

# Studying Reliability Evaluation of Multi-State Communication Systems

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**ABSTRACT:** Reliability measures the ability of a system to perform its intended functions. It is one of the most critical performance measures of today's complex systems, such as transportation systems, power systems, and communication systems. This paper studies the Reliability Evaluation of Multi-State Communication Systems. All systems are designed to perform their intended tasks in each environment. Some systems can perform their tasks with various distinctive levels of efficiency usually referred to as performance rates. A system that can have a finite number of performance rates is called a multi-state system (MSS). Usually, MSS is composed of elements that in their turn can be multi-state. A binary system is the simplest case of a MSS having two distinctive states (perfect functioning and complete failure).

The paper places a particular stress on systems used to model communication systems such as the multi-state k-out-of-n system. The reliability metrics studied include reliability per se, its derivatives (for assessment of importance metrics) and its integral (for assessment of MTTF). Out of many approaches for evaluation of reliability for multi-state systems.

**Keywords:** Multi State Systems, Multi-Valued Karnaugh Map, (Binary Decision Diagram), Multiple Decision Diagram, Importance measure, Probability ready expression, Brute Force Solution Exhaustive.

## I. INTRODUCTION

The reliability of a device is defined to be the probability that it will perform its intended functions satisfactorily for a specified period of time under specified operating conditions. Today's engineering systems are sophisticated in design and powerful in function. Examples of such systems include aircrafts, space shuttles, telecommunication networks, robots, and manufacturing facilities.

Reliability is a critical performance measure of these systems, and it has been emphasized more and more by the industry and the

government. The reliability issue exists because of the uncertainties in engineering systems. There are uncertainties in manufacturing processes. There are also uncertainties in the external operating environment and internal operations of an engineering system. Because of these uncertain factors, that cannot be completely controlled or predicted, an engineering system cannot be 100% guaranteed to always perform its intended function. Failures in one form or another are sometimes unavoidable.

All systems are designed to perform their intended tasks in a given environment. Some systems can perform their tasks with various distinctive levels of efficiency usually referred to as performance rates. A system that can have a finite number of performance rates is called a multi-state system (MSS). Usually, MSS is composed of elements that in their turn can be multi-state. Actually, a binary system is the simplest case of a MSS having two distinctive states (perfect functioning and complete failure).

The basic concepts of MSS reliability were primarily introduced in the mid of the 1970's by Murchland (1975), El-Neveih et al. (1978), Barlow and Wu (1978), and Ross (1979). Natvig (1982), Block and Savits (1982), and Hudson and Kapur (1982) extended the results obtained in these works. Since that time MSS reliability began intensive development. Essential achievements that were attained up to the mid 1980's were reflected in Natvig (1985) and in El-Neveih and Prochan (1984) where one can find the state of the art in the field of MSS reliability at this stage.

In practice there are many different situations in which a system should be considered a MSS:

- Any system consisting of different binary-state units that have a cumulative effect on the entire system performance has to be considered a MSS. Indeed, the performance rate of such a system depends on the availability of its units, as the different

numbers of the available units can provide different levels of task performance.

The simplest example of such a situation is the well-known k-out-of-n systems. These systems consist of n identical binary units and can have n+1 states depending on the number of available units. The system performance rates is assumed to be proportional to the number of available units. It is assumed that performance rates corresponding to more than k-1 available units are acceptable. When the contributions of different units to the cumulative system performance rate are different, the number of possible MSS states grows dramatically as different combinations of k available units can provide different performance rates for the entire system.

- The performance rate of elements composing a system can also vary because of their deterioration (fatigue, partial failures) or because of variable ambient conditions. Element failures can lead to the degradation of the entire MSS performance.

In general, the performance rate of any element can range from perfect functioning up to complete failure. The failures that lead to a decrease in the element performance are called partial failures. After partial failure, elements continue to operate at reduced performance rates,

and after complete failure the elements are totally unable to perform their tasks.

## II. EXAMPLES OF MSSS:

- Consider a wireless communication system consisting of transmission stations. The state of each station is defined by the number of subsequent stations covered in its range. This number depends not only on the availability of station amplifiers, but also on the conditions for signal propagation that depend on weather, solar activity, etc.
- In a power supply system consisting of generating and transmitting facilities, each generating unit can function at different levels of capacity. Generating units are complex assemblies of many parts. The failures of different parts may lead to situations in which the generating unit continues to operate, but at a reduced capacity. This can occur during outages of several auxiliaries such as pulverizers, water pumps, fans, etc. For example, Billiton and Allan (1996) describe a three-state 50 MW generating unit. The performance rates (generating capacity) corresponding to these states and probabilities of the states are presented in Table 1.1

Table 1.1. Capacity distribution of 50 MW generator

Number of state (MW)	Generating capacity	(MW) State probability
1	50	0.960
2	30	0.033
3	0	0.007

- Multi-state models are used in medicine (Giard et al. 2002; van den Hout and Matthews 2008; Marshall and Jones 2007; Putter et al. 2007), etc. In (van den Hout and Matthews 2008) is considered a cognitive ability during old age. An illness-death model is presented in order to describe the progression of an illness over time. The model considers three states: the healthy state, an illness state, and the death state. The model is used to derive the probability of a transition from one state to another within a specified time interval.

## III. GENERIC MULTI-STATE SYSTEM MODEL

In order to analyze MSS behavior one has to know the characteristics of its elements. Any system element j can have k<sub>j</sub> different states

corresponding to the performance rates, represented by the set

$$g_i = \{ g_{j1}, g_{j2}, g_{j3}, \dots, g_{jkj} \} \quad (1.1)$$

Where  $g_{ji}$  is the performance rate of element j in the state i,  $i \in \{1, 2, k_j\}$ . The performance rate  $G_j(t)$  of element j at any instant  $t \geq 0$  is a random variable that takes its values from  $g_j : G_j(t) \in g_j$ . Therefore, for the time interval  $[0, T]$ , where T is the MSS operation period, the performance rate of element j is defined as a stochastic process. In some cases, the element performance cannot be measured only by a single value, but by more complex mathematical objects, usually vectors. In these cases, the element performance is defined as a vector stochastic process  $G_j(t)$ . The probabilities associated with the different states (performance rates) of the system

element  $j$  at any instant  $t$  can be represented by the set

$$P_j(t) = \{ P_{j1}(t), P_{j2}(t), \dots, P_{jkj}(t) \} \quad (1.2)$$

Where

$$P_{ji}(t) = \Pr\{G_j(t) = g_{ji}\} \quad (1.3)$$

**Example 1**

Consider a 2-out-of-3 MSS. This system consists of three binary elements with the performance rates  $G_i(t) \in \{g_{i1}, g_{i2}\} = \{0, 1\}$ , for  $i=1,2,3$ , where

$$g_{i1} = \begin{cases} 0, & \text{if element is in a state of complete failure} \\ 1, & \text{if element functions perfectly. } i \end{cases}$$

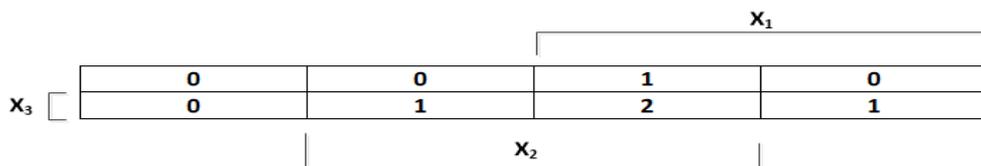
The system output performance rate  $G(t)$  at any instant  $t$  is

$$G(t) = \begin{cases} 0, & \text{if there is more than one failed element} \\ 1, & \text{if there is only one failed element} \\ 2, & \text{if all the elements function perfectly.} \end{cases}$$

The values of the system structure function  $G(t) = f(G_1(t), G_2(t), G_3(t))$  for all the possible system states are presented in Table 1.2

**Table 1.2** Structure function for 2-out of-3 system

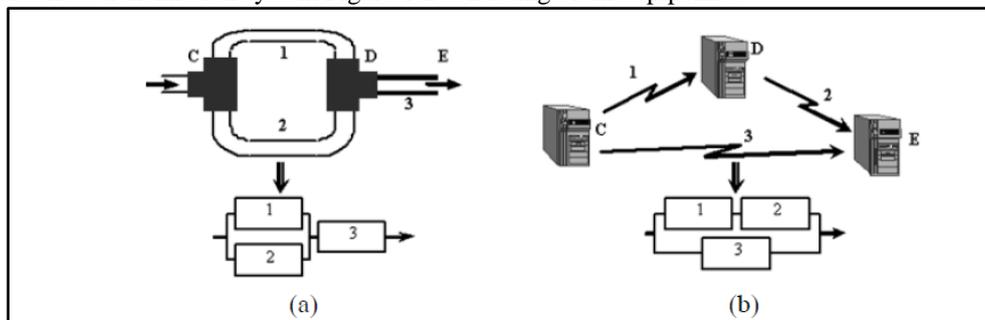
$G_1(t)$	$G_2(t)$	$G_3(t)$	$f(G_1(t), G_2(t), G_3(t))$
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	2



**Figure. 1.1.**Karnaugh Map 2-out of-3 system

**Example 2**

a) Consider a flow transmission system Figure 1.2 consisting of three pipelines



**Figure. 1.2.** Two different MSSs with identical structure functions

The oil flow is transmitted from point C to point E. The pipes' performance is measured by their transmission capacity (ton per minute). Elements 1 and 2 are binary. A state of total failure for both elements corresponds to a transmission

capacity of 0 and the operational state corresponds to the capacities of the elements 1.5 and 2 tons per minute, respectively, so that  $G_1(t) \in \{0, 1.5\}$ ,  $G_2(t) \in \{0, 2\}$ . Element 3 can be in one of three states: a state of total failure corresponding to a

capacity of 0, a state of partial failure corresponding to a capacity of 1.8 tons per minute, and a fully operational state with a capacity of 4 tons per minute so that  $G_3(t) \in \{0, 1.8, 4\}$ . The system output performance rate is defined as the maximum flow that can be transmitted from C to E. The total flow between points C and D through parallel pipes 1 and 2 is equal to the sum of the flows through each of these pipes. The flow from point D to point E is limited by the transmitting

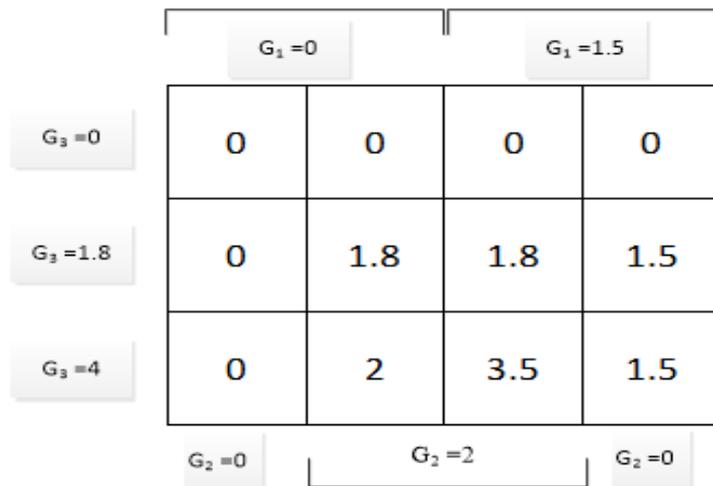
capacity of element 3. On the other hand, this flow cannot be greater than the flow between points C and D. Therefore, the flow between points C and E (the system performance) is

$$G(t) = f(G_1(t), G_2(t), G_3(t)) = \min \{(G_1(t) + G_2(t), G_3(t))\}$$

The values of the system structure function  $G(t) = f(G_1(t), G_2(t), G_3(t))$  for all the possible system states are presented in Table 1.3.

**Table 1.3.** Possible states of oil transmission system

$G_1(t)$	$G_2(t)$	$G_3(t)$	$f(G_1(t), G_2(t), G_3(t))$
0	0	0	0
0	0	1.8	0
0	0	4	0
0	2	0	0
0	2	1.8	1.8
0	2	4	2
1.5	0	0	0
1.5	0	1.8	1.5
1.5	0	4	1.5
1.5	2	0	0
1.5	2	1.8	1.8
1.5	2	4	3.5



**Fig. 1.3.** Karnaugh Map Two different MSSs with identical structure functions.

b) Consider a data transmission system Figure 1.2b consisting of three fully reliable network servers and three data transmission channels (elements). The data can be transmitted from server C to server E through server D or directly. The time of data transmission between the servers depends on the state of the corresponding channel and is the

channel performance rate. This time is measured in seconds.

Elements 1 and 2 are binary. They may be in a state of total failure when data transmission is impossible. In this case data transmission time is formally defined as  $\infty$ . They may also be in a fully operational state when they provide data transmission for 1.5 s and 2 s, respectively:  $G_1(t) \in$

$\{\infty, 1.5\}$ ,  $G_2(t) \in \{\infty, 2\}$ . Element 3 can be in one of three states: a state of total failure, a state of partial failure with data transmission for 4 s, and a fully operational state with data transmission for 1.8 s:  $G_3(t) \in \{\infty, 4, 1.8\}$ . The system performance rate is defined as the total time the data can be transmitted from server A to server C. When the data is transmitted through server D, the total time of transmission is equal to the sum of times  $G_1(t)$  and  $G_2(t)$  it takes to transmit them from server C to server D and from server D to server E, respectively. If either element 1 or 2 is in a state of total failure, data transmission through server D is impossible. For this case we formally state that  $(\infty+2) = \infty$  and  $(\infty+1.5) = \infty$ . When the data are transmitted from server C to server E directly, the transmission time is  $G_3(t)$ . The minimum time needed to transmit the data from C to E directly or through D determines the system transmission time. Therefore, the MSS structure function takes the form.

$$G(t) = f(G_1(t), G_2(t), G_3(t)) = \min \{(G_1(t) + G_2(t), G_3(t))\}$$

Note that the different technical systems in this Example a and b, even when they have different reliability block diagrams (Figures 2a and 2b), correspond to the identical MSS structure functions.

#### IV. TYPES OF MULTI-STATE SYSTEM

According to the generic model (1) and (2), one can define different types of MSS by determining the performance distribution of its elements and defining the system's structure function. It is possible to invent an infinite number of different structure functions to obtain different models of MSS. The question is whether the MSS model can be applied to real technical systems. This section presents different application inspired MSS models that are most used in reliability engineering.

##### a) Series Structure

The series connection of system elements represents a case where a total failure of any individual element causes an overall system failure. In the binary system the series connection has a purely logical sense. The topology of the physical connections among elements represented by a series reliability block diagram can differ, as can their allocation along the system's functioning process. The essential property of the binary series system is that it can operate only when all its elements are fully available.

When MSS is considered and the system performance characteristics are of interest, the series

connection usually has a "more physical" sense. Indeed, assuming that MSS elements are connected in a series means that some processes proceed stage by stage along a line of elements. The process intensity depends on the performance rates of the elements. Observe that the MSS definition of the series connection should preserve its main property: the total failure of any element (corresponding to its performance rate equal to zero) causes the total failure of the entire system (system performance rate equal to zero).

One can distinguish several types of series MSS, depending on performance and the physical nature of the interconnection among the elements.

First, consider a system that uses the capacity (productivity or throughput) of its elements as the performance measure. The operation of these systems is associated with some media flow continuously passing through the elements.

Examples of these types of system are power systems, energy or materials continuous transmission systems, continuous production systems, etc. The element with the minimal transmission capacity becomes the bottleneck of the system. Therefore, the system capacity is equal to the capacity of its "weakest" element. If the capacity of this element is equal to zero (total failure), then the entire system capacity is also zero.

##### b) Parallel Structure

The parallel connection of system elements represents a case where a system fails if and only if all of its elements fail. Two basic models of parallel systems are distinguished in binary reliability analysis. The first one is based on the assumption that all of the elements are active and work sharing. The second one represents a situation where only one element is operating at a time (active or standby redundancy without work sharing).

A MSS with a parallel structure inherits the essential property of the binary parallel system so that the total failure of the entire system occurs only when all of its elements are in total failure states. The assumption that MSS elements are connected in parallel means that some tasks can be performed by any one of the elements. The intensity of the task accomplishment depends on the performance rate of available elements.

For a MSS with work sharing, the entire system performance rate is usually equal to the sum of the performance rates of the parallel elements for both flow transmission and task processing systems. Indeed, the total flow through the former type of system is equal to the sum of flows through its parallel elements. In the latter type of

MSS, the system processing speed depends on the rules of the work sharing. The most effective rule providing the minimal possible time of work completion shares the work among the elements in proportion to their processing speed. In this case, the processing speed of the parallel system is equal to the sum of the processing speeds of all of the elements.

