Seismic Signal Denoising Based on Improved VMD Algorithm

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ABSTRACT: Seismic data usually contains a lot of noise. In order to effectively remove noise and improve the signal-to-noise ratio of seismic signals, a combined bacterial foraging optimization algorithm BFOA and VMD denoising algorithm is proposed. Aiming at the problem that the decomposition of traditional variational modal analysis (VMD) is mainly affected by the number of components K and the penalty factor alpha in the process of seismic signal denoising, resulting in low resolution, the bacterial foraging optimization algorithm optimizes the VMD parameters and combines the sample Entropy denoising algorithm. Firstly, the parameters K and a of VMD are optimized by the bacterial foraging optimization algorithm, and the optimal parameter combination is obtained. The seismic signal is decomposed into several intrinsic modal components with different center frequencies through the VMD algorithm, and then set the threshold according to the difference of seismic signal information combined with the sample entropy function, directly eliminate the high noise component, and finally reconstruct the signal, and finally complete the seismic signal denoising. The experimental results show that VMD has good adaptability to the decomposition of non-stationary signals, and can be applied to seismic signals well. Combining the two methods can achieve better denoising effect. And after processing the actual seismic data, the signal-to-noise ratio is obviously better than the traditional single denoising method.

KEYWORDS: Variational modal decomposition, seismic signal denoising, signal-to-noise ratio, bacterial foraging optimization algorithm, sample entropy, K-L divergence.

I. INTRODUCTION

With the large-scale exploitation of oil and gas, the acquisition environment becomes more and more complex, and a large amount of noise will occur during the acquisition process, and seismic data with high fidelity and low signal-to-noise ratio cannot be obtained, which will affect the subsequent analysis of seismic signals. Since most of the noise contained in seismic data is Gaussian white noise, this paper mainly discusses the removal of random noise. Initially, the processing of random noise was in Yilmaz Ö’s work [1], which proposed that multiple coverage techniques can be used to remove random noise, deal with. This method performs the denoising effect by averaging the measurement data by taking multiple measurements on the same measurement point. Facing the increasingly complex geographical environment, this method increases the acquisition cost and difficulty. The seismic signal is a low-frequency signal, while the random noise is often a high-frequency signal. According to this feature, the current seismic signal denoising methods are mainly divided into the following two categories: The filter filters the noise signal and achieves the effect of denoising through the transformation of different domains [2]. Both methods achieve the purpose of restoring the signal by sacrificing a small amount of effective signal. The variational modal decomposition proposed by Konstantin et al. [3] is an adaptive and non-recursive decomposition method, which decomposes the signal into a finite number of modal components, and determines the center frequency and bandwidth of each modal component. This decomposition method can reduce the residual noise in each mode and further reduce redundant modes. Compared with EMD, it has a solid mathematical foundation and overcomes the problem of mode aliasing well. However, the number of modes and the parameters of the penalty terms required by VMD depend on experience, and the choice of parameters has a great influence on the effect of decomposition. The bacterial foraging optimization algorithm was introduced into the

VMD algorithm, and the parameters of the VMD algorithm were adaptively optimized. Therefore, a new seismic signal denoising method is proposed by combining the two methods. Firstly, the earthquake is carried out by bacterial foraging optimization algorithm to find the optimal parameter combination, and then the seismic signal is subjected to variational modal decomposition with the optimal parameter combination to obtain a limited number of modal components. Combined with the sample entropy, these modal components are subjected to threshold classification. Then, the dominant modal components of the noise are discarded, and the remaining modes are reconstructed to obtain the denoised signal, which can retain the effective signal in the seismic signal as much as possible and suppress random noise.

