Structures, Electronic, and Transport Properties of M₂PbC (M=Zr, V)

S. T. Ahamsa,b, A. Shaarić, N. F. Abdul Pattahá, M. Bubá,ć

áDepartment of Physics, Universiti Teknologi Malaysia, Johor Bahru
bDepartment of Pure and Applied Physics, Adamawa State University, Mubi, Nigeria.
cDepartment of Science Laboratory Technology, Federal Polytechnic, Mubi, Nigeria

ABSTRACT: This study investigates the electronic, and transport properties of M₂PbC (M=Zr, V) MAX phases based on density functional theory density functional perturbation theory, and Boltzmann transport theory, respectively. The electronic and density of states show the V, and Zr containing MAX phases have metallic characteristics. The energy bands at the Fermi level (EF) and elsewhere within its vicinity, are mainly contributed by the V 3d, Zr 4d, and the Pb 6p states. A V-Pb, Zr – Pb, hybridizations occur just below the EF and stretch through to the conduction band, the effects of which are more strongly felt than the Zr-C, V-C bonds. The phonon contribution to thermal conductivity at finite temperature is low compared to electron contribution. From the calculated transport coefficients, a complete replacement of Zr with V expands the thermolectric figure of merit from 0.026 for Zr₂PbC to 0.13 for V₂PbC, respectively.

KEYWORDS: MAX phase, metallic, hybridization, thermal conductivity, thermolectric figure of merit

I. INTRODUCTION

In the universal quest to fight global warming, clean energy bases are eminent [1], [2]. Thermoelectricity directly converts waste heat or temperature gradient to an electrical potential difference using the Seebeck effect[3]–[5]. The birth of the Seebeck effect in the early 1800s has many applications in today's engineering requirements, however, a lot of challenges thwart so many efforts of maximizing the efficiency of thermolectric devices[6]–[9]. Statistical results show that over 60% of energy is laid waste globally, mostly in the form of waste heat[10]. Research interest in thermoelectricity (TE) has attracted growing attention recently [4]. An efficient TE device is determined by the (ZT) [11] expressed in equation (1).

\[
ZT = \frac{S^2 \sigma}{k_{el} + k_{lat}} \frac{1}{T}
\]

where ZT is a dimensionless quantity, S represents the Seebeck coefficient, \( \sigma \) and \( k_{el} \) the electrical and thermal conductivities, and \( S^2 \sigma \) the power factor, respectively. To attain the best performance, ZT must be enhanced by maximizing the power factor \( S^2 \sigma \) which minimizes the \( k_{el} \). The \( S^2 \sigma \) is a key factor in understanding the thermolectric properties of a material [12]. However, these three parameters (S, \( \sigma \), and\( k_{el}, k_{lat} \)) depend on each other. Therefore, the challenge to establish support for a TE material with high ZT is to simultaneously achieve a high Seebeck coefficient, high electrical conductivity, low electronic thermal conductivity, and low phonon thermal conductivity[13].

In recent years, MAX phase materials have propelled the focus of researchers for their enormous potential and TE may just be one of the novels upcoming. Magnuson et al. (2011) have studied the Seebeck factor in Ti₃SiC₂ single-crystal thin films and have traced the source to anisotropies in element-specific electronic states. They found that the bulk form of Ti₃SiC₂ has a Seebeck factor of almost zero over a large temperature spectrum. On the contrary, however, the in-plane (basal ab) Seebeck coefficient of Ti₃SiC₂, calculated on single-crystal films, has a substantially positive Seebeck coefficient of 4 to 6 \( \mu V/K \)[14]. Series of ab-initio computations were conducted to investigate the transportation in monolayers and multilayers of the functionalized M₂C (M = Sc, Ti, V, Zr, Nb, Mo, Hf, Ta), and M₂N (M = Ti, Zr, Hf) using F, O, and OH[15]. Though relaxation time (\( \tau \)) is an undefined factor in their
evaluations, a comparison of the transport coefficients of MXenes is still justifiable. The simple reason is that the MXenes here have structures analogous to MAX phase structures as they were initially etched from a family of nanolaminates [16].