### Example 3

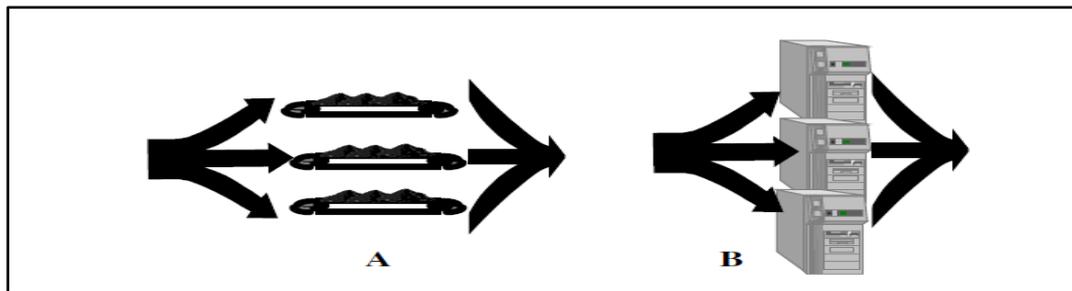


Figure 1.4. Examples of parallel systems with work sharing.

(A: flow transmission system; B: task processing system)

In a MSS without work sharing the system performance rate depends on the discipline of the elements' activation. Unlike binary systems, where all the elements have the same performance rate, the choice of an active element from these of different ones affects the MSS performance. The most common policy in both flow transmission and task processing MSSs is to use an available element with the greatest possible performance rate. In this case, the system performance rate is equal to the maximal performance rate of the available parallel elements.

### c) k-out-of-n Structure

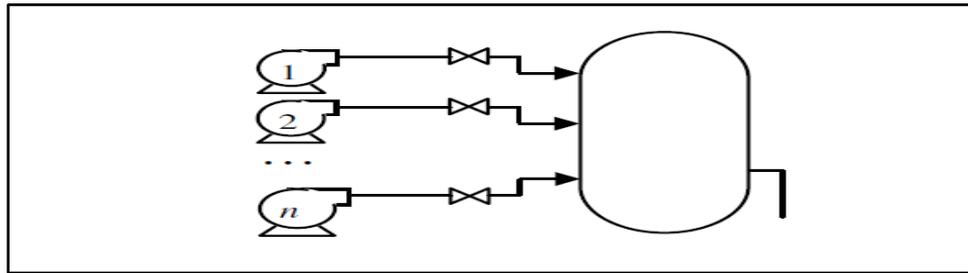
The parallel MSS is not only a multi-state extension of the binary parallel structure, but it is also an extension of the binary k-out-of-n system. Indeed, the k-out-of-n system reliability is defined as a probability that at least k elements out of n are in operable condition (note that  $k = n$  corresponds to the binary series system and  $k = 1$  corresponds to the binary parallel one). The reliability of the parallel MSS with work sharing is defined as the probability that the sum of the elements' performance rates is not less than the demand. Assuming that the parallel MSS consists of n identical two-state elements having a capacity of 0 in a failure state and a capacity of 1 in an operational state and that the system demand is equal to k, one obtains the binary k-out-of-n system.

Consider a system of several parallel coal conveyors supplying the same system of boilers (Figure 1.4A) or a multi-processor control unit (Figure 1.4B), assuming that the performance rates of the elements in both systems can vary. In the first case the amount of coal supplied is equal to the sum of the amounts supplied by each one of the conveyors. In the second case the unit processing speed is equal to the sum of the processing speeds of all of its processors.

The first generalization k-out-of-n system to the multi-state case was suggested by Singh. His model corresponds to the parallel flow transmission MSS with work sharing. Rushdi and Wu and Chen in suggested models in which the system elements have two states but can have different values of nominal performance rate. A review of the multi-state k-out-of-n models. Huang et al. suggested a multi-state generalization of the binary k-out-of-n model that cannot be considered as a parallel MSS. In this model, the entire system is in state j or above if at least  $k_j$  multi-state elements are in state  $m(j)$  or above.

### Example 4

Consider a chemical reactor to which reagents are supplied by n interchangeable feeding subsystems consisting of pipes, valves, and pumps (Figure 1.5). Each feeding subsystem can provide a supply of the reagents under pressure depending on the technical state of the subsystem. Different technological processes require different numbers of reagents and different pressures. The system's state is determined by its ability to perform certain technological processes. For example, the first process requires a supply of  $k_1 = 3$  reagents under pressure level  $m(1) = 1$ , the second process requires a supply of  $k_2 = 2$  reagents under pressure level  $m(2) = 2$ , etc.



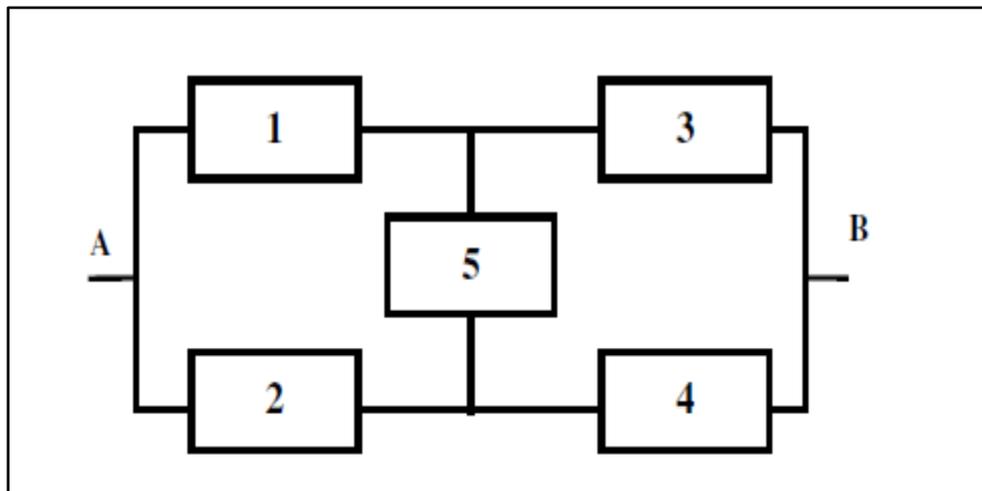
**Figure 1.5.** Example of multi-state k-out-of-n system that can be reduced to a parallel one

This multi-state model can be easily reduced to a set of binary k-out-of-n models. Indeed, for each system state  $j$ , every multi-state element  $i$  having the random performance  $G_i$  can be replaced with a binary element characterized by the binary state variable  $X_i = 1(G_i \geq m(j))$  and the entire system can be considered as  $k_j$ -out-of- $n$ .

**d) Bridge Structure**

Many reliability configurations cannot be reduced to a combination of series and parallel

structures. The simplest and most commonly used example of such a configuration is a bridge structure (Figure 1.6). It is assumed that elements 1, 2 and 3, 4 of the bridge are elements of the same functionality separated from each other by some reason. The bridge structure is spread in spatially dispersed technical systems and in systems with vulnerable components separated to increase the entire system survivability. When the entire structure performance rate is of interest, it should be considered as a MSS.



**Figure 1.6.** Bridge structure

**e) Systems with Two Failure Modes**

Systems with two failure modes consist of devices that can fail in either of two different modes. For example, switching systems not only can fail to close when commanded to close, but they can also fail to open when commanded to open. Typical examples of a switching device with two failure modes are a fluid flow valve and an electronic diode. The binary reliability analysis considers only the reliability characteristics of elements composing the system. In many practical cases, measures of element (system) performance must be taken into account. For example, fluid-transmitting capacity is an important characteristic of a system containing fluid valves (flow

transmission system), while operating time is crucial when a system of electronic switches (task processing system) is considered. The entire system with two failure modes can have different levels of output performance in both modes depending on the states of its elements at any given moment. Therefore, the system should be considered to be multi-state.

When applied to a MSS with two failure modes, reliability is usually considered to be a measure of the ability of a system to meet the demand in each mode (note that demands for the open and closed modes are different). If the probabilities of failures in open and closed modes are respectively  $Q_0$  and  $Q_c$  and the probabilities of

both modes are equal to 0.5, then the entire system reliability can be defined as  $R = 1 - 0.5(Q_0 + Q_c)$ , since the failures in open and closed modes are mutually exclusive events.

An important property of systems with two failure modes is that redundancy, introduced into a system without any change in the reliability of the individual devices, may either increase or decrease the entire system's reliability.

