An Application of Schrödinger Time Independent Perturbation Theory in Determining the Energy Level (To Second Order Correction) Of Atomic Spectral

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ABSTRACT: In this research proposal title "An application of Schrödinger Time-Independent Perturbation theory in determining the Energy level (up to second order correction) of atomic spectral" the exact solutions to the schrodinger equation for first and second order perturbation will be determine. However, for most problems in quantum mechanics, it is extremely difficult to obtain exact solutions of the schrodinger equation and one has to resort to approximate methods. The three most important approximate methods are the perturbation methods, the variational methods and STIWE. For the purpose of this study, the perturbation methods will be discuss

I. I INTRODUCTION

Time-independent perturbation theory is sometime called stationary perturbation theory. Basically perturbation actually means small disturbance and we all know that such disturbance occurs in some time [1]. Perturbation theory can refer to many different techniques in different fields, for instance differential equation, Hamiltonian mechanics, operator theory. Generally you may have situation in which you have good knowledge of the system, or a situation in which your system is easy to describe and you want to know the behavior of the system in a more difficult situation by splitting your system in your known part plus a perturbation that may or may not be time [2].
Electrons are also sensitive to other properties of the nucleus they are bound to, such as nuclear mass, charge distribution or spin [3]. These nuclear properties which are simultaneously, distort the atomic spectra in different ways. Small distortions in the atomic spectra can give insight into the structure of the atom, molecules or other chemical species (Eisbarg, 1974). To account for these nuclear effects, one must often applied nuclear theory and any additional experimental information which may provide useful calibration for the observed values.

Statement of the Problem
There have been series of studies on perturbation theory that means it is a system that the equations are known and that is a change in atomic spectral due to fine structure, the hyperfine interaction and lamp shift of the nuclear structure, it is still not possible to predict how it will behave in the feature. There is no more information on nuclear structure but not even a single study has been carry out on nuclear structure effect on atomic spectral.

Aim and objectives of the study
The main aim of this work is to apply the time independent perturbation theory on the nuclear structure effects on atomic spectra on the specific objectives of the study are;
    i.   to view the classical and quantum mechanical equation of perturbation theory,
    ii.   to apply the time independent perturbation theory and calculate some nuclear structure effect and
    iii.   generating function of hermit polynomials (Rodrigure formula).

Significance of the study
        The study of the nuclear perturbation theory is of immense benefit to the physics department in university and other tertiary institution in Nigeria as the findings of the study will educate the entire population under the umbrella of the study on the nuclear structure, the

factors affecting the orbit coupling of the electron, the demonstration of the hyperfine interaction effect and also the demonstration on the appearance of relativistic correction of the kinetic energy in the nuclear structure. The study will serve as a respiratory of information to other, researchers on the above topic. Finally the study still contributes to the body of existing literature and knowledge in this field of study and provides a basic for further research.

Scope of the Study
In this research work, the determination of time-independent perturbation theory, there are many factors that affect the nuclear structure these are the energy correction of perturbation, fine structure, hyperfine structure, lamb shift, isotope shift, finite size module. In this work we deal only with perturbation theory.

Materials and Methods
The materials that will be use are Schrödinger, first and second order perturbation theory.

Methods
First and second order Perturbation theory
The problem is formulated as follows:
Suppose there is for a system
H = H_0 + V   (1)
Where H_0 is the unperturbed Hermittonian the solution of which are known and V is small perturbation (disturbance).
To emphasize that V in equation (1) is small recall that
H = H_0 + λV

Where λ is some parameter that emphasize the smallness of V
The unperturbed problem is
$H_0\psi_0 = \varepsilon_0\psi_0$
Where ψ_0 and ε_0 are known
We now concentrate on some particular unperturbed state. ε_0Represent energy of some unperturbed state not necessary the ground state
$\therefore$ H_0 evolves continuously into H as λ goes from 0 to 1 and ψ_0 goes continuously into wave function ψ which are perturbed wave function.
Hψ = $\varepsilon\psi$ (2)
The unperturbed energy ε_0 goes continuously into perturbed energy ε.
The assumption "going continuously into" is express mathematically by the facts that we can express the energy and wave function in power series of λ.
$\psi = \psi_0 + \lambda\psi_1 + \lambda^2\psi_2 + \lambda^2\psi_2 + \lambda^3\lambda_3 + \dots$ (3)

$\varepsilon = \varepsilon_0 + \lambda\varepsilon_1 + \lambda^2\varepsilon_2 + \lambda^3\varepsilon_3 + \dots$ (4)
Where $0 \leq \lambda \leq 1$
The subscript on $\varepsilon^{'s}$ and $\psi^{'s}$ refers to the order of perturbation ε_1 is the first order correction tomthe energy. ε_2 is the second order, similarly ψ_1 and ψ_2 are first and second order correction to wave function
From equation (2)
$H\psi = \varepsilon\psi$
Substitute equation (3) and (4) into equation (2) above
$(H_0 + \lambda v)(\psi_0 + \lambda\psi_1 + \lambda^2\psi_2 + \cdots) = (\varepsilon_0 + \lambda\varepsilon_1 + \lambda^2\varepsilon_2 + \cdots)(\psi_0 + \lambda\psi_1 + \lambda^2\psi_2 + \cdots)$
By expanding equation 3.7 we get
$H_0\psi_0 + \lambda H_0\psi_1 + \lambda^2 H_0\psi_2 + \lambda v\psi_0 + \lambda^2 v\psi_1 + \lambda^3 v\psi_2 + \dots$
$=$
$\varepsilon_0\psi_0 + \lambda\varepsilon_0\psi_1 + \lambda^2\varepsilon_0\psi_2 + \lambda\varepsilon_1\psi_0 + \lambda^2\varepsilon_1\psi_1 + \lambda^3\varepsilon_1\psi_2 + \lambda^2\varepsilon_2\psi_0 + \lambda^3\varepsilon_2\psi_1 + \cdots$
Rearranging
$H_0\psi_0 + \lambda(H_0\psi_1 + v\psi_0) + \lambda^2(H_0\psi_2 + v\psi_1) + \lambda^3(\dots.) + \cdots = \varepsilon_0\psi_0 + \lambda(\varepsilon_0\psi_1 + \varepsilon_1\psi_0) + \lambda^2(\varepsilon_0\psi_2 + \varepsilon_1\psi_1 + \varepsilon_2\psi_0) + \lambda^3(\dots)$ (5)
Equating coefficient of power of λ on both side
Zero order-λ^0: $H_0\psi_0 = \varepsilon_0\psi_0$

First order-λ^1 : $H_0\psi_1 + v\psi_0 = \varepsilon_0\psi_1 + \varepsilon_1\psi_0$ (6)
Second order-λ^2 : $H_0\psi_2 + v\psi_1 = \varepsilