Viscosity and Drift Flux Analysis in a Two Phase System

Ramiro Escudero G[1]., Rosa E. Pérez S[2].

[1]*Institute of Research in Metallurgy and Materials.* [2] *Faculty of Pharmacobiology-Chemistry. Universidad Michoacana de San Nicolás de Hidalgo. Santiago Tapia 403, C.P. 58000. Morelia, Michoacán, México.*
Author to whom correspondence should be addressed

ABSTRACT

Column flotation are enjoying renewed interest due to new applications such as. It is well known that efficiency of a given dispersion process depends on the characteristics of the dispersed phase (gas holdup, bubble surface area flux, and bubble size). In order to predict these dispersion characteristics for a given duty (i.e., de-inking of recycled paper, industrial effluents treatment, de-oiling of water, mineral processing, and so on) a mathematical model known as Drift Flux Analysis is currently applied.

Drift Flux Analysis assumes a constant dynamic viscosity of the continuous phase (one cP or one gram/centimetre-second) not matter the changes as result of the pulp consistency or solids content. This paper shows the relevance of considering the real value of the dynamic viscosity in terms of the characteristics of a gas dispersion. Viscosity of water was varied by using a polymer and the bubble size, bubble surface area, and gas holdup were calculated through the Drift Flux Model.

Results show a good agreement between the calculated and measured bubble diameter once the true value of viscosity of the continuos phase is considered during the solution of the drift flux model. For assumptions of unchanged viscosity, both the calculated and the measured bubble diameters show a great disagreement, as result of viscosities different from one centipoise.

KEYWORDS: flotation, columns, rigid spargers, bubble size, gas holdup, bubble surface area flux.

I. INTRODUCTION

Drift flux theory, which was first introduced by Wallis (1964), has been used to relate the fraction of gas with the gas and liquid rates in bubble columns. The concept has been applied to multi-phase systems (Masliyah, 1979), and additional parameters have been introduced to try to define the effect of the environment on bubble formation (Dobby et al, 1988; Yianatos et al, 1988; Xu and Finch, 1990, Dobby and Finch, 1990). The result is a model to estimate mean bubble size in a system involving a dispersed phase (i.e, flotation columns).

The use of flotation column in applications such as de-inking of recycled paper (Watson et al., 1996), de-oiling of water (Takahashi et al., 1979; Strickland, 1980; Van Ham et al., 1983), and for metal ion recovery from hydrometallurgical solutions (Castro Silva et al., 1993; Rybas et al., 1993; Tavera et al., 2000) is observing renewed interest. In these applications the formation of small bubbles is crucial to collect the fine ink particles or oil droplets. A literature review showed that there is almost no published information regarding the best dispersion characteristics for a given flotation application. On the other hand, the dependence of bubble surface area flux with parameters as flotation rate, recovery, and the metallurgical performance is well documented in the literature (Gorain, 1996; Tavera et. Al., 2000), where a linear relationship between all the former parameters is reported.

As mentioned by Jameson et al., (1977), and O´Connor and Mills (1995), the bubble size and the bubble surface area flux should be well predicted (or calculated) in order to consider a proper value of the flotation rate constant for a given column duty.

Dobby and Finch (1986) suggest that small bubbles are desirable for fine particle flotation because they increase particle-bubble collision probability. They create more stable froth phases and a greater column carrying capacity. Nevertheless, small bubbles (or particle-bubble aggregate) are prone to be trapped by the tailings stream due to their weak buoyancy force, they remain in the pulp and can not lift themselves (Escudero, 1998).

During the development of the drift flux model the following two assumptions are made:
- The liquid observes Newtonian behaviour, and
- Small spherical bubbles ascend uniformly, homogeneously distributed over the cross-section of the column.

Another assumption made when the drift

flux model is solved is the value of the viscosity of the continuous phase. The viscosity of tap water is considered despite the fact that either mineral pulp or any liquid different from water observes a viscosity larger or smaller than one centipoise.

A direct method to measured bubble size is by cinephotography (Davidson and Schuler, 1960; Wraith, 1971; Yianatos et al., 1988; Geary and Rice, 1991). Images of bubbles are obtained and the size is measured. Today, this usually involves an image analyser to help process the large number of bubbles (at least 500) that must be examined for statistical reliability. The volume of the bubble is calculated assuming symmetry about the vertical axis. There are other direct methods for measuring bubble volume, such as X-ray cinephotography, γ-ray absorption, and laser techniques, although their application is restricted by the sophistication of the devices (Drew et al., 1970).

Drift flux analysis is a method to estimate average bubble diameter based on the knowledge of the gas holdup and phase velocities (Drift Flux model and its iterative solution is described in the appendix). Good agreement between the bubble size determined through drift flux analysis and that from photographic evidence has been reported (Yianatos et al., 1988; Dobby et al., 1988; Xu and Finch, 1990; Escudero et al., 2000) for gas-water systems. Comparisons between experimental and calculated bubble size for liquids with viscosity larger than that of water has not been tested to date.

This paper compares the mean bubble size measured using photography with that estimated from drift flux analysis for a gas-liquid systems varying the viscosity from 1 to 4.7 centipoises.

Experimental set-up

The apparatus is shown in Figure 1. A rectangular section of column made with transparent plexiglas was placed at the top of a 5.7 cm (0.057 m) diameter column. This provided a flat section with a cross sectional area equal to that of the circular column. The section was wide enough (0.02 m) to allow the free movement of the rising bubbles, but spread them out to facilitate photographic analysis.

Air was fed through a vertical sparger at the bottom of the column into tap water containing 20 ppm of dowfroth 250C and a certain amount of poly (acrylamide-CO- acrylic acid) to vary the viscosity of the liquid. After conditioning the water in a tank, the column was filled and photographs were taken for every Jg value.
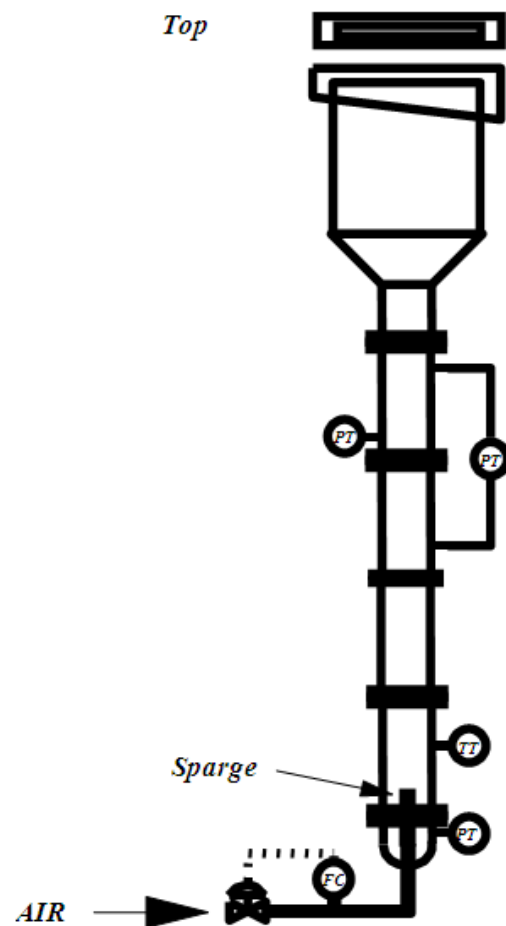

Figure 1. Experimental set-up to obtain photographic measurement of d_b. The ε_g (necessary to estimate d_b from drift flux analysis) was measured over the mid portion of the column.

The surface area (A_s) of the sparger used during the test was 79.8 cm^2 (79.8x10^{-4} m^2)with a nominal pore size 2 μm and permeability 1.6 darcy (1.6x10^{-5} m^2). The variables monitored were: air flowrate (**q**, L/min), pressure drop between the taps (1.81 m) at which the gas holdup was measured (Δp, cm H$_2$O), temperature (**T**, °C), and head pressure at the bottom of the column (p_t, cm H$_2$O). All tests were run under batch conditions.

Pressure drop was measured using differential pressure transducers (Bailey, model PTSDDD1221B2100). The air rate was measured and controlled using a mass flowmeter/controller (MKS Instruments, model 1562A-40L-SV). Corrections to the air flowrate for temperature were made. The temperature was measured by using a ICTD temperature detector (Transduction Ltd., model ICTDP/N1662).

Kinematic viscosities of liquids were

determined by using a Cannon-Fenske Routine viscometer type for transparent liquids (Cannon Instrument Company, mod. 50Z185).

Pictures of the bubbles at the flat section were taken for each value of J_g using a stationary digital camera (Sony, mod. Mavica MVC-FD95). Bubble diameters were measured manually employing an image analyzer (Media Cibernetics LP, Image Pro 4.0). The image analyzer was

calibrated according to a milimetric tape secured to the inside-front of the flat section. For each condition, about 800 bubbles were measured.

II.   RESULTS AND DISCUSSION
Measured bubble size distribution and mean size.
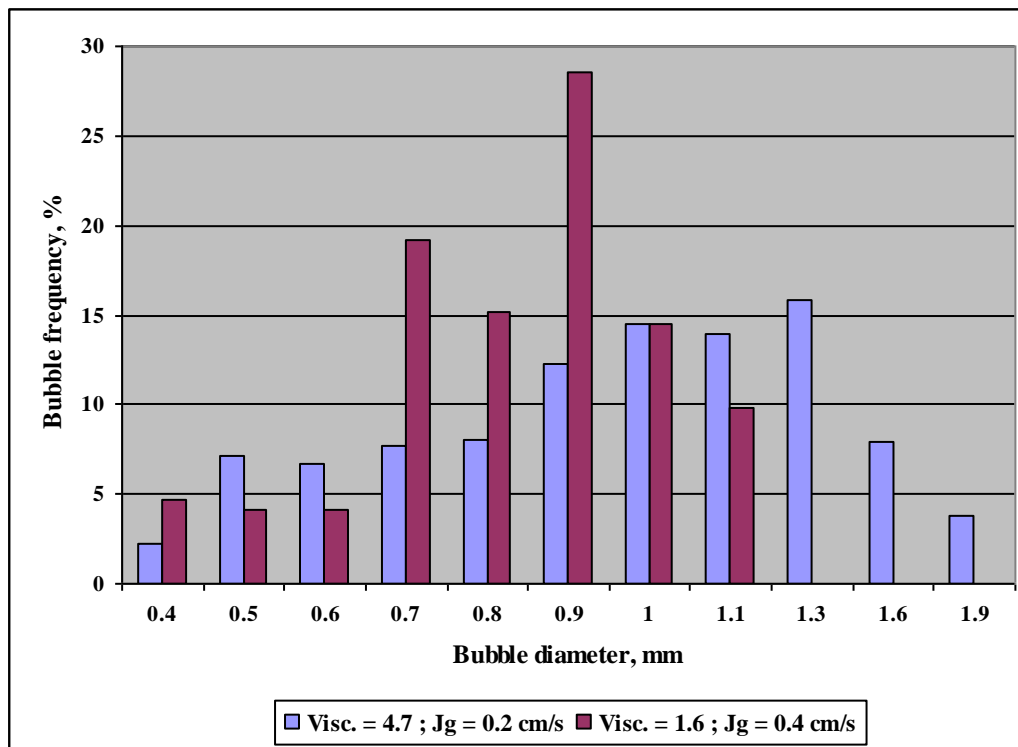A typical distribution of bubble sizes as obtained from the image analyzer is shown in Figure 2.

Figure 2.- Typical bubble size distributions as obtained from the image analyzer. Data for $\sigma = 65$ dyn/cm^2, SS sparger 2 μm. Viscosities 1.6 and 4.7. $J_g = 0.2$ and 0.4, respectively.

From the distribution, a Sauter mean and a number mean bubble diameter were calculated. The number mean diameter was calculated from:

$$d_b = \frac{\Sigma \text{ diameters}}{\text{number of bubbles measured}} \qquad (1)$$

The Sauter diameter, often considered the appropriate one for flotation (Laplante et al., 1983; Yianatos et al.,

$$d_b = 6 \frac{\Sigma \text{ bubble volume}}{\Sigma \text{ bubble area}} \qquad (2)$$

1988; Gorain et al., 1995; Gomez et al., 2000) was calculated as follows:

From the above figure, differences between the two mean sizes, number and Sauter, is not significant when the bubble size distribution is relatively narrow. For viscosity equal to 4.7 cP the bubble Sauter and bubble number diameters are 0.125 and 0.115 cm, respectively. For the case of viscosity 1.6 cP the bubble Sauter and bubble number diameters are 0.084 and 0.80 cm, respectively. However, the Sauter diameter will be designated as that measured and compared with the calculated diameter.

Comparison between measured and calculated bubble size.

Drift flux analysis was used to estimate a mean bubble size knowing the air and liquid flowrates, the gas holdup, and the true value of viscosity of the liquid, measured as mentioned in a previous chapter of the paper. Tables 1 shows the Sauter bubble diameter compared with that mean estimated from drift flux analysis for a stainless steel sparger with nominal pore size (as quoted by the manufacturer) 2 μm and viscosity ranging from 1.0 to 4.7 g/cm-s (1.0 to 4.7 centipoise.).

The observed trend of bubble diameter increasing with the viscosity is as expected if we consider that the viscous drag force retards the bubble formation since it is opposite to the buoyancy force of the bubble.

Table 1.- Sauter (measured) bubble diameter and calculated mean diameter from drift flux analysis. Data for a SS sparger with nominal pore size 2 μm. Surface tension of the liquid 65 Dyn/cm^2.

J_g cm/s	Viscosity = 1.0 cP			Viscosity = 1.6 cP		
	ε_g, %	bubble diameter, cm		ε_g, %	bubble diameter, cm	
		Sauter measured	Drift Flux calculated		Sauter measured	Drift Flux calculated
0.2	3.48	0.054	0.053	3.33	0.073	0.065
0.4	6.00	0.061	0.062	5.59	0.084	0.078
0.6	7.95	0.072	0.070	7.42	0.091	0.089
J_g cm/s	Viscosity = 3.1 cP			Viscosity = 4.7 cP		
	ε_g, %	bubble diameter, cm		ε_g, %	bubble diameter, cm	
		Sauter measured	Drift Flux calculated		Sauter measured	Drift Flux calculated
0.2	2.97	0.098	0.093	2.75	0.125	0.115
0.4	5.05	0.118	0.108	4.90	0.145	0.130
0.6	6.19	0.155	0.132	5.58	0.178	0.167

The bubble diameters are also compared in Figure 3. A complete agreement is observed for the case of viscosity of 1 cP, and for all the air flowrates tested here.
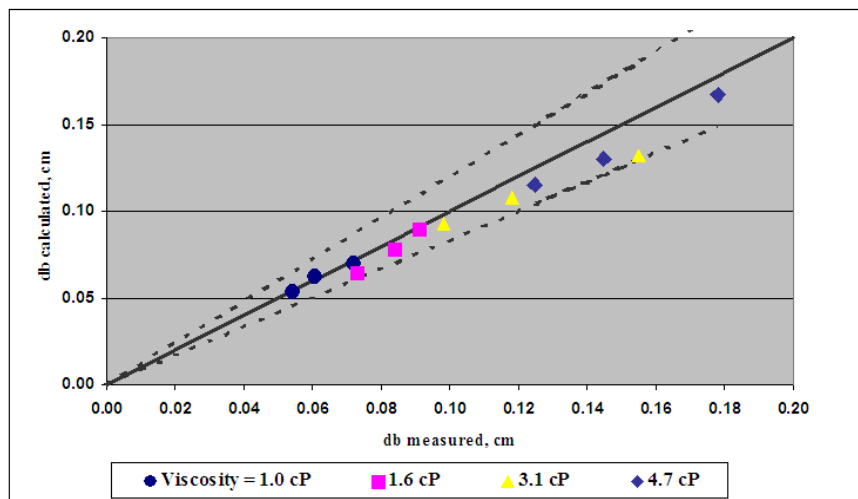

Figure 3.- Bubble size measured, and estimated. Sparger pore size 2 μm. Surface tension of the liquid 65 dyn/cm^2. The dotted lines means ± 20% error.

In accord with previous experimental evidence, d_b from drift flux analysis compares well with the Sauter mean d_b (Yianatos et al., 1988; Dobby et al., 1988; Xu and Finch, 1990). The rest of the compared data converges well enough within a +/- 20 % error, after considering the measured value of viscosity of the liquid. In other words, the Drift Flux model calculates with acceptable accuracy a mean bubble diameter once the right values of the physicochemical characteristics of the liquid are considered.

As mentioned above, in most of the processes involving gas dispersions the true value of the viscosity of the liquid is quite different from that of water. The implication in considering the viscosity of the liquid always being as one centipoise, leads to miscalculate or wrongly predict an average bubble size and then the characteristics of the dispersion (gas holdup, bubble size, and bubble surface area flux). An example of the former statement can be demonstrated through the Figure 4. As can be observed all the calculated bubble sizes (for true viscosities different from 1.0 cP) disagree from those measured using the photographic technique, drawing diameters with an error larger than 20%. The consequences of mistakes in predicting a bubble size were pointed out in the introduction.
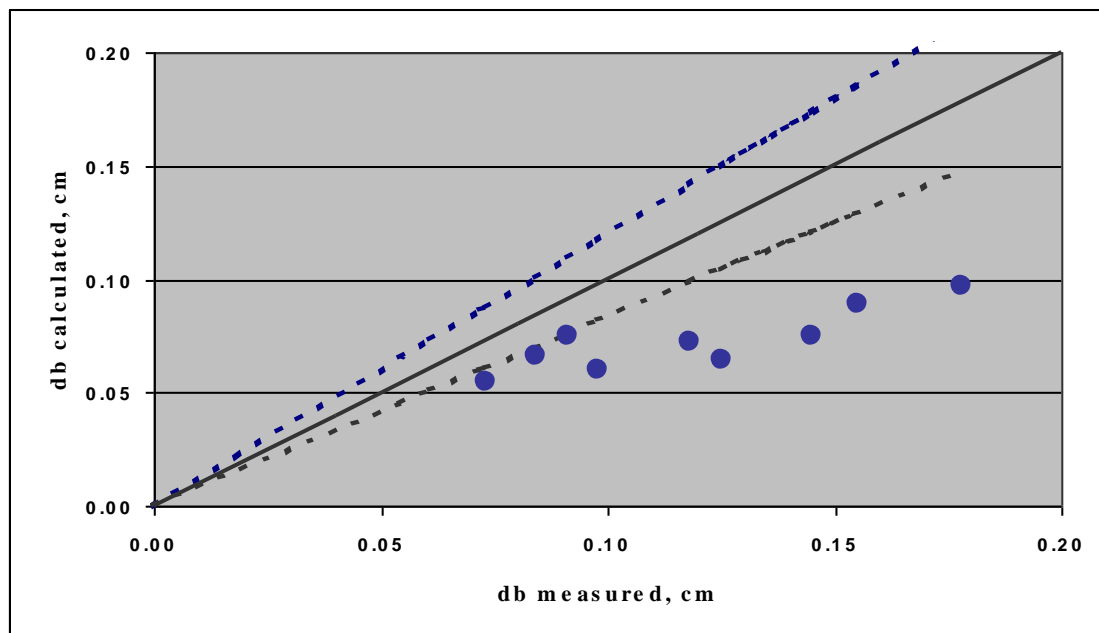

Figure 4.- **Comparison between the mean diameters, number and Sauter, for data in Table 4.1. The dotted line represents an error interval of +/- 20%.**

III. CONCLUSIONS

Comparisons between measured (using photographic technique), and calculated bubble diameters (through the solution of the Drift Flux model) drawn the following conclusions:

The relevance of considering the true value of the dynamic viscosity in terms of the characteristics of a gas dispersion was determined. The Drift Flux model calculates with acceptable accuracy a mean bubble diameter once the right values of the physicochemical characteristics of the liquid are considered.

If changes in viscosity are not considered and it is taken as one centipoise, an average bubble size and then the characteristics of the dispersion (gas holdup, bubble size, and bubble surface area flux) will be miscalculated. This error in predicting bubble size leads to wrongly predict both the gas holdup the bubble surface area flux.

As is mentioned in the literature, the knowledge of the properties of a gas dispersion are needed in order to design a column flotation for a given duty.

Appendix
Bubble flow model: drift flux analysis
Drift flux analysis (Wallis, 1969) considers the relative phase velocity (slip velocity) and has been widely used to estimate mean bubble size in flotation columns (Banisi & Finch, 1994; Yianatos et al., 1988b). In the case of a flotation column, the appropriate expression for the relative slip velocity (U_{sb}) between the gas phase and the liquid phase is:

$$U_{sb} = \frac{J_g}{\varepsilon_g} + \frac{J_l}{1 - \varepsilon_g} \tag{3}$$

where:

ε_g = Fractional gas holdup

J_g, J_l = Superficial velocities of the gas and liquid respectively, cm/s

The slip velocity is given in terms of the system properties by an expression due to Masliyah (1979) after Richardson & Zaki (1954):

$$U_{sb} = \frac{g * d_b^2 * (\rho_{sl} - \rho_b) * (1 - \varepsilon_g)^{m-1}}{18 * \mu_{sl} * (1 + 0.15 * \mathrm{Re}_s^{0.687})} \tag{4}$$

where:

μ_{sl} = Viscosity of the slurry (pulp), g/cm-s

ρ_{sl}, ρ_b = Densities of the slurry, and the bubble respectively, gr/cm^3.

The bubble swarm Reynolds number is calculated as:

$$\mathrm{Re}_s = \frac{d_b * U_{sb} * \rho_{sl} * (1 - \varepsilon_g)}{\mu_{sl}} \tag{5}$$

and **m** is a function of the Reynolds number of the bubble ($\mathbf{Re_b}$):

$$m = \left(4.45 + 18 * \frac{d_b}{d_c}\right) * \mathrm{Re}_b^{-0.1} \qquad\qquad 1 < \mathrm{Re}_b < 200 \tag{6a}$$

$$m = 4.45 * \mathrm{Re}_b^{-0.1} \qquad\qquad 200 < \mathrm{Re}_b < 500 \tag{6b}$$

$$\mathrm{Re}_b = \frac{U_t * \rho_{sl} * d_b}{\mu_{sl}} \tag{7}$$

The value of **m** in most cases is approximately 3 and Banisi and Finch (1994), and Shah *et al.* (1982) suggest that for ε_g less than 30% the drift flux relationship of Richardson and Zaki (Equation (4)) is the suitable expression for relating slip velocity to terminal velocity $\mathbf{U_t}$:

$$U_{sb} = U_t * (1 - \varepsilon_g)^{m-1} \tag{8}$$

Combining Equations 3 and 8, and rearranging yields:

$$U_t = \frac{\left(\dfrac{J_g}{\varepsilon_g} + \dfrac{J_l}{(1 - \varepsilon_g)}\right)}{(1 - \varepsilon_g)^{m-1}} \tag{9}$$

or

$$U_t = \frac{g * d_b^2 * (\rho_{sl} - \rho_b)}{18 * \mu_{sl} * (1 + 0.15 * \mathrm{Re}_s^{0.687})} \tag{10}$$

Equation (10) is one form of the relationship:

$$U_t = \sqrt{\frac{4 * g * d_b}{3 * C_D}} \qquad (11)$$

where $\mathbf{C_D}$ is the drag coefficient of the gas bubbles. According to Karamanev *et al*. (1992):

$$U_t = \sqrt{\frac{4 * g * d_b}{3 * 0.95}} \qquad (12)$$

There are several ways to resolve these equations and estimate the bubble size (Banisi & Finch, 1994; Xu & Finch, 1990; Dobby *et al.*, 1988; Yianatos *et al.*, 1988). All of the methods involve an iterative procedure.

1.- Assume initial $\mathbf{d_b}$ and $\mathbf{Re_b}$ (typically 0.1 cm and 100, respectively)
2.- Calculate $\mathbf{U_{bt}}$ using Masliyah expression (Equation (10))
3.- Calculate $\mathbf{U_t}$ using $\mathbf{Re_b}$ (Equation (7))
4.- Compare $\mathbf{U_t}$ from step 3 and 4; iterate on $\mathbf{Re_b}$
5.- Calculate $\mathbf{m}$ from Equation (6a) or (6b)
6.- Assume % $\mathbf{\varepsilon_g}$ (10%)
7.- Calculate $\mathbf{U_{sb}}$ in the swarm using Equation (4)
8.- Calculate $\mathbf{Re_{bs}}$ (Equation (5))
9.- Calculate $\mathbf{U_{sb}}$ using Masliyah expression (Equation (10))
10.- Compare $\mathbf{U_{sb}}$ from step 8 and 10; iterate on % $\mathbf{\varepsilon_g}$
11.- Match experimental % $\mathbf{\varepsilon_g}$ iterating on $\mathbf{d_b}$

REFERENCES

[1]. Banisi, S., and Finch, J.A. 1994. Technical Note: Reconciliation of Bubble Size Estimation Methods Using Drift Flux Analysis. Minerals Engineering. Vol. 7, pp 1555-1559.

[2]. Castro Silva M. E. M., Gazzi Salum M. J., and Correa de Araujo A. 1993 Precipitate Flotation of the Removal Heavy Metals and Cyanide from Metallurgycal Effluents. Proceedings of the XVIII International Mineral Processing Congress. pp 1429-1434.

[3]. Davidson, J.F., and Schuler, B.O.G. 1960. Bubble Formation at an Orifice in a Viscous Liquid. Transactions International of Chemical Engineering. Vol. 38, pp 144-154.

[4]. Dobby G. S., Yianatos J. B., and Finch J. A. 1988. Estimation of Bubble Diameter in Flotation Columns from Drift Flux Analysis. Canadian Metallurgical Quarterly. Vol.27, No.2, pp 85-90.

[5]. Drew, T.B.; Cokelet, G.R.; Hoopes, J.W., and Vermeulen, T. 1970. "Advances in Chemical Engineering". Academic Press Editors. Vol. 8, pp 255-368.

[6]. Escudero G. R. 1998. "Characterization of Rigid Spargers and their Selection for Flotation Columns". PhD Thesis. Department of Mining and Metallurgical Engineering. McGill University. Montreal, Canada.

[7]. Finch J. A., and Dobby G. S. "Column Flotation". 1990. Pergamon Press. pp 9-58.

[8]. Geary, N.W., and Rice, R.G. 1991. Bubble Size Prediction for Rigid and Flexible Spargers. AIChE Journal. Vol. 37, No. 2, pp 161-168.

[9]. Gomez C. O., Escudero R., and Finch J. A. 2000. Determining Equivalent Pore Diameter for Rigid Porous Spargers. The Canadian Journal of Chemical Engineering. Vol.78, pp 785-792.

[10]. Gorain B. K., Manlapig E. V., and Franzidis J. P. 1996. The Effect of Gas Dispersion Properties on the Kinetics of Flotation. Proceedings of the Symposium "Column 96". Gomez and Finch Eds. August 26-28. Society of CIM, Montreal, QC, Canada.

[11]. Karamanev D.J. y Nikolov L.N. 1992. Freely Rising Spheres Do Not Obey Newton's Law for Free settling. *A.I.Ch.E. Journal*. Vol. 38, 1992, pp. 1843 – 1846.

[12]. Laplante R.R., Toguri J.M., and Smith H.W. 1983. The Effect of Air Flow Rate on the Kinetics of Flotation, Part I: The Transfer of Material from the Slurry to the Froth. International Journal of Mineral Processing. Vol.11, pp 203-219.

[13]. Masliyah, J.H. 1979. Hindered Settling in a Multi-Species Particle Systems Chemical Engineering Science, Vol.34, pp 1166-1168.

[14]. Richardson, J.F., and W.N. Zaki. 1954. Sedimentation and Fluidization Part I. Transactions Institution of Chemical

Engineering, Vol.32, pp 35-53.

[15]. Rybas V., Volkov V., Ivanov V., Mantsevich M., and Ponomarov G. 1993. Ecological Outlook for the New Technology of the Copper-Nickel Ore Beneficiation Based on the Flotation with Nitrogen. Proceedings of the XVIII International Mineral Processing Congress. pp 997-998.

[16]. Strickland W. T. 1980. Laboratory Results of Cleaning Produced Water by Gas Flotation. Society of Petroleum Engineers Journal. June. pp 175-190.

[17]. Takahashi T., Miyahara T., and Nishizaki Y. 1979. Separation of Oily Water by Bubble Column. Journal of Chemical Engineering of Japan. Vol.12, No.5. pp 394-399.

[18]. Tavera F. J., Escudero R., Finch J. A., and Uribe A. 2000. Flotación de Ni-Deta en Medios Acuosos: Aplicación de Columnas de Flotación. Revista de Química Teórica y Aplicada AFINIDAD. Barcelona, España. Vol. 57, No. 490, pp 415-423.

[19]. Van Ham N. J. M:, Behie L. A., and Svreck W. Y. 1983. The Effect of Air Distribution on the Induced Air Flotation of Fine Oil in Water Emulsions. The Canadian Journal of Chemical Engineering. Vol. 61, August, pp 541-547.

[20]. Watson J. A., Gómez C.O., and Finch J. A. 1996. De-inking of Recycled Parer Using Column Flotation. "Column'96". Proceedings of the International Simposium on Flotation Columns. Montreal, Canada. pp 195-207.

[21]. Wraith, A.E. 1971. Two Stage Bubble Growth at a Submerged Plate Orifice. Chemical Engineering Science. Vol. 26, pp 1659-1671.

[22]. Yianatos J.B., Finch J.A., Dobby G.S., and Xu M. 1988. Bubble Size Estimation in a Bubble Swarm. Journal of Colloid and Interface Science, Vol.126, No.1, pp 37-44.