#### f) Weighted Voting Systems

Voting is widely used in human organizational systems, as well as in technical decision making systems. The use of voting for obtaining highly reliable data from multiple unreliable versions was first suggested in the mid-1950s by von Neumann. Since then the concept has been extended in many ways.

A voting system makes a decision about propositions based on the decisions of  $n$  independent individual voting units. The voting units can differ in the hardware or software used and/or by available information. Each proposition is a priori right or wrong, but this information is available for the units in implicit form. Therefore, the units are subject to the following three errors:

- Acceptance of a proposition that should be rejected (fault of being too optimistic).
- Rejection of a proposition that should be accepted (fault of being too pessimistic).
- Abstaining from voting (fault of being indecisive).

This can be modelled by considering the system input being either 1 (proposition to be accepted) or 0 (proposition to be rejected), which is supplied to each unit. Each unit  $j$  produces its decision (unit output), which can be 1, 0, or  $x$  (in the case of abstention). The decision made by the unit is wrong if it is not equal to the input. The errors listed above occur when:

- the input is 0, the decision is 1,
- the input is 1, the decision is 0,
- the decision is  $x$  without regard to the input.

Accordingly, the reliability of each individual voting unit can be characterized by the probabilities of its errors.

To make a decision about proposition acceptance, the system incorporates all unit decisions into a unanimous system output which is equal to  $x$  if all the voting units abstain, equal to 1 if at least  $k$  units produce decision 1, and otherwise equal to 0 (in the most commonly used majority voting systems  $k = n/2$ ).

Note that the voting system can be considered as a special case of a  $k$ -out-of- $n$  system with two failure modes. Indeed, if in both modes

(corresponding to two possible inputs) at least  $k$  units out of  $n$  produce a correct decision, then the system also produces the correct decision. (Unlike the  $k$ -out-of- $n$  system, the voting system can also abstain from voting, but the probability of this event can easily be evaluated as a product of the abstention probabilities of all units.)

Since the system output (number of 1-opting units) can vary, the voting systems can also be considered as the simplest case of an MSS.

A generalization of the voting system is a weighted voting system where each unit has its own individual weight expressing its relative importance within the system. The system output is  $x$  if all the units abstain. It is 1 if the cumulative weight of all 1-opting units is at least a prespecified fraction  $\tau$  of the cumulative weight of all non-abstaining units. Otherwise the system output is 0.

Observe that the multi-state parallel system with two failure modes is a special case of the weighted voting system in which voting units never abstain. Indeed, in both modes (corresponding to two possible inputs) the total weight (performance) of units producing a correct decision should exceed some value (demand) determined by the system threshold. The weighted voting systems have been suggested by Gifford for maintaining the consistency and the reliability of the data stored with replication in distributed computer systems. The applications of these systems can be found in imprecise data handling, safety monitoring and self-testing, multi-channel signal processing, pattern recognition, and target detection.

#### g) Multi-state Sliding Window Systems

The sliding window system model is a multi-state generalization of the binary consecutive  $k$ -out-of- $r$ -from- $n$  system, which has  $n$  ordered elements and fails if at least  $k$  out of any  $r$  consecutive elements fail. In this generalized model, the system consists of  $n$  linearly ordered multi-state elements. Each element can have a number of different states: from complete failure to perfect functioning.

A performance rate is associated with each state. The system fails if an acceptability function of performance rates of any  $r$  consecutive elements is equal to zero. Usually, the acceptability function is formulated in such a manner that the system fails if the sum of the performance rates of any  $r$  consecutive elements is lower than the demand  $w$ . The special case of such a sliding window system in which all the  $n$  elements are identical and have two states with performance rates 0 and 1 is a  $k$ -out-of- $r$ -from- $n$  system where  $w = r - k + 1$ .

As an example of the multi-state sliding window system, consider a conveyor typeservice system that can process incoming tasks simultaneously according to a first-in-first-out rule and share a common limited resource. Each incoming task can have different states and the amount of the resource needed to process the task is different for each state of each task. The total resource needed to process  $r$  consecutive tasks should not exceed the available amount of the resource. The system fails if there is no available resource to process  $r$  tasks simultaneously.

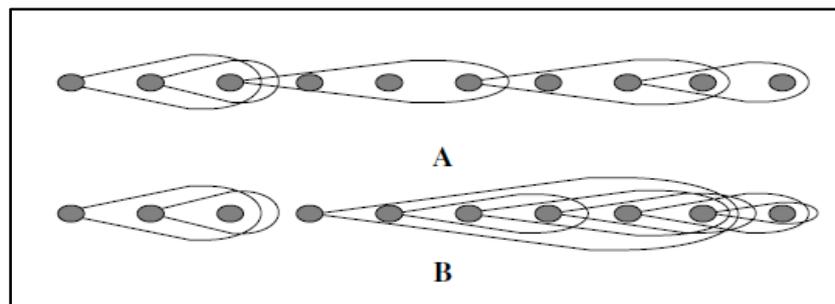
#### h) Multi-state Consecutively Connected Systems

A linear consecutively connected system is a multi-state generalization of the binary linear consecutive  $k$ -out-of- $n$  system that has  $n$  ordered elements and fails if at least  $k$  consecutive elements fail. In the multi-state model, the elements have different states, and when an element is in state  $i$  it is able to provide connection with  $i$  following elements ( $i$  elements following the one are assumed to be within its range). The linear multi-state consecutively connected system fails if its first and last elements are not connected (no path exists between these elements).

The first generalization of the binary consecutive  $k$ -out-of- $n$  system was suggested by Shanthikumar. In his model, all of the elements can have two states, but in the working state different elements provide connection with different numbers of following elements. The multi-state generalization of the consecutive  $k$ -out-of- $n$  system was first suggested by Hwang and Yao. Algorithms for linear multi-state consecutive  $k$ -out-of- $n$  system reliability evaluation were developed by Hwang & Yao, Kossow and Preuss, Zuo and Liang, and Levitin.

#### Example 5

Consider a set of radio relay stations with a transmitter allocated at the first station and a receiver allocated at the last station (Figure 1.7). Each station  $j$  has transmitters generating signals that reach the next  $k_j$  stations. A more realistic model should take into account differences in the retransmitting equipment for each station, different distances between the stations, and the varying weather conditions. Therefore,  $k_j$  should be considered to be a random value dependent on the power and availability of retransmitted amplifiers as well as on the signal propagation conditions. The aim of the system is to provide propagation of a signal from transmitter to receiver.



**Figure 1.7.** Linear consecutively connected MSS in states of successful functioning (A) and failure (B)

A circular consecutively connected system is a multi-state generalization of the binary circular consecutive  $k$ -out-of- $n$  system. As in the linear system, each element can provide a connection to a different number of the following elements ( $n$ th element is followed by the first one). The system functions if at least one path exists that connects

any pair of its elements; otherwise there is a system failure (Figure 1.8). Malinowski and Preuss have shown that the problem of reliability evaluation for a circular consecutively connected system can be reduced to a set of problems of reliability evaluation for linear systems.

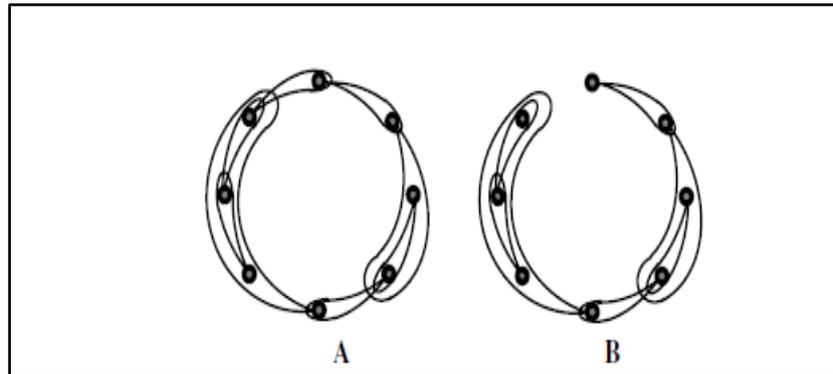


Figure 1.8. Circular consecutively connected MSS in states of successful functioning (A) and failure (B)

**i) Multi-state Networks**

Networks are systems consisting of a set of vertices (nodes) and a set of edges that connect these vertices. Undirected and directed networks exist. While in the undirected network the edges merely connect the vertices without any consideration for direction, in the directed network the edges are ordered pairs of vertices. That is, each edge can be followed from one vertex to the next.