II. BACTERIA OPTIMAL FORAGING ALGORITHMS AND VMD THEORY

Bacterial foraging optimization algorithm

The Bacterial Foraging Optimization Algorithm belongs to the field of Bacteria Optimization Algorithms and Swarm Optimization, and more broadly to the fields of Computational Intelligence and Metaheuristics. It is related to other Bacteria Optimization Algorithms such as the Bacteria Chemotaxis Algorithm [4], and other Swarm Intelligence algorithms such as Ant Colony Optimization and Particle Swarm Optimization. The Bacterial Foraging Optimization Algorithm is inspired by the group foraging behavior of bacteria such as E.coli and M.xanthus. Specifically, the BFOA is inspired by the chemical behavior of bacteria that will perceive chemical gradients in the environment (such as nutrients) and move toward or away from specific signals [5].

According to the principle of Escherichia coli finding food in the environment, an intelligent algorithm, namely the bacterial foraging optimization algorithm, is abstracted, including three basic processes: tropism operation, replication operation and migration operation.

Step 1: Initialization of parameters including the total number of bacteria $S$, the migration probability $P_{ed}$; the number of chemotaxis $N_c$; the maximum number of steps $N_s$ of unidirectional movement in the chemotaxis operation; the number of replication operation steps $N_{re}$; the number of migration operations $N_{ed}$;

Step 2: Determine the position of the initial bacterial population, and determine whether the migration times, reproduction times, and chemotaxis times have reached the set maximum value in turn;

Step 3: Elimination:

Let $\mathbf{P}^i(j, k, l)$ be the j-th tropism operation of the i-th bacterium, the k-th replication operation, and the coordinates after the first migration operation, there are:

$$\mathbf{P}^i(j, k, l) = \mathbf{P}^i(j, k, l) + \mathbf{C}^i(j, k, l)\mathbf{Y}(j, k, l)$$

(1)

In the formula, $\mathbf{C}^i(j, k, l)$ is the trend step vector;

$$\mathbf{Y}(j, k, l)$$

is the direction vector, and:

$$\mathbf{C}^i(j, k, l) = \mathbf{C}^i(j, k, l + 1)/N_1$$

(2)

$$\mathbf{C}^i(j, k + 1, l) = \max(\mathbf{F}(j, k, l)) - \min(\mathbf{F}(j, k, l))/N_2$$

(3)

If the individual bacteria i does not migrate, there are:

$$\mathbf{C}^i(j + 1, k, l) = \mathbf{C}^i(j, k, l)$$

(4)

else,

$$\mathbf{C}^i(j + 1, k, l) = \mathbf{C}^i(j, k + 1, l)$$

(5)

where $N_i$ is the number of times of bacterial i trending operations and $N_2$ is the scaling factor.

Step 4: Reproduce: Let the bacterial populations $M$ and $\mathbf{F}^i(j, k, l)$ be the fitness values of bacteria i, first sort the fitness values of all bacteria in descending order, and then replace the top-ranked $M/2$ individual bacteria with the lower-ranked individual bacteria of $M/2$.

Step 5: Dispersal: At a certain probability, when a bacterial individual meets the conditions of migration, the bacterial individual disappears, and a new bacterial individual replaces the original individual to maintain a balanced number of bacteria.

Step 6: The number of migration reaches the predetermined value and the operation is over.

VMD theory

The VMD (Variational Mode Decomposition) algorithm [6] is a process of decomposing a signal into several finite-bandwidth sub-signals. It is an adaptive, non-recursive, quasi-orthogonal decomposition method, in which each sub-signal, i.e., the IMF component, surrounds its respective center frequency. The essence of VMD is to construct a variational problem and find the optimal solution. It decomposes the input signal $f(t)$ into k modal components $\mathbf{u}_k(t)$ such that the sum of the estimated bandwidths of the k modes is minimized. The constraint is that the sum of the modal components equals the input signal.
where \( u_k(t) \) is the AM and FM signal, and the expression is

\[
u_k(t) = A_k(t) \cos(\phi_k(t))
\]

The variational problem is constructed according to the following steps [7].

