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Application of Raj Transform For Solving Mathematical Models Occurring in the Health Science and Biotechnology

Dinkar P. Patil[1], Sakshi T. Bhat[2]
Department of Mathematics , K.R.T. Arts , B.H.Commerce and A.M. Science College , Nashik

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ABSTRACT
Lot of Mathematical Models including differential equations play an important role in healthcare and biotechnology. One of them in Malthus model. This model was developed by Thomas Malthus, in his essay on world Population Growth and resource supply. Another interesting equation is Advection diffusion equation and predator prey model. We use a integral transform called Raj Transform to obtain the solutions of these models which are important in biotechnology and health sciences.
Key Words: Integral Transform, Raj Transform, Mathematical Models, Health Sciences, Biotechnology.

I. INTRODUCTION:

Mathematical models are very much useful for society. Many methods are used to solve these models. Integral transforms play an important role to solve such models.

Recently, S.R. Kushare [2] introduced Kushare transform.Further Savita Khakale and Dinkar Patil [3] introduced Soham transform in November 2021. As researchers are interested in introducing the new integral transforms at the same time they are also interested in applying the transforms to various fields, various equations in different domain. In January 2022, Patil et al [4, 5, 13, 32, 33, 35, 38, 41]used Kushare transform solving different problems. D.P. Patil [6, 7, 9] used Sawi transform for solving various types of problems. Laplace transforms and Shehu transforms are used in chemical sciences by Patil [8]. Using Mahgoub transform, parabolic boundary value problems are solved by D. P. Patil [10] D.P. Patil [11] used double Laplace and double Sumudu transforms to obtain the solution of wave equation. Further Dr. Patil [12] also obtained dualities between double integral transforms. D. P. Patil [14] solved boundary value problems of the system of ordinary differential equations by using Aboodh and Mahgoub transforms. Double Mahgoub transformed is used by Patil [15] to solve parabolic boundary value problems. Laplace, Sumudu, Aboodh, Elazki and Mahagoub transforms are compared and used it for solving boundary value problems by Dinkar Patil [16]. Further, Patil with Tile and Shinde [18] used Anuj transform and solved Volterra integral equations for first kind. Rathi sisters and D. P. Patil [19] solved system of differential equations by using Soham transform. Vispute, Jadhav and Patil [20]used Emad Sara transform for solving telegraph equation. Kandalkar, Zankarand Patil [21] evaluate the improper integrals by using general integral transform of error function. Dinkar Patil, Prerana Thakare and Prajakta Patil [22] obtained the solution of parabolic boundary value problems by using double general integral transform. D. P. Patil et al [23, 24, 26] used various integral transform to obtain the solution of Newton's law of cooling. Dinkar Patil et al [25, 29, 30, 31, 40, 47, 48] used integral transforms for handling growth and Decay problems, D.P. Patil et al [27] used Emad-Falih transform for general solution of telegraph equation. Dinkar Patil et al [28] introduced double kushare transform. Recently, Wagh sisters and Patil used Soham [34] transform in chemical Sciences. Raundal and Patil [36] used double general integral transform for solving boundary value problems in partial differential equations. Rahane, Derle and Patil [37] developed generalized double rangaig integral transform. Patil et al [39, 49] used new general integral transform and Soham transform to solve Abel's integral equations.Thakare and Patil [42] used general integral transform for solving mathematical models from health sciences. Rathi sisters used Soham transform for analysis of impulsive response of Mechanical and Electrical oscillators with Patil [43]. Patil [44, 46] used KKAT transform for solving growth and decay problems and Newton's law of cooling. Suryawanshi et al [17, 45] used Soham transform for solving volterra integral equations and mathematical models occurring in health science and biotechnology.

We organized this paper as follows. Introduction is in first section. Preliminary concepts about Raj transform is in second section and Third section is for Raj transform for solving logistic growth models in Health science. Raj transform is used for predator Prey model in fourth section. Problems are solved in fifth section. Conclusion is in sixth and acknowledgement is in seventh section.

1. Useful Results And Formulae : In this section we include some required definitions , some useful formulae and theorems based on Raj transform.

Definition :[50] The Raj transform of the function $f(t)$ is defined as

Z($f(t)$) = $\int_o^\infty f(\frac{t}{s})e^{-t}dt$, where t is in between zero and infinity (0< $k_1 \le t \le k_2$), here k_1 and k_2 are either finite or infinite. ……(2.1)

Raj Transform Of Some Standard functions:[50]

Raj transform of some functions are stated in following table.

Sr.No.	F(t)	R{F(t)}
1.	1	1
2.	t	$\frac{1}{s}$
3.	t^2	$\frac{2}{s^2}$
4.	t^n	$\frac{n!}{s^n}$
5.	e^{at}	$\frac{s}{s-a}$
6.	sinat	$\frac{as}{a^2+s^2}$
7.	cosat	$\frac{as}{a^2+s^2}$

Raj transform of Derivatives : [50] Raj transform of deriative of function f(t) is given by,

$R(f'(t)) = s\,R(s) - s\,f(0)$

2. Raj transform for Logistic Growth Model

Consider the Logistic growth model equation

$$\frac{du}{dt} = u - f(u), u(0) = u_0 \qquad \text{…………(3.1)}$$

Here f is non linear function of u . Suppose that solution u of equation (3.1) is of infinite power series as follows ,

$$u = u(t) = \sum_{n=0}^{\infty} a_n t^n \qquad \text{…………(3.2)}$$

Further (3.2) also satisfies the conditions for Existence of Raj Transform.

Applying Raj transform on both sides of (3.1) we get

$$s\,R(s) - s\,u(0) = R(s) - F(s) \qquad \text{………(3.3)}$$

Where R(s)=R(u(t)) and F(s) = R(f(u))

Rearranging the terms in (3.3) we get,

$$R(s) = u_0 \frac{s}{s-1} - \frac{F(s)}{s-1} \qquad \textbf{.........(3.4)}$$

If we suppose (u) = u^2 Then

$$f(u) = \left[\sum_{n=0}^{\infty} a_n t^n\right]^2$$

$$= \left[a_o + a_1 t + a_2 t^2 + a_3 t^3 + \ldots\ldots + a_n t^n \right]^2$$

$$= (a_0)^2 + 2a_o a_1 t + (2a_o a_2 + a_1^2)t^2 + (2a_o a_3 + 2a_1 a_2)t^3 + \ldots.. \qquad \textbf{..........(3.5)}$$

Taking Raj transform on both sides of (3.5) ,

$$F(s) = (a_o)^2 + \frac{2a_o a_1}{s} + \frac{2!\,(2a_o a_2 + a_1^2)}{s^2} + \frac{3!\,(2a_o a_3 + 2a_1 a_2)}{s^3} + \ldots..$$

$$\therefore \frac{F(s)}{s-1} = \frac{a_o{}^2}{s-1} + \frac{2a_o a_1}{s(s-1)} + \frac{2\,(2a_o a_2 + a_1^2)}{s^2(s-1)} + \frac{6(2a_o a_3 + 2a_1 a_2)}{s^3(s-1)} + \ldots.$$

Applying Partial Fractions On R.H.S Of The Above Equation ,

$$\frac{F(s)}{s-1} = \frac{e^t}{s}\left(a_0^2 + 2a_o a_1 t + 4a_o a_2 + 2a_1^2 + 12a_o a_3 + 12a_o a_2 + ..\right)$$
$$- t\left(2a_o a_1 t + 4a_o a_2 + 2a_1^2 + 12a_o a_3 + 12a_o a_2 + \ldots\right)$$
$$- \frac{t^2}{2}\left(4a_o a_2 + 2a_1^2 + 12a_o a_3 + 12a_o a_2 + \ldots\right)$$
$$- \frac{t^3}{6}\left(12a_o a_3 + 12a_o a_2 + \ldots.\right)$$

Applying inverse raj transform to both sides of the above equation and rearranging the terms we get ,

$$u(t) = u_0\left(1 + t + \frac{t^2}{2} + \frac{t^3}{6} + \ldots.\right) - \left[a_0^2 t + \left(\frac{a_0^2}{2} + a_o a_1\right)t^2 + \left(\frac{a_0^2}{6} + \frac{a_0 a_1}{3} + \frac{a_0 a_2}{3} + \frac{a_1^2}{3}\right)t^3\right] + \ldots\ldots$$

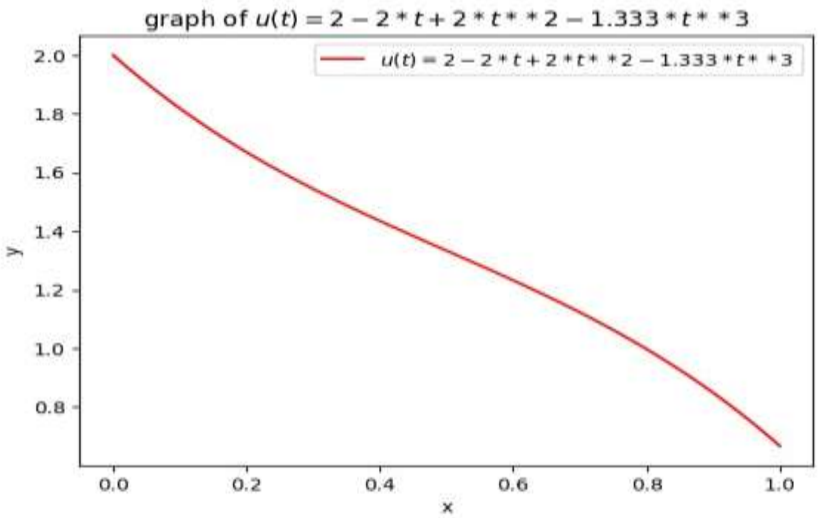
$$\therefore \boldsymbol{u(t) = u_0 + (u_0 - a_0^2)t + \left(\frac{u_0}{2} - \frac{a_0^2}{2} - a_o a_1\right)t^2 + \left(\frac{u_0}{6} - \frac{a_0^2}{6} - \frac{a_0 a_1}{3} - \frac{2a_0 a_2}{3} - \frac{a_1^2}{3}\right)t^3}$$

If we consider u_0= 2 and compare with (3.2) we obtain

a_o = 2 , a_1= -2 , a_2= 3, $a_3 = \frac{-4}{3}$

$$\therefore u(t) = 2 - 2t + 2t^2 + \frac{-4}{3}t^3 \ldots..$$

It is required solution and the graph of this solution is



This figure is a graph showing that how the population of a species changes when a hazard function is acting in life of the species.

From the graph, we can conclude that , if there is more competition in life and more hazards then the population decreases. Here f(u) that is hazard function is taken as square of the population that means more hazard , so population of insects reaches to zero in less than one unit interval of time.

3. Raj Transform For Predator Prey Model:

In this section we use Raj Transform For Predator Prey Model .

The interaction between two species and their effect on each other is called as predator

Prey relationship. In this one species is feeding on the other species. An organism that eats or hunts other organism as food is called as predator and an organism that is killed by other organism for food is called as prey. Fox and Rabbit, Lion and Zebra are examples of predator and prey. This concept of predator prey is not only applicable for animals but it is applicable for plants too. Grasshopper and Leaf is an example of this.

Consider the system of differential equations governing predator prey model ,

$$\frac{du}{dt} = u - f(u,v) \qquad \text{.......(4.1)}$$

$$\frac{dv}{dt} = \beta[g(u,v) - v] \qquad \text{.......(4.2)}$$

With initial conditions u(0) = u_0 and v(0) = v_0, f and g are nonlinear functions of u and v. β is a positive constant. Let u and v be the solutions of this system , which are infinite series of the form $u = (t) = \sum_{n=0}^{\infty} a_n t^n$, $v = (t) = \sum_{n=0}^{\infty} b_n t^n$ and they both also satisfy the required conditions for existence of Raj Transform.

Applying Raj Transform to both sides of (4.1) and (4.2),

$$R\left(\frac{du}{dt}\right) = R(u) - R(f(u,v))$$

$$R\left(\frac{dv}{dt}\right) = \beta[R[g(u,v)] - R(v)]$$

Using the Raj transform of Derivative theorem

$s(U(s)) - su(0) = U(v) - F(s)$

$s(V(s)) - sv(0) = \beta G(s) - \beta F(s)$

Rearranging the terms and simplifying we get,

$U(s) = u_0 \frac{s}{s-1} - \frac{F(s)}{s-1}$

$V(s) = \frac{\beta G(s)}{s+\beta} + \frac{s.v_0}{s+\beta}$

Applying inverse Raj Transform

$u(t) = u_0 e^t - R^{-1}\left(\frac{F(s)}{s-1}\right)$ ....(4.3)

$v(t) = v_0 e^{-\beta t} + \beta R^{-1}\left(\frac{G(s)}{s+\beta}\right)$ ....(4.4)

Equations (4.3) and (4.4) represent the solution of system of equations (4.1) and (4.2) ,

4. Applications and Results : In this section we use results in the above section to solve some systems of differential equations arising in biotechnology and health of sciences.

Example 1: Consider the system of differential equations governing predator prey model .

$\frac{du}{dt} = u - uv$ ……(5.1)

$\frac{dv}{dt} = uv - v$ ……(5.2)

With initial conditions $u(0) = 1.3,\ v(0) = 0.6$

Suppose,

$u = \sum_{n=0}^{\infty} a_n t^n$, $v = \sum_{n=0}^{\infty} b_n t^n$ be solutions of the system (5.1) and (5.2).