An acyclic network is a network in which no path (a list of vertices where each vertex has an edge from it to the next one) starts and ends at the same vertex. The directed networks considered in reliability engineering are usually acyclic. The networks often have a single root node (source) and

one or several terminal nodes (sinks). Examples of directed acyclic networks are presented in Figure 1.9 A and B. The aim of the networks is the transmission of information or material flow from the source to the sinks. The transmission is possible only along the edges that are associated with the transmission media (lines, pipes, channels, etc.). The nodes are associated with communication centers (retransmitters, commutation, or processing stations, etc.) The special case of the acyclic network is a three-structured network in which only a single path from the root node to any other node exists (Figure 1.9 C). The three-structured network with a single terminal node is the linear consecutively connected system.

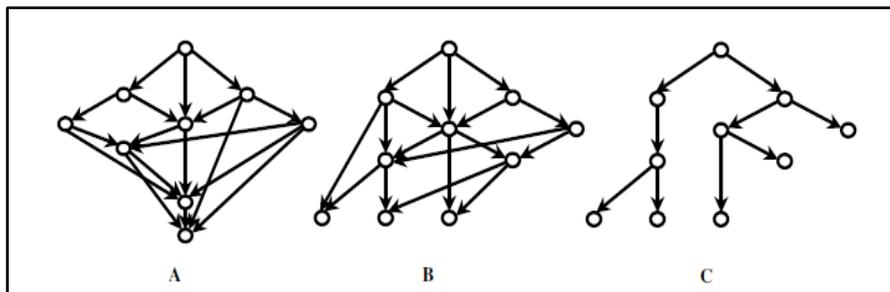


Figure 1.9. Examples of acyclic networks: A: a network with single terminal node, B: a network with several terminal nodes, C: a tree-structured network

Each network element can have its transmission characteristic, such as transmission capacity or transmission speed. The transmission process intensity depends on the transmission characteristics of the network elements and on the probabilistic properties of these elements. The most commonly used measures of the entire network performance are:

- The maximal flow between its source and sink (this measure characterizes the maximal amount of material or information that can be transmitted from the source

to the sink through all of the network edges simultaneously).

- The flow of the single maximal flow path between the source and the sink (this measure characterizes the maximal amount of indivisible material or information that can be transmitted through the network by choosing a single path from the source to the sink).
- The time of transmission between the source and the sink (this measure characterizes the delivery delay in networks having edges and/or vertices with limited transmission speed).

In binary stochastic network theory, the network elements (usually edges) have a fixed level of the transmission characteristic in its working state and limited availability. The problem is to evaluate the probability that the sinks are connected to the source or the probable distribution of the network performance. There are several possible ways to extend the binary stochastic network model to the multistate case.

In the multi-state edges models, the vertices are assumed fully reliable and edge transmission characteristics are random variables with a given distribution. The models correspond to:

- Communication systems with spatially distributed fully reliable stations and channels affected by environmental conditions or based on deteriorating equipment.

- Transportation systems in which the transmission delays are a function of the traffic.

In the multi-state vertices models, the edges are assumed fully reliable and the vertices are multi-state elements. Each vertex state can be associated with a certain delay, which corresponds to:

- Discrete production systems in which the vertices correspond to machines with variable productivity.

- Digital communication networks with transmitters characterized by variable processing time.

These networks can be considered as an extension of task processing series-parallel reliability models to the case of the network structure.

The vertex states can also be associated with transmitting capacity, which corresponds to:

- Power delivery systems where vertices correspond to transformation substations with variable availability of equipment and edges to represent transmission lines.

- Continuous production systems in which vertices correspond to product processing units with variable capacity and edges represent the sequence of technological operations.

These networks can be considered as an extension of simple capacity-based series-parallel reliability models in the case of network structure (note that networks in which the maximal flow between its source and sink and the single maximal flow path between the source and the sink are of interest extend the series-parallel model with work sharing and without work sharing respectively).

In some models, each vertex state is determined by a set of vertices connected to the given one by edges. Such random connectivity models correspond mainly to wireless communication systems with spatially dispersed stations. Each station has transmitters generating signals that can reach a set of the next stations.

Note that the set composition for each station depends on the power and availability of the retransmitted amplifiers as well as on variable signal propagation conditions. The aim of the system is to provide propagation of a signal from an initial transmitter to receivers allocated at terminal vertices. (Note that it is not necessary for a signal to reach all the network vertices in order to provide its propagation to the terminal ones). This model can be considered as an extension of the multi-state linear consecutively connected systems in the case of the network structure.

The last model is generalized by assuming that the vertices can provide a connection to a random set of neighboring vertices and can have random transmission characteristics (capacity or delay) at the same time.

In the most general mixed multi-state models, both the edges and the vertices are multi-state elements. For example, in computer networks the information transmission time depends on the time of signal processing in the node computers and the signal transmission time between the computers (depending on transmission protocol and channel loading).

The earliest studies devoted to the multi-state network reliability were by Doulliez, and Jamouille, Evans and Somers. These models were intensively studied by Alexopoulos and Fishman, Lin, Levitin, Yeh. The three-structured networks were studied by Malinowski and Preuss.

#### j) Fault-tolerant Software Systems

Software failures are caused by errors made in various phases of program development. When the software reliability is of critical importance, special programming techniques are used in order to achieve its fault tolerance. Two of the best-known fault-tolerant software design methods are n-version programming (NVP) and recovery block scheme (RBS). Both methods are based on the redundancy of software modules (functionally equivalent but independently developed) and the assumption that coincidental failures of modules are rare. The fault tolerance usually requires additional resources and results in performance penalties (particularly with regard to computation time), which constitutes a tradeoff between software performance and reliability.

The NVP approach presumes the execution of n functionally equivalent software modules (called versions) that receive the same input and send their outputs to a voter, which is aimed at determining the system output. The voter produces an output if at least k out of n

outputs agree (it is presumed that the probability that  $k$  wrong outputs agree is negligibly small). Otherwise, the system fails. Usually, majority voting is used in which  $n$  is odd and  $k = (n+1)/2$ .

In some applications, the available computational resources do not allow all of the versions to be executed simultaneously. In these cases, the versions are executed according to some predefined sequence and the program execution terminates either when  $k$  versions produce the same output (success) or after the execution of all the  $n$  versions when the number of equivalent outputs is less than  $k$  (failure). The entire program execution time is a random variable depending on the parameters of the versions (execution time and reliability), and on the number of versions that can be executed simultaneously.

## V. RELIABILITY OF OPTICAL FIBERS, CABLES, AND SPLICES

An optical fiber cable, also known as a fiber optic cable, is an assembly similar to an electrical cable, but containing one or more optical fibers that are used to carry light. The optical fiber elements are typically coated individually with plastic layers and contained in a protective tube suitable for the environment where the cable will be deployed. Different types of cable are used for different applications, for example long distance telecommunication, or providing a high-speed data connection between different parts of a building. Fiber design and transmission technology have collaboratively evolved to increase bandwidth

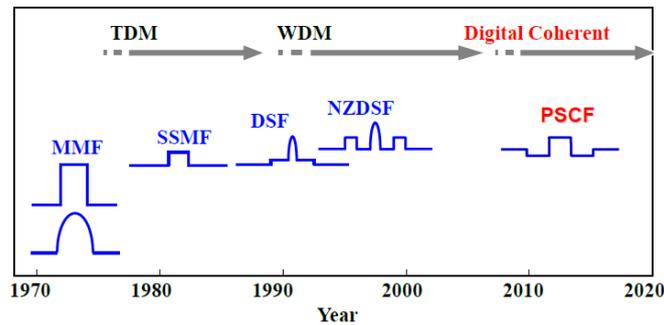


Figure 1.10: shows the evolution demand for bandwidth increasing

There are two types of fibers available on the market today; they are called single mode (SM) and multimode (MM). Single mode is an optical wavelength in which light travels in one mode. In multimode it travels in multiple modes. The single mode has a higher bandwidth than the multimode, thereby allowing larger amounts of data to be sent and over longer distances. The fiber core used in single mode fibers has a very small diameter, usually only 8-10  $\mu\text{m}$  and is made of glass. Laser

light is used to counteract the chromatic dispersion that occurs due to the small core in the single mode fibers, if laser light is not used it would not be possible to send data over long distances. Two of the more common single mode fibers are called OS1 and OS2, the most common sizes are 8/125 or 10/125. The first number represents the diameter of the core in  $\mu\text{m}$  and the second number represents the diameter of the cladding in  $\mu\text{m}$