1. Calculate the analytical signal of each modal component through the Hilbert-Huang transform, and obtain its corresponding unilateral spectrum.
2. The exponential term \( e^{-j\omega_k t} \) is introduced to adjust the spectrum of each modal component to its corresponding baseband.
3. The bandwidth corresponding to each modal component is estimated by Gaussian smoothing.

The expression of the constructed constrained variational problem is,

\[
\min_{\delta(t)} \int \left[ \left( \delta(t) + \frac{j}{\pi t} \right) u_k(t) \right] e^{-j\omega_k t} dt 
\]

\[
\sum_k u_k = f
\]

In the formula, \( u_k = \{u_1, u_2, u_3, ..., u_k\} \) is the set of modal components, \( w_k = \{w_1, w_2, w_3, ..., w_k\} \) is the set of center frequencies corresponding to each modal component.

In order to ensure the reconstruction accuracy of the signal and the strictness of finding the optimal solution, for the constraints obtained by equations (7) and (8), the penalty factor \( \alpha \) and the Lagrange operator \( \tilde{\lambda}(t) \) are introduced, and then the multiplier algorithm in alternating directions is used. The saddle point of the augmented function is obtained, and then the optimal solution of each parameter is obtained by the method of selection, and finally each modal component is obtained by inverse transformation. The penalty factor can ensure that the decomposed and reconstructed signal has high accuracy when the signal receives noise interference. The Lagrange operator \( \tilde{\lambda}(t) \) can ensure the strictness of the constraints [5]. After the two parameters are brought in, it can be used quadratic Penalty factor \( \alpha \) and Lagrangian penalty operator \( \tilde{\lambda}(t) \).

The augmented Lagrangian expression is

\[
L(\{u_k\}, \{w_k\}, \lambda) = \sigma \sum_k \| \delta(t) + \frac{j}{\pi t} \| u_k(t) \| e^{-j\omega_k t} \|^2_2 + \| f(t) - \sum_k u_k(t) \|^2
\]

\[
+ \| f(t) - \sum_k u_k(t) \|^2 + < \lambda(t), f(t) - \sum_k u_k(t) >
\]

Equation (4) is iteratively solved using the alternating direction multiplier algorithm to determine its saddle point. The decomposed AM and FM component signals can be expressed as

\[
\hat{u}_k^{t+1}(w) = \frac{\int_0^T \omega \mu_k(w) dl}{\int_0^T \int_0^T \mu_k(w) dl}
\]

\[
\omega_k^{t+1} = \frac{\int_0^T \omega \mu_k(w) dl}{\int_0^T |\mu_k(w)| dl}
\]

### III. DENOISING ALGORITHM BASED ON IMPROVED VMD ALGORITHM

#### Kullback-Leibler divergence

Kullback-Leibler divergence can judge the similarity of two probability distributions, which is equivalent to the difference in information entropy between the two probability distributions. The formula is as follows,

\[
D_{KL}(p \parallel q) = \sum_{i=1}^N p(x_i) \log \frac{p(x_i)}{q(x_i)}
\]

where: \( p(x_i) \) is the probability distribution of the real data, and \( q(x_i) \) is the theoretical probability distribution. The closer the two sets of data are, the smaller the relative entropy, and the greater the difference in the distribution of the two sets of data, the greater Kullback-Leibler divergence.

#### Correlation coefficient

Through VMD decomposition, the seismic signal can be transformed into a finite number of modal components. The modal component with small order corresponds to the low-frequency component of the signal, which can be considered as an effective component. The modal component with a large order corresponds to the high-frequency component of the signal, and is greatly affected by noise. This paper uses the correlation coefficient criterion and the sample entropy to find the demarcation point of the noise signal. The correlation coefficient is a statistical indicator that measures the degree of correlation between variables. The Pearson correlation coefficient method is used to solve the linear relationship between each component and the original data in turn. The calculation formula is:

\[
\text{Corr}(x,y) = \frac{\text{cov}(x,y)}{\sqrt{\text{var}(x) \cdot \text{var}(y)}}
\]

The smaller the value of \( |\text{Corr}(x,y)| \), the smaller the correlation between the two variables.

#### Sample Entropy
Sample entropy is an improved method for measuring the complexity of time series based on approximate entropy. Compared with approximate entropy, sample entropy has two advantages: the calculation of sample entropy does not depend on data length; sample entropy has better consistency, that is, the changes of parameters \( m \) and \( r \) have the same degree of influence on the sample entropy. The lower the value of the sample entropy, the higher the sequence self-similarity; the larger the value of the sample entropy, the more complex the sample sequence. Since the decomposed signal components include signal-dominant components and noise-dominant components, in this paper, the sample entropy of the signal components is calculated to determine the size of the noise content in the signal components. The calculation principle of the sample entropy is as follows:

For a signal sequence \( \{x(i), i = 1, 2, ..., N\} \), assume an \( m \)-dimensional vector sequence

\[
X_m(i) = \{x(i), x(i+1), ..., x(i+m-1)\},
\]

1 \( \leq i \leq N - m + 1 \)

(14)

**Seismic Signal Denoising Algorithm Based on Improved VMD Algorithm**

According to the variational mode decomposition method and theoretical derivation, although the variational mode decomposition algorithm overcomes the shortcomings of the traditional empirical mode decomposition EMD and its improvement methods, it needs to set several parameter values before decomposition to obtain the best decomposition effect, where the number of decomposition layers \( K \) and the penalty factor \( \alpha \) have the greatest influence on the decomposition results. Under normal circumstances, it is necessary to manually select the values of \( K \) and \( \alpha \) before the variational mode decomposition, but the manual selection cannot make the variational mode decomposition achieve the best effect. The set number of decomposition \( K \) and the penalty factor \( \alpha \) are different, and the result of signal processing will be greatly affected. If the set \( \alpha \) is smaller, the bandwidth of the obtained \( K \) IMF components will be larger. Therefore, simply and directly selecting parameters is an urgent problem to be solved. Reference\(^3\) proposed a variational modal decomposition parameter optimized by genetic algorithm. Reference\(^6\) uses particle swarm optimization to optimize the selection of parameters in variational modal decomposition.

Therefore, it is proposed to optimize VMD parameters using joint BFOA and relative entropy to improve the performance of VMD. The key to BFOA optimizing VMD parameters lies in the selection of its fitness function, and the relative entropy reflects the similarity of the two sets of data. Therefore, when BFOA is used to optimize VMD parameters, the fitness function is selected as the relative entropy, and the fitness function value is calculated each time. Then compare and update each other to finally determine the parameter combination, and then combine the sample entropy and relative coefficient to determine the threshold value of the modal component among all the IMF components obtained by VMD decomposition, determine the IMF component dominated by noise, and perform filtering processing. The remaining useful signals are reconstructed to obtain denoised signals.

The specific steps of BFOA to optimize VMD algorithm are as follows:

1. Set the parameters of BFOA.
2. Use BFOA to optimize VMD parameters, and use relative entropy as an adaptive function to find a set of optimal parameter combinations \( K \) and \( \alpha \). The specific optimization process is shown in Fig. 1, and then set the number of VMD algorithm decompositions to \( K \) respectively, and the penalty factor \( \alpha \) decomposes the signal to obtain \( K \) IMF components.
3. Combine the sample entropy and correlation coefficient, determine the mode dominated by noise and the mode dominated by signal, discard the mode dominated by noise directly, and reconstruct the remaining useful signal to obtain the final denoised signal. The process is shown in Fig. 2.
IV. EXPERIMENTATION