$uv = a_0b_0 + (a_0b_1 + a_1b_0)t + (a_0b_2 + a_1b_1 + a_2b_0)t^2 + (a_0b_3 + a_1b_2 + a_2b_1 + a_3b_0)t^3 + \ldots..$

Applying Raj Transform ,

R(uv) = $a_0b_0R(1) + (a_0b_1 + a_1b_0)$ R(t) + $(a_0b_2 + a_1b_1 + a_2b_0)$ R (t^2) + $(a_0b_3 + a_1b_2 + a_2b_1 + a_3b_0)R(t^3) + \ldots..$

$\therefore$ R(uv) = $a_0b_0 + (a_0b_1 + a_1b_0)\frac{1}{s} + (a_0b_2 + a_1b_1 + a_2b_0)\frac{2}{s^2} + (a_0b_3 + a_1b_2 + a_2b_1 + a_3b_0)\frac{6}{s^3} + \ldots..$

Suppose , R(uv) = F(s) = G(s)

By previous section we have ,

$$R(v) = u_0\frac{s}{s-1} - \frac{F(s)}{s-1}$$

$$R(s) = 1.3\frac{s}{s-1} - \left[\frac{a_0b_0}{s-1} + \frac{(a_0b_2+a_1b_1+a_2b_0)}{s(s-1)} + \frac{2(a_0b_2+a_1b_1+a_2b_0)}{s^2(s-1)} + \frac{6(a_0b_3+a_1b_2+a_2b_1+a_3b_0)}{s^3(s-1)} + \ldots..\right]$$

Rearranging the terms ,

$$R(s) = 1.3\frac{s}{s-1} - \left[\frac{a_0b_0}{s-1} + \frac{(a_0b_2+a_1b_1+a_2b_0)s}{s(s-1)s} + \frac{2(a_0b_2+a_1b_1+a_2b_0)s^2}{s^2(s-1)s^2} + \frac{6(a_0b_3+a_1b_2+a_2b_1+a_3b_0)s^3}{s^3(s-1)s^3} + \ldots..\right]$$

Applying partial fractions and Rearranging the terms

$$R(s) = 1.3\frac{s}{s-1} - [a_0b_0\left(\frac{1}{s-1}\right) + (a_0b_1 + a_1b_0)\left(\frac{-1}{s} + \frac{1}{s-1}\right)$$
$$+2\,(a_0b_2 + a_1b_1 + a_2b_0\ )\left(\frac{-1}{s} - \frac{-1}{s^2} + \frac{1}{s-1}\right)$$
$$+6\,(a_0b_3 + a_1b_2 + a_2b_1 + a_3b_0)\left(\frac{-1}{s} - \frac{1}{s^2} - \frac{1}{s^3}\right) + \ldots.]$$

Applying inverse Raj Transform,

$$u(t) = 1.3 + (1.3 - a_0b_0)t + (1.3 - a_0b_0 - a_0b_1 - a_1b_0)\frac{t^2}{2} + (1.3 - a_0b_0 - a_0b_1 - a_1b_0 - a_0b_2 + a_1b_1 + a_2b_0)\frac{t^3}{6} + \ldots.$$

……(5.3)

Similarly, we can obtain ,

$$v(t) = 0.6 + (0.6 - a_0b_0)t + (0.6 - a_0b_0 - a_0b_1 - a_1b_0)\frac{t^2}{2} + (-0.6 - a_0b_0 - a_0b_1 - a_1b_0 - a_0b_2 + a_1b_1 + a_2b_0)\frac{t^3}{6} + \ldots.$$

…….(5.4)

From (5.3)

$\sum_{n=0}^{\infty} a_n t^n$ = 1.3 + $(1.3 - a_0b_0)$ t + $(1.3 - a_0b_0 - a_0b_1 - a_1b_0)\frac{t^2}{2}$ + $(1.3 - a_0b_0 - a_0b_1 - a_1b_0 - a_0b_2 + a_1b_1 + a_2b_0)\frac{t^3}{6} + \ldots.$

Hence $a_0 = 1.3$, $a_1 = (1.3 - a_0b_0)$, $a_2 = (1.3 - a_0b_0 - a_0b_1 - a_1b_0)$ $a_3 = (1.3 - a_0b_0 - a_0b_1 - a_1b_0 - a_0b_2 + a_1b_1 + a_2b_0)$, …… From (5.4)