MAX phases have a universal stoichiometry of $M_{n+1}AX_n$, where $n = 1, 2, 3$ and represents an integer, $M$ is a transition metal, $A$ is found in group A element which is largely found in group III-A, IV-A, $X$ stands for C/N [20]. The group crystalizes into a hexagonal shape of space group P63/mmc [21]–[24]. This family of compounds has been widely explored recently due to their exceptional and dual characteristics as a metal and as a ceramic [17], [25]–[27]. Akin to metals, they serve as excellent thermal and electric electrodes having a remarkable thermal shock and damage tolerance [28]–[30]. Nanolaminates are known to show endurance to corrosion and oxidation [31], [32]. Mostly elastic and yet extremely easy to machine [33], [34], which are some of the salient physical characteristics that make the MAX phases suitable for many potential applications where resistance to ultra-high-temperature, radiation damage tolerance, self-lubrication, and so on are some of the key factors to look for [35]–[37]. Reliant on the integer, $n$, MAX phases are grouped into a class of 211 ($n = 1$), 312 ($n = 2$), 413 ($n = 3$) and 514 ($n = 4$) phases and so on [38].

High-performance ZT is one recent area of research interest in condensed matter due to its practical applications as a clean power generation material from waste-heat supplies like power plants and automobile machines [39]. Because of the presence of both electronic and thermal transport processes, as well as non-equilibrium transport processes, the properties required to understand electrical transport and thermoelectric properties of MAX phases are difficult to calculate from the first principles [40]. Whence, MAX phases have been studied as precursors for MXenes of family formula $(M_{n+1}X_n)_2$, a 2D nanosheet with properties analogous to that of graphene currently only synthetizable through a selective chemical etching of, largely the Al-based MAX phases [41]. In a related review on the structural, elastic, thermodynamic, and optical properties of Zr$_2$PbC, the structure has been reported to be unstable at normal pressure and temperature [42]. As there are just but few pieces of works of literature that have either directly or otherwise studied the ZT of Zr$_2$PbC and V$_2$PbC, this research paper aims at providing a systematic cross-examination of their electronic and thermoelectric properties respectively, for the first time.

II. COMPUTATIONAL FRAMEWORK

The ternaries Zr$_2$PbC and V$_2$PbC MAX phases have been investigated based on first principles using density functional theory [43][44] as implemented in Quantum Espresso (QE) [45], [46] simulation package and the Boltzmann transport theory [47], [48]. The generalized gradient approximation (GGA-PBE) [49] exchange energy was used. Throughout this study, ultrasoft pseudopotentials (UPP) have been employed and an energy cut-off of 800 Ry was applied to limit the number of components in the plane wave expansion. A Gaussian smearing technique has been used to a smearing width of 2.0 $\times 10^{-2}$ Ry. A 16 $\times$ 16 $\times$ 6 k-points mesh has been produced utilizing the Monkhorst-Pack program for the Brillouin zone integration, [50] to generate a uniform k-points grid along with the a, b, c-axes in reciprocal space. The BFGS (Broyden–Fletcher–Goldfarb-Shanno) method has been used to optimize the lattice parameters and was fully optimized after relaxing the atomic positions through several volumes to a threshold of approximately about $1.0 \times 10^{-8}$ Ry [50]. Transport properties were evaluated using Boltzmann transport theory as implemented in the BoltzTraP2 simulation code [47], [48], while the phonon contribution to thermal conductivity was calculated using the D3Q simulation pack as implemented in quantum espresso 6.7 [46]. Within this approximation, the Seebeck coefficient becomes $\tau$-independent without any other adjustable parameter, but the electrical conductivities and the power factors are obtained for $\tau$. BoltzTraP2 computes the transport coefficients centered on the rigid-band approximation (RBA), which accepts that changing the temperature, or alloying (doping) a system, does not change the band structure. In the rigid band approximation (RBA) the density of the materials of states ($n(\varepsilon)$) can be calculated by applying Equation (2):

$$n(\varepsilon) = \int \frac{d\mathbf{k}}{B\pi^3} \sum_b \delta(\varepsilon - \varepsilon_{b,\mathbf{k}})$$

where the $b$ operates over the bands. A linearized edition of the BTE has been employed under the RTA, where the energy and temperature-dependent transport distribution function, $\sigma(\varepsilon, T)$ given by Equation (3) is applied for calculating the moments, Equation (4) of the generalized transport coefficients, $\mathcal{E}^{(0)}(\mu; T)$ [48], [51]
\[ \sigma(\varepsilon, T) = \sum_b \mathbf{v}_{b,k} \times \mathbf{v}_{b,k} \tau_{b,k} \delta(\varepsilon - \varepsilon_{b,k}) \frac{d\mathbf{k}}{8\pi^3} \]  