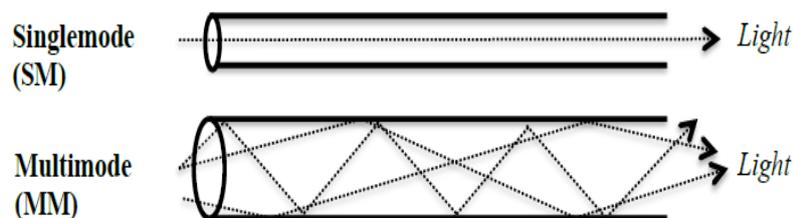


Figure 1.11. Illustration of light travelling through SM and MM fibers,

### 5.1 Introduction to Reliability

Reliability of a product (component, device, system or a chain of network components and devices) is defined as the probability that the product will meet a set of specified properties for a given period of time in service. Very high reliability, i.e. high survivability, is demanded for telecommunications and other communications networks. Thus a relatively low failure probability, such as  $10^{-3}$ -  $10^{-5}$ , for 25 - 40 years lifetime is required for communications networks components. In order to estimate the failure probability of a component for 25 - 40 years' service, it is required that it is known, which type of the failure mechanisms of the component is the dominant failure mechanism during usage (in service). It is also required to be known by which mathematical equation the long term failure rate during service (usage) can be estimated from the test data of the component for this dominant failure mechanism, or for a known combination of several competing failure mechanisms.

It can be mentioned, that average lifetime for the whole population of installed component is in many cases tremendously longer than the lifetime required at the low failure probability for the first failures, and therefore average lifetime may not be considered as a reliability issue.

For example for optical fibers in cables it is assumed that the fractures of fibers caused by stress corrosion at the weak flaws of the fibers under a low static stress is the dominant failure mechanism. The estimated lifetime or failure rate is calculated by using the weak flaw distribution data surviving the proof test and the parameters of the mechanical behavior of the fibers. Furthermore, a slow crack growth leading to the fractures, following a single power law theory (alternatively two-region power law theory or exponential theory) is assumed to occur at the weak: flaws. This kind of consideration must be done for both fibers inside the cables and for the fiber in the splice boxes. Some of the weak spots may, in practice, locate in the splice boxes caused by the defects on the fiber during splicing procedure. The parameters of reliability are defined and characterized, in general, for all communications network components, including optical fibers, cables, passive and active optical components and devices by using the following functions

### 5.2 Reliability Function, Survival Probability

A reliability function, also called a survival function or survival probability

$$S(t) = \frac{n(t)}{n(0)} \quad (1.4)$$

Where  $n(0)$  is the original population and  $n(t)$  is the surviving population, is a mathematical equation, which describes the probability of surviving until time  $t$ , i.e. a function of the population expected to survive until time  $t$ .

### 5.3 Failure Function, Failure Probability

Failure function, also called as unreliability function or cumulative distribution function (CDF)

$$F(t) = 1 - S(t) = 1 - \frac{n(t)}{n(0)} \quad (1.5)$$

where  $S(t)$  is the survival probability. This equation describes the probability of failing before time  $t$ , i.e. the fraction of the population expected to fail before time.

### Probability Density Function (Failure Probability per unit Time)

Probability density function (PDF)  $f(t)$  describes the probability of failure per unit time at time  $t$  for any member of the original population  $n(0)$

$$f(t) = \frac{dF(t)}{dt} = -\frac{dS(t)}{dt} = -\frac{1}{n(0)} \frac{dn(t)}{dt} \quad (1.6)$$

where  $F(t)$  is the failure function as defined above.

### Failure Rate

Failure rate  $\lambda(t)$ , also called as hazard rate, failure intensity, force of mortality and instantaneous failure rate, describes the probability of failure per unit time at time  $t$ , for the members of the original population which survived until time  $t$ .

$$\lambda(t) = -\frac{1}{S(t)} \frac{dS(t)}{dt} = \frac{f(t)}{S(t)} = -\frac{1}{n(t)} \frac{dn(t)}{dt} \quad (1.7)$$

where  $S(t)$ ,  $f(t)$  and other parameters are defined as given above. The units used for failure rate are: %/time unit and FITs. The probability of failure  $f(t)dt$  is the instantaneous failure probability during a very short time period from  $t$  to  $t+dt$ , but  $\lambda(t)d(t)$  is the probability of failure during a longer time after  $t$ , from  $t$  to  $2t$  or more.

The failure rate in FITs can be calculated using equation

$$\lambda = \frac{\Delta n}{n(0) \Delta t [\text{h}]} \quad (1.8)$$

If now  $\frac{\Delta n}{n(0)} = 10^{-2} = 1\%$  and  $\Delta t = 10^4 \text{ h} = 1 \text{ year}$ , then  $\lambda = 10^{-6} / \text{h}$ , which is an inconvenient dimension. Therefore if we calculate the same by using the time unit of Gig hours, so failure rate in FITs can be calculated

$$\lambda[\text{FITs}] = \frac{\Delta n}{n(0)} \frac{1}{\Delta t[\text{Gh}]} = \frac{\Delta n}{n(0)} \frac{10^9}{\Delta t[\text{h}]} \quad (1.9)$$

For the above given example failure rate  $\lambda = 1000 \text{ FITs}$ , which corresponds to 1 % of a population fails in about 1 year. If the reliability requirement for a component type is defined so that failure probability  $F \leq 10^{-3}$  is required for 30 years, the allowed maximum failure rate  $\lambda = 4.6 \text{ FITs}$ .

The reliability requirements vary depending on country, operating company and application. For example the national reliability requirement for optical fibers in telecommunications cables including the splices of the fibers, in Sweden is defined: the allowed failure probability  $F \leq 10^{-3}$  for 100 km fibers for 40 years lifetime. This equals to the failure rate requirement: less than 1 failure/100 000 km fiber is allowed during 40 years lifetime. Thus the maximum allowed failure rate is 0.029 FITs/km.

Failure rate  $\lambda(t)$  as a function of time (usually looks like a bath tub curve) is a sum of the failure rate functions of infant mortality rate (which decreases as a function of usage time) and wear-out failure rates of failure mechanisms (which increase as a function of time). In addition, there might be failures due to accidents and natural catastrophes and so called freak failures, which are caused by temporary manufacturing process mistakes or other odd reasons which are not statistically enough frequent to be obtained at any tests.

### Lifetime

The lifetime of optical fiber, cable, active or passive component etc. is the period of usage (service) time, from the installation to the point the allowed highest failure rate (or fracture probability) is reached. This means that the lifetime is defined for a large population of installed components in service, not for single components.

Because usually a very low fracture probability is required and distribution of the lifetimes within population varies within a huge range, the lifetime of an individual component on average is much longer than specified and it is also very different between individual components.

### 5.4 Statistical Methods and Parameters

Usually there is a large variation of the weakness magnitudes/effect in a component population. It is even possible that there are several different variation dimensions in the weakness population. For example in fibers we can find at least three types of distributions: a flaw size distribution, a flaw distance/location frequency distribution, a flaw type distribution and a variation along fiber length. Thus it is important to measure the statistical parameters of the distribution of the weaknesses, because the final failure time distribution is dependent on, a function of, the original weakness distribution. Usually some statistical analysis methods are used for both. It can be the same type of equation or completely different statistical equation. The original weakness population can be normally, or Weibull or exponentially or log-normally distributed. In addition, the failure time distribution itself in service may also show a normal, exponential, log-normal or Weibull distribution with another set of parameters. Usually the lifetime of passive and active components, such as light bulbs or lasers, can be estimated from a single statistical data of service failure time distribution. But for optical fibers a two level analysis is required: a weak spot distribution which is a Weibull distribution (including both flaw size, flaw frequency and length variation), and the final fracture time distribution under static service stress according to a power law. For the case where fibers are proof tested, the modified weak flaw distribution is needed.

### Time to Failure and Between Failures

Furthermore, two parameters, the median time to failure (MTTF) and mean time between failures (MTBF), are used to describe the lifetime distribution. In addition, the variation of failure rate may be given as a standard deviation and variance (symmetric distributions) or by other distribution parameters (such as slope for a Weibull type of failure distribution) or as confidence intervals.

The Median Time To Failure (MTTF) is the time from the moment of installation to the point when 50 % of the component population have failed. This time is usually much longer than the lifetime defined at a low failure probability.