$\sum_{n=0}^{\infty} b_n\, t^n = 0.6 + (0.6 - a_0 b_0)\, t + (0.6 - a_0 b_0 - a_0\, b_1 - a_1 b_0)\frac{t^2}{2} + (-\, 0.6 - a_0 b_0 - a_0\, b_1 - a_1 b_0 - a_0 b_2 + a_1 b_1 + a_2 b_0)\frac{t^3}{6} + \ldots.$

Hence $b_0 = 0.6$, $b_1 =$ (0.6 $- a_0 b_0$) , $b_2 =$ (0.6 $- a_0 b_0 - a_0\, b_1 - a_1 b_0$) , $b_3 =$ (- 0.6 $- a_0 b_0 - a_0\, b_1 - a_1 b_0 - a_0 b_2 + a_1 b_1 + a_2 b_0$) , ……

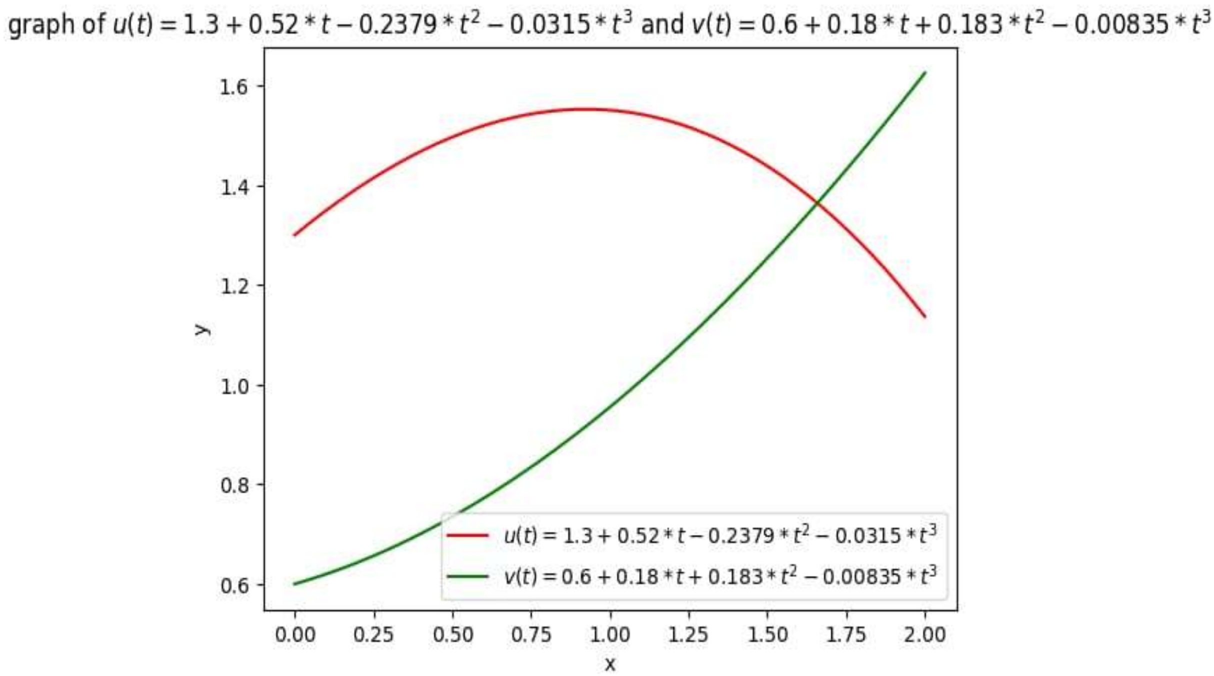
Obtaining values of a_0 , a_1 , a_2 , a_3 ,…. ,b_0 , b_1 , b_2 , b_3 ,….
We get required solution of the system of equations

$u(t) = 1.3 + 0.52t - 0.237t^2 - 0.03153t^3 + \ldots$

And

$v(t) = 0.6 + 0.18t + 0.183t^2 - 0.00835t^3 + \ldots$

Graph of the system (5.1) and (5.2) with given initial conditions is ,



This graph showing effect of predators on preys.

From this graph we can conclude that the number of predators and prey is maintained (conserved) in some limit. That means if the number of prey increases then the number of predators will also increases due to increase in food supply. Increase in the predators consumes more food. It results reduction in food supply which means number of prey reduces. A times come when the number of predators and prey becomes equal. Then increase in predator results decrease in prey. Hence there is shortage of food for predators. Thus the chain is continued and number of predators and prey always remains in some specific limit.

II. CONCLUSION :

By using Raj transform we can easily solve the mathematical models in biochemistry , health sciences , and environmental sciences , containing ordinary differential equations.

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