where \( \mathbf{v}_{b,k} \) represents the group velocities, \( \tau_{b,k} \) relaxation time approximation (RTA)

\[ \mathcal{L}^{(\alpha)}(\mu; T) = q^2 \int \sigma(\varepsilon, T)(\varepsilon) - \mu \left( \frac{\partial f^{(0)}(\varepsilon, \mu; T)}{\partial \varepsilon} \right) d\varepsilon \]  

where \( \mathcal{L} \) denotes the generalized transport coefficients, and \( f^{(0)} \) represents the Fermi distribution function. The Seebeck coefficient (S), electrical, and thermal conductivities (\( \sigma_e, k_{el} \)) due to charge carrier contribution at and around the Fermi can thus be calculated by applying Equations (5) to (7), respectively.

\[ \sigma_e = \mathcal{L}^{(0)} \]  

\[ S = \frac{1}{qT} \mathcal{L}^{(1)} \]  

\[ k_{el} = \frac{1}{qT} \left[ \frac{(\mathcal{L}^{(1)})^2}{\mathcal{L}^{(0)}} - \mathcal{L}^{(2)} \right] \]

Table 1. Once self-consistency, or the minimum, has been reached, the total energy [52] of the system is calculated.

3.2. Electronic properties

All the electronic and bonding physical characteristics were investigated to ascertain the electrical behaviours of Zr\(_2\)PbC and V\(_3\)PbC. The calculated energy bands are depicted in Figures 2 (a), and (b), respectively. With practically zero bandgaps at the Fermi level (represented with the horizontal black dash line, there is a lot of overlapping of the valence and conduction bands

\[ E_0 = \sum_i f_i \varepsilon_i - E_{ii}[n(r)] + E_{XC}[n(r)] \]  

\[ - \int n(r)V_{XC}[n(r)] dr + E_{ion} - \text{ion} \]

III. RESULTS AND DISCUSSIONS.

3.1. Structural properties.

The 211 layered ternaries, Zr\(_2\)PbC, and V\(_3\)PbC crystallize into the hexagonal structure of space group P6\(_3\)mmc like any other MAX compound. Their unit cell contains two formula units of eight atoms each. The atomic coordinates of atoms which are located thus: Zr/V (0.33, 0.67, z), Pb (0.67, 0.33, 0.25) and C (0, 0, 0), respectively. Figure 1 shows the unit cell structures of Zr\(_2\)PbC, and V\(_3\)PbC, respectively. The equilibrium crystal structure of these compounds has been attained by minimizing the total energy[52] Equation (8).

where \( E_0 \) the total ground state total energy, \( \varepsilon_i \) is one-electron contribution, \( E_{ii} \) is the Hartree contribution, \( E_{XC} \) is the exchange contribution, and the fourth and fifth terms are the ewald contributions.

The calculated results for V\(_3\)PbC have been compared with literature data of the parent material Zr\(_2\)PbC. A Series of geometry optimizations were conducted on the completely relaxed coordinates of V\(_3\)PbC and Zr\(_2\)PbC to obtain the structural parameters from the optimized volume. During such calculations, fixed unit cell volume has been considered, and several readings of separate axial ratios of c/a are obtained to evaluate the total and minimum energy \( E_{\text{min}} \) as recorded in inherent to the two studied nanolaminate which is akin to their metallic behaviours. With these flash characteristics, Zr\(_2\)PbC, and V\(_3\)PbC are projected to exhibit electrical, and thermal conductivities just like any other MAX phase. There is an important feature foreseen of the band structure which is a solid anisotropy with less energy dispersion on the c-axis observed from a reduced dispersion down the short Γ₋A, L₋M, and K₋H paths as displayed in Figure 2 (a) and (b), respectively.
To understand the atomic bonding behaviours in the ternary layered materials, the total, and partial densities of states (TDOS, and PDOS) have been computed and compared as presented in Figures(a) and (b), respectively. This spans across a range of -12 to +5 electronvolts, while C -2p and Pb -6p states demonstrate satisfactory impact in the valence band within the energy -5 eV to -1 eV and -7 eV to -1 eV, respectively in the two cases for Zr\textsubscript{PbC} and V\textsubscript{PbC}. The peaks in the TDOS for V\textsubscript{PbC} are manifest from the enormous contributions of the V -3d, Pb-6p, and C -2p, states as depicted in Figure 3(b). The TDOS peak at the E\textsubscript{F} is a result of the practically visible contributions from V -3d and Pb -6p, while C -2p and Pb -6s are the builders of the TDOS in the range of -7 eV to -2 eV. On the other hand, the V -3d, Pb -6p C -2p, are the results we see manifesting in the TDOS in the conduction band. The calculated TDOS for each of the ternaries at the E\textsubscript{F} is 1.84 and 4.62 states/eV, respectively. The results imply that V\textsubscript{PbC} conducts more readily than Zr\textsubscript{PbC}. Generally, the nanolaminates have remarkably solid M-X bonds and moderately weak M-A bonds. Generally, the Zr/V d-orbitals and Pb p-orbital, respectively share major factors responsible for the conduction properties of the studied MAX compounds.