The Mean Time Between Failures (MTBF) is the average time between failures. In order to calculate this, the distribution of failure times must be known. If the failure rate is a constant the mean time between failures is the inverse of the failure rate. MTBF parameter is used

for low failure rate components with a huge range of failure times, such as fibers in cables.

In most cases of electronic component failures occurring after a certain time during a certain period, and the failure time distribution is not symmetric. In these cases the failure time distribution must be known in order to estimate the MTTF and other failure time distribution parameters.

### Mean Time to Repair

This parameter, the mean time to repair (MTTR) is estimated from the field failure repair times including the measurement in order to localize the failed component. At least some kind of experience of the repair and maintenance processes is needed. Usually, a complete device or a component circuit board is exchanged at failure. For cable failures the time to repair can be significantly longer.

### Network Reliability

Reliability of a network is dependent on the individual failures of system and exchange devices, cable and component network, power suppliers, computers and software. In this work we considered only the reliability of optical passive components, including fibers cables and outside plant components and fiber amplifiers.

In general, telecommunications networks are built using parallel and series chains of components, and are to some extent backed up with alternative routes of chains of components. However, the extent to which the security is built into backup systems depends on the operator. The network reliability, the probability of failure outside of the specified performance within a lifetime, is a function of for example a combination of sums and/or products of the component failure probabilities.

## 5.5 Standards for Optical Fiber, Component and Network Reliability

### Background

Photonic components have now been installed in commercial fiber optic systems for over 20 years. In the simplest early systems, reliability concentrated around the opto-electronic transmitter, which was a light-emitting diode or a laser diode. These were semiconductor components directly modulated with electric

current, with performance and reliability that was very sensitive to wavelength and to a number of environmental factors, such as temperature. The receiver, which utilized a PIN photodiode or avalanche photodiode, was of less

concern. The fiber was protectively contained within a cable, and sections were joined by splices or connectors. In this scenario, the reliability aspects were rather limited and understood to a degree.

Recently, the optical fiber and some of the optoelectronics are coming closer to the customer, where temperature, humidity, and chemical interaction are less controlled than in a central office or headend. With more frequent rearrangements to provide service flexibility, there is more handling by craft personnel. Both these aspects magnify reliability concerns.

As system architectures became more complicated, so did other active and passive components. Branching components were introduced, for example as optical splitters that took the light from one fiber and distributed it amongst several output fibers or operated as optical combines in the reverse direction. Wavelength selective splitters/combiners, termed optical multiplexers/ demultiplexers or wavelength division multiplexers/demultiplexers (WDMs), were recently introduced to increase capacity by sending several wavelength channels of information on one fiber. In the future they will be used in optical networking to switch and route wavelength channels. There are several technologies that perform these functions, and the reliability of these passive components is only beginning to be studied.

Optical power budgets were becoming strained by several factors: longer lengths (and attenuation) between regenerators, higher bit-rates (which lead to lower receiver sensitivity), and the optical power loss due to the complex components above. Fortunately, this problem was addressed by the optical amplifier. The optical fiber amplifier (OFA) contains an active fiber pumped by a diode laser, along with a WDM, an optical filter at the output, isolators (which allow light to pass in only one direction) at the input and output, and possibly other components. The OFA is a complex subsystem of active and passive components, each with its own reliability concerns. Other active components have become more complex. With higher bit-rates, the optical source is often externally modulated, either as a separate component, or as part of the same opto-electronic chip. With WDM, sources and detectors are being produced in the laboratory that have simultaneous generation/detection of several wavelengths on a single chip.

The semiconductor optical amplifier (SOA), which in some ways resembles a laser

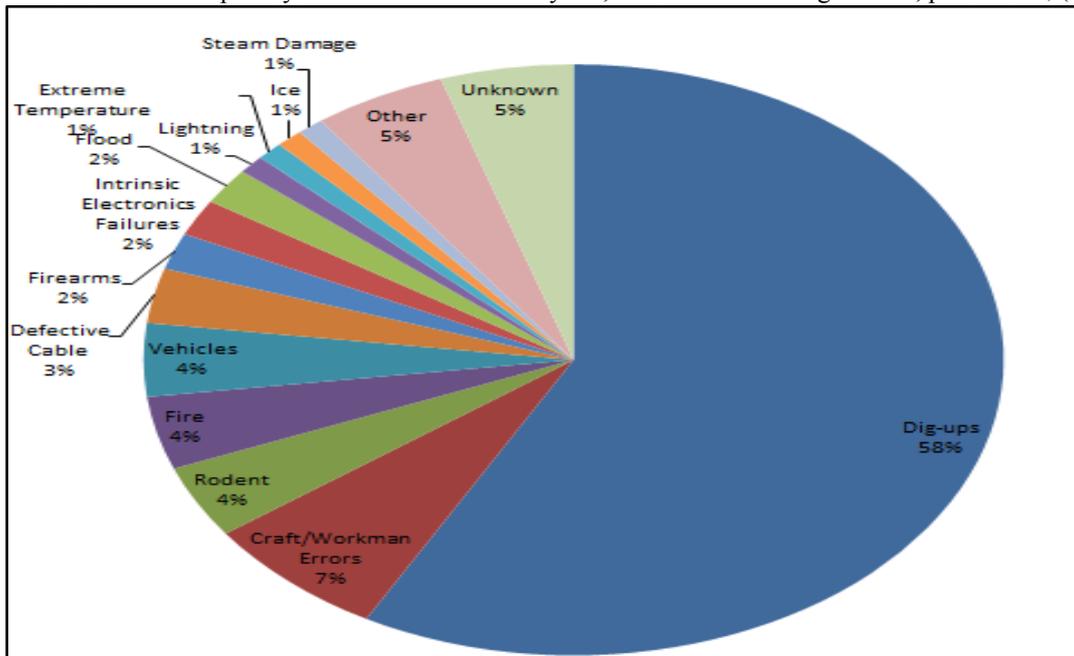
diode, is being increasingly used. Reliability of all these is so far unknown. In an operating photonic system, the reliability of the chain of complex components is crucial. And with an escalating amount of information on each fiber, a failure at the

source, along the fiber at a branching component or amplifier, or at the receiver, can be very costly to the service operator. As well as the cost of lost service revenue and of repairs, a major outage will encourage customer migration to a competitor.

**Table 1.4.** Studies of Historical fiber optic cable failure

Causes	Reported Failures	% Percentage
Dig-ups	172	57
Craft/Workman Errors	22	7
Rodent	13	4
Fire	11	4
Vehicles	11	4
Defective Cable	8	3
Firearms	7	2
Intrinsic Electronics Failures	6	2
Flood	6	2
Lightning	4	1
Extreme Temperature	3	1
Ice	3	1
Steam Damage	3	1
Other	15	5
Unknown	16	5
Total	300	100

Table 1.4 shows studies of Historical fiber optic cable failure which as study done by Ref: V. Hou, "Update on Interim Results of Fiber Optic System Field Failure Analysis", NFOEC Proceedings Vol. 1, p. 539-545, (1991)



**Figure 1.12.** Results of studies of Historical Fiber Optic System Field Failure Analysis

“Ref: V. Hou, “Update on Interim Results of Fiber Optic System Field Failure Analysis”, NFOEC Proceedings Vol. 1, p. 539-545, (1991)”

### 5.6 Fiber Characteristics

There are three key characteristics of optical fibers which may be affected by environmental conditions: strength, attenuation, and resistance to losses caused by micro bending. The reliability of fibers for a given application is ensured by selection of the appropriate fiber design, coating, control of the manufacturing process, and extensive environmental testing prior to qualification.

#### A Strength and Fatigue

Contrary to common perception, glass is an inherently strong material. Optical fibers elongate nearly elastically under tensile loading until brittle failure occurs. The strength of optical fibers is mainly determined by randomly distributed surface defects, most often mechanically or chemically induced cracks or flaws. The fracture probability depends on the fiber stress, fiber length, and the loading time, and is usually represented as a Weibull distribution: The parameters of this distribution are obtained by experimentation, i.e., by loading a statistically significant amount of randomly selected samples. Because there is not a single value of strength, but rather a probability of strength, fibers must be tested to guarantee a minimum strength or a corresponding minimum lifetime. In fiber manufacturing, all fibers are proof-tested for a short test time so that flaws larger than a given size are detected by failure at the position of the flaw. For 125  $\mu\text{m}$  O.D. fibers proof-tested to 0.35 GN/m<sup>2</sup>, the critical flaw size is about 1.6  $\mu\text{m}$ . In addition to the initial strength of the fiber, fatigue must be considered. It is generally accepted that under normal conditions, stress-enhanced interaction of the glass with moisture allows the flaw to grow, thus reducing the stress at which fracture can occur. Long life is achieved by minimizing both stress and moisture. To illustrate the sensitivity of static fatigue to initial strain capability, one model predicts that a fiber with an initial strain capability of 0.33 percent, when exposed to 0.2 percent strain at 2 percent relative humidity, will break in a few hours, whereas a fiber with 0.47 percent initial strain capability under the same conditions will survive 100 years.

