i) \quad (c > 0) \text{ is the only}$$

function satisfied the following conditions:

- (i) H is the continuous function of p_1, p_2, \dots, p_n ,
- (ii) H get the maximum if and only if $p_1 = p_2 = \dots = p_n$;

$$(iii) \quad H(X) = H(Y) + H(X/Y), \quad \text{where,}$$

$Y = f(X), H(X/Y)$ is conditional entropy of X under the condition we know Y .

At the point, $H(X)$ is called the entropy of X . Let $c = 1$, people called

$$H(X) = -\sum_{i=1}^n p_i \log(p_i) \text{ to the traditional entropy.}$$

If the distribution of random variable X is continuous, which distribution density function is $f(x)$, the entropy of X is defined as follows:

$$H(X) = -\int_R f(x) \log(f(x)), \text{ where, } R \text{ is the}$$

definition domain of $f(x)$.

Since the entropy is introduced, as a measure of the system stability, it has become an important tool for studying the complex system and been extensively studied. In this paper, we study the change trend of the entropy of the HTCN and the result is shown in section 3.

III. RESULTS AND DISCUSSION

First, we build the HTCN with tourism number in 2020, and give the degree distribution frequency of the HTCN in Figure 1. We find that the proportion of nodes with large degree is small, i.e, most cities have only several connections whereas a small percent of the total number of nodes have many connections in HTCN of China, the hubs can make the HTCN more robust.

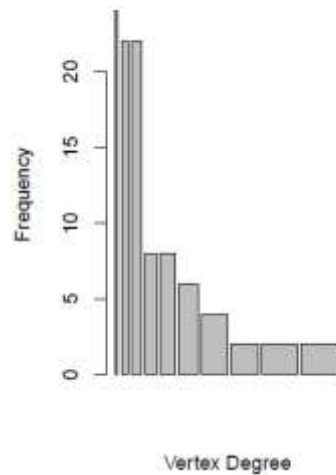


Figure 1 The degree distribution of the HTCN

In figure 2, we give the Log-Log degree distribution of HTCN, it shows that the HTCN has the property of the scale-free network. The scale-free network shows a non-homogeneous nature and a kind of sequence the complex network emerges. It is not uniform, which degree distribution curve is continuously decreasing and the probability connected to other k nodes is proportional to $k^{-\gamma}$. γ characterizes non-homogeneity to some extent, γ increases, the decreasing speed of the degree distribution curve increases, then the non-

homogeneity of networks is clearer. Otherwise, the growth characteristics of the HTCN are similar to the scale-free network. City economy and tourist development make possible the growth of HTCN networks. The number of tourism cities increases, and is an increasing function of time within a certain period. The development of economics and tourism of cities decides the rule pattern of links, which satisfy the preferential attachment condition. Therefore, the degree distribution of the networks can be consistent with the power-law.

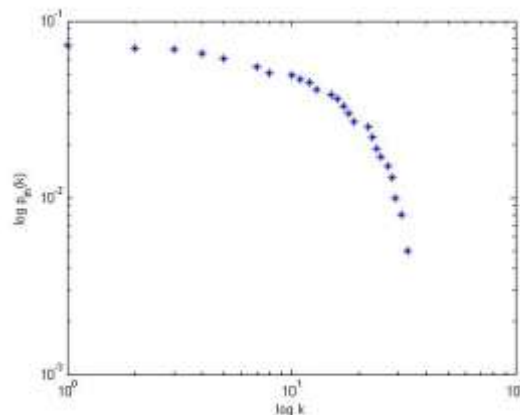


Figure2 Log-Log degree distribution of HTCN

Finally, we calculate the entropy of tourism network from 2016 to 2020 in Table 1. From the table, we find the entropy from 2016 to 2019 is below 3 and changes a little bit, but there is a significant increase in 2020. As we all know, the higher the entropy, the more unstable the matter. The entropy of an isolated system always tends to increase, and eventually reaches the maximum state of entropy, which is the most chaotic state of the system. However, for the open system, due to the intervention of human factors and the influence of

the external environment, the entropy of the open system decreases and reaches the ordered state. The HTCN is an open system, its entropy from 2016 to 2019 changes a little bit, but there is a significant increase in 2020. The possible reason is that the outbreak of the epidemic in 2020 led to great changes in tourism. In the future, we will further study the change trend of tourism network through entropy, which is beneficial for us to formulate corresponding strategies to guide the healthy development of tourism.

Table 1 The entropy of the tourism network

Year	2016	2017	2018	2019	2020
Entropy	2.9063	2.9692	2.9343	2.9836	3.5762

IV. CONCLUSION

In this paper, we build the HTCN and study its property by the degree distribution and entropy. We find that it has the scale-free property and the small-world effect by data analysis. The number of the tourist cities is increasing and the possibility of a city where the people tour is proportional to the popularity (the connection level) of the city.

With the improvements of the living standards recently, the people's demand for travel is increasing, which forces us to study the tourist and get its internal law, so that, we can gain a reasonable solution to ensure the safety of people traveling and form a good state of the tourism operation.

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