### 3.3. Transport properties

The Seebeck coefficient (S), power factor (S\textsuperscript{2}\sigma\textsubscript{e}), electrical conductivity(\sigma/\tau), and electronic thermal conductivity (k\textsubscript{el}) of Zr\textsubscript{PbC} and V\textsubscript{PbC} has been calculated from the semiclassical Boltzmann transport theory. The calculated electronic structure data, E(n,k), Equations (3) and (4) have been used in calculating the transport coefficients of k\textsubscript{el}, \sigma, and S using Equations (5), (6), and (7), respectively[51] while the phonon contribution (k\textsubscript{ph}) which is very insignificant is calculated using D3Q and is compared to the electronic contribution.

The figure of merit ZT has been evaluated using Equation (1). An energy range of E-\mu = \pm 0.15 Ry has been used to evaluate the integrals in Equations (2) to (4). The temperature range of interest for all the reported results is 200 K to 800 K with a step size of 200. Plots and results for the calculated TPs for Zr\textsubscript{PbC} and V\textsubscript{PbC} are displayed in Table 2, Figures 5 (a) to (d), and Figures 6(a) to (d), respectively.

The magnitude of S for bulk Zr\textsubscript{PbC} and V\textsubscript{PbC} MAX phases varies directly to the temperature which is clearly shown in Table 2 and Figure 5 (a) and Figure 6 (a), respectively. The Seebeck has increased significantly from 25.5 \muVK\textsuperscript{-1} for Zr\textsubscript{PbC} at E-\mu =0.075 Ry to 64.4 for V\textsubscript{PbC} at E-\mu = -0.125 Ry, respectively. Which is a good indication that the nanolaminates V\textsubscript{PbC} has a more promising stand compared to Zr\textsubscript{PbC} as future thermoelectric material.

The calculated power factor, electrical and thermal conductivities, and figure of merit show that the substitution of Zr with V rises the electrical and thermal conductivities as well as increases the ZT as recorded in Table 2 and depicted in Figure 8 respectively. Hence, the calculated results at temperature 800 K, shows Zr\textsubscript{PbC} has a lower value of S, S\textsuperscript{2}\sigma\textsubscript{e}, \sigma, k\textsubscript{el}, and ZT, respectively compared to V\textsubscript{PbC} which is depicted in Table 2 and Figure 5, and Figure 7. The difference in the electrical conductivity agrees well with the simulated density of state (TDOS).

The calculated ZT Figure 8 shows that Zr\textsubscript{PbC} and V\textsubscript{PbC} can be tuned through substitution or alloying on the M site with an
appropriate transition metal. Figures 8 (a) and (b). The calculated band structures, TDOS, and PDOS completely validate the transport of our results.

As there are no literature results on the new ordered V₂PbC MAX phase available, the results of the parent material Zr₂PbC have been used for comparison where necessary and available. The contribution to thermal conductivity is greater due to the substitution of Zr with V on the M-site in bulk V₂PbC than in the bulk Zr₂PbC. For the given operating temperature range, the calculated ZT for V₂PbC is 0.133, far superior to that of the parent Zr₂PbC (ZT = 0.026) for the same temperature results which are shown in Figure 8 (b) and (a) plots and recorded in Table 2, respectively.

Table 1. Equilibrium dimensional constants a, and c, the ratio c/a, volume V, and minimum energy Eₘᵢₙ.