Experimental simulation

To evaluate the effectiveness of the denoising method, the denoising effect is compared with other traditional single denoising effects. First generate a single-channel synthetic seismic record, add Gaussian white noise of different decibels to the signal, and then use hard threshold wavelet denoising, soft threshold wavelet denoising, VMD denoising, and the proposed improved VMD method to denoise the noise. Calculate the signal-to-noise ratio to compare the denoising effect. Figure 1 is a synthetic single-channel seismic record, Figure 2 is the signal after adding Gaussian white noise, and its signal-to-noise ratio is SNR=5dB. Figure 3 and Figure 4 are the improved VMD denoising method combined with BFOA selection for noisy signals. Good optimal combination [K, Alpha], Figure 5, Figure 6, and Figure 7 are the wavelet hard threshold denoising results and the wavelet soft threshold denoising results, respectively. The denoising results of VMD and the signal-to-noise ratio have been improved to a certain extent. However, it can be seen that the waveform amplitude is also attenuated after wavelet hard threshold denoising and soft threshold
denoising, and some effective signals are lost. Figure 9 shows the denoising results of the improved method proposed by the author. It can be seen from Figure 9 that the noise has been removed to a large extent, and the problem of amplitude loss in wavelet hard threshold denoising and soft threshold denoising has also been overcome. By calculating the SNR, the SNR obtained by the improved denoising method proposed by the author after denoising is significantly higher than that of the other three methods. In order to evaluate the denoising effect of the improved denoising method in this paper under different signal-to-noise ratios, the following experiments were carried out. Four kinds of white noise with different energies and noise signals with different signal-to-noise ratios are added to the signal, and the wavelet hard threshold method is used for denoising, the wavelet soft threshold method for denoising VMD decomposition and reconstruction denoising, and the improved method in this paper for denoising. Then calculate the signal-to-noise ratios after denoising with different noise intensities. The signal-to-noise ratios before and after denoising are shown in Table 1. It can be seen from Table 1 that the SNR improves the most after denoising by the improved method in this paper, followed by VMD Decomposition and reconstruction denoising, wavelet soft threshold denoising, wavelet hard threshold denoising. After comparing the denoising effects of various methods under different noise energies, the effectiveness of the improved denoising method proposed in this paper is confirmed.
Fig. 3 BFOA find [BestK, BestAlpha]

Fig. 4 BFOA find [BestK, BestAlpha]
**Fig. 5** Wavelet hard threshold denoising

**Fig. 6** Wavelet soft threshold denoising

**Fig. 7** VMD decomposition and reconstruction denoising
Fig. 8 IMFs with improved VMD decomposition

Fig. 9 Improved VMD decomposition reconstruction denoising
Tab.1 The denoising effect of adding different decibel Gaussian white noise

<table>
<thead>
<tr>
<th>Noise+signal dB</th>
<th>Wavelet hard threshold dB</th>
<th>Wavelet soft threshold dB</th>
<th>VMD dB</th>
<th>Improved decomposition dB</th>
</tr>
</thead>
<tbody>
<tr>
<td>5dB</td>
<td>7.6656</td>
<td>9.6044</td>
<td>8.4426</td>
<td>10.9781</td>
</tr>
<tr>
<td>7dB</td>
<td>9.6569</td>
<td>10.6763</td>
<td>10.2505</td>
<td>11.9184</td>
</tr>
<tr>
<td>11dB</td>
<td>14.8859</td>
<td>15.1193</td>
<td>15.4929</td>
<td>17.6141</td>
</tr>
</tbody>
</table>

V. CONCLUSION

By improving the selection of VMD adaptive parameters, combining the advantages of BFOA and overcoming the shortcomings of traditional soft threshold functions and hard threshold functions, a new VMD that can adjust parameters adaptively combined with sample entropy and correlation coefficient is proposed to detect earthquakes. The algorithm for denoising the signal, so as to keep the effective signal as much as possible while denoising. At the same time, the VMD decomposition is combined with the threshold denoising method, and the multi-scale adaptive decomposition characteristics of VMD are used to further improve the denoising effect. It is proved that the improved method proposed in this paper can effectively denoise the seismic signal, improve the signal-to-noise ratio, and the denoising effect is better than the other three single denoising methods, and can retain more detailed and effective information during denoising.

REFERENCES