#### B. Attenuation

Recently, a good deal of interest has concentrated on the effects of hydrogen on the

optical attenuation of fiber. Initial research was driven by submarine applications because undersea installations may expose the fiber to hydrogen gas, but the results are also applicable to terrestrial systems. It has been found that in hydrogen atmospheres, the fiber's attenuation increases at the longer wavelengths due to the diffusion of the hydrogen into the glass core. At least three components have been identified. The effect of interstitial molecular hydrogen (H<sub>2</sub>) is reversible and is insensitive to glass composition. Reactions forming short wavelength edge effects and hydroxyl (OH) effects are permanent. The latter effect is also composition dependent. Permanent effects are associated with defects in glass structure caused by composition and processing conditions.

Multimode fibers doped with high percentages of phosphorus (7 percent by weight) are particularly susceptible. By reducing the P<sub>2</sub>O<sub>5</sub> concentration, loss increases at 1300 nm can be suppressed to a sufficient extent without any hindrance for practical usage. For low phosphorus doped multimode fiber (less than 1 percent by weight), up to approximately 0.06 dB/km at 1300 nm can be expected after 30 years. For single-mode fibers, slow longterm loss increments at 1310 nm are expected to remain less than 0.02 dB/km after 25 years at 20°C in one atmosphere of hydrogen.

#### C. Micro bending

Micro bending, sharp curvatures involving local axial displacements of a few micrometers, and spatial wavelengths of a few millimeters, can cause significant losses in optical fibers. The fiber coating, which is applied during manufacturing to protect the fiber surface and preserve the intrinsic high strength, plays a dominant role in minimization of micro bending.

Contributing factors include improperly selected coating, improperly applied coating, variation of coating modulus or dimension with temperature or humidity, coating asymmetry, inclusions, or outside distortions transferred to coating. Some of the desirable properties of fiber coatings are good abrasion resistance, ease of stripping, chemical resistance, ease of subsequent processing during cabling, and environmental stability.

The environmental tests performed on optical waveguides primarily examine the effectiveness of the coating materials. If the coating is not performing its function, degradation in the strength or micro bend resistance of the fiber will be quite evident. Fiber treatments include acidic, basic and water soaks, temperature humidity

cycling, fungus testing, extended dry heat and abrasion, and flammability testing.

The objective of fiber optic cabling is to package the fibers without degrading their initial transmission characteristics in such a way that these characteristics remain stable throughout the design life of the cable under specified environmental and mechanical loading conditions. Unlike copper cables where electrical isolation is an important design parameter, the only factors affecting the design of fiber optic cables are mechanical and environmental. All forces, whether axial or radial, acting on an optical fiber, and of course any bending moment, will cause the

transmission characteristics to deviate. To prevent this, the cabled design must either substantially isolate the fibers from forces acting externally on the cables or at least cushion the fibers so that these forces are not converted into serious deformations.

## VI. MULTI-STATE PRACTICE PHYSICAL STRUCTURE

Transmission links connection between main central exchange (switch) and 3 different branch exchange, connecting by four separates fiber optics cables, and we need to study multi state system for this case.

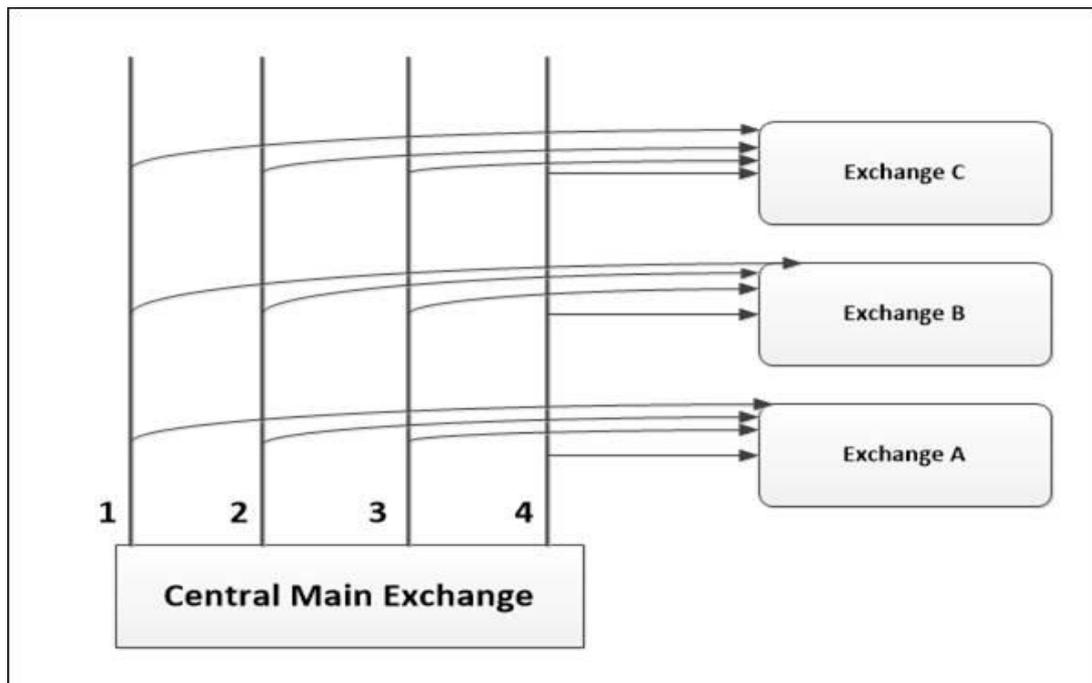


Figure1.13: Communication Multi-State physical structure

Consider for example a central Main exchange, as above Figure, Laser beam contains a stream of bits is delivered from the main source (Central Main Exchange ) to three branch exchanges (A, B, and C) through four Fiber optics cables . A fiber optics cable is considered to be a multi-state component (thus  $n = 4$ ).

Each Exchange has different demands on protection links.

1. Exchange A: requires at least four fiber optics cable to meet its demand of protection (full protection) (4 ON)
2. Exchange B: requires at least two fiber optics cable to meet its demand of protection (half protection) (2 ON)

3. Exchange C requires at least three fiber optics cable to meet its demand of protection (Three-quarters protection ) (3 ON)

Four different system states

1. System state 0: it cannot meet the protection demand of **Exchange A**
2. System state 1: it can meet the protection demand of up to **Exchange A**. That is, the system can meet the protection demand of **Exchange A**, but cannot meet the protection demand of **Exchange B**.
3. System state 2: it can meet the protection demand of up to **Exchange B**. That is, the system can meet the protection demands of **Exchange A** and **Exchange B**, but cannot meet the protection demand of **Exchange C**.

4. System state 3: it can meet the protection demands of up to **Exchange C**. That is, the

protection demands of **Exchange A, B and C** can all be met the protection demands.

**Table 1.5:** A comparison of two possible interpretation of a MSS system.

Interpretation	Communication example	Oil example
Commodity	Data & voice traffic	oil
source	Central main exchange	Supply tank
links	Fiber optic cable	pipes
sink	Branch switch or exchange	station
purpose	Protection for connection	Satisfy a demand for oil
Interpretation used in	This Thesis	Tian et al., and Mo et al.

## VII. CONCLUSION

Reliability measures the ability of a system performing its intended functions. It is one of the most critical performance measures of today's complex systems, such as transportation systems, power systems, communication systems and aircraft systems, and has been emphasized more and more by academia, industry and government. Reliability of a system needs to be evaluated accurately.

In traditional binary reliability framework, both systems and components can only take two possible states: completely working and totally failed. However, engineering systems typically have multiple partial failure states in addition to the above-mentioned completely working and totally failed states. Reliability analysis considering multiple possible states is known as multi-state reliability analysis. Multi-state reliability analysis recognizes the multiple possible states of engineering systems, and enables more accurate system reliability analysis.

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