<table>
<thead>
<tr>
<th>MAX Phase</th>
<th>a (Å)</th>
<th>c (Å)</th>
<th>c/a</th>
<th>V (Å³)</th>
<th>Eₘᵢₙ (Ry)</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Zr₂PbC</td>
<td>3.402</td>
<td>14.93</td>
<td>4.39</td>
<td>150.5</td>
<td>-693.73</td>
<td>This work</td>
</tr>
<tr>
<td></td>
<td>3.38</td>
<td>14.66</td>
<td>4.34</td>
<td></td>
<td></td>
<td>[53]</td>
</tr>
<tr>
<td></td>
<td>3.42</td>
<td>14.95</td>
<td>4.37</td>
<td></td>
<td></td>
<td>[36]</td>
</tr>
<tr>
<td>V₂PbC</td>
<td>3.153</td>
<td>11.52</td>
<td>3.65</td>
<td>116.7</td>
<td>-877.96</td>
<td>This work</td>
</tr>
</tbody>
</table>

Table 2. Calculated Seebeck (S), power factor (S²σₜ), electrical and thermal conductivities (σ and kₑ), and figure of merit (ZT) for Zr₂PbC and V₂PbC as a function of energy.

<table>
<thead>
<tr>
<th>Comp.</th>
<th>S (μV/K)</th>
<th>S²σₜ (10⁹ W/K·m)</th>
<th>σₑ (10¹⁸Ω·m⁻¹)</th>
<th>kₑ (10¹⁵ W/m·K)</th>
<th>ZT</th>
</tr>
</thead>
<tbody>
<tr>
<td>Zr₂PbC</td>
<td>25.50</td>
<td>2.00</td>
<td>6.20</td>
<td>11.81</td>
<td>0.026</td>
</tr>
<tr>
<td>V₂PbC</td>
<td>64.40</td>
<td>6.20</td>
<td>4.00</td>
<td>7.30</td>
<td>0.133</td>
</tr>
</tbody>
</table>

Figure 2. Calculated energy bands of (a) Zr₂PbC, and (b) V₂PbC

Figure 3. Total, and partial density of states of (a) Zr₂PbC, and (b) V₂PbC
Figure 4. Electronic thermal conductivity with an inset of lattice contribution for Zr$_2$PbC

Figure 5. Variation of (a) Seebeck ($S$) (b) power factor ($S^2\sigma$) (c) electrical conductivity ($\sigma$) (d) thermal conductivity ($k$) of Zr$_2$PbC
Figure 6. Variation of (a) Seebeck (S) (b) power factor ($S^2\sigma$) (c) electrical conductivity ($\sigma_e$) (d) thermal conductivity ($k_\text{el}$) of V$_2$PbC

Figure 7. Variation of (a) Seebeck (S) (b) power factor ($S^2\sigma$) (c) electrical conductivity ($\sigma_e$) (d) thermal conductivity ($k_\text{el}$) of M$_2$PbC with temperature
The relationship between the efficiency of thermoelectric power generation and the average ZT at 800 K shows that V$_2$PbC is a more promising TE material than the bulk Zr$_2$PbC.

IV. CONCLUSIONS

We have studied the electronic and thermoelectric properties of Zr$_2$PbC, and V$_2$PbC using first-principles density functional theory, density functional perturbation theory, and Boltzmann transport theory, respectively. The electronic bands and TDOS show that both ternary MAX phases are metals with V$_2$PbC having the highest DOS at the E$_F$. The calculated Seebeck coefficients and ZT show that V$_2$PbC is a likely thermoelectric material in extreme conditions than Zr$_2$PbC and future thermoelectric material. Due to their relatively high electrical conductivities, these materials are potentially high-temperature conductive components manufacturing by-products. It is hoped that further studies will help in the discovery of more new MAX phases that can replace the dominance of semiconducting devices as potential thermoelectric materials used in engineering technology.

V. ACKNOWLEDGMENT

The authors wish to thank MOHE Malaysia Grant: R.J13000.7854.5F231, ADSU, Mubi, Nigeria and TETFund, Nigeria. And acknowledge Dr. Yap Yung Szen, of the Physics Department, Universiti Teknologi Malaysia, for his technical advice and suggestions.

REFERENCES


Figure 8. Energy dependence of ZT for (a) Zr$_2$PbC and (b)V$_2$PbC at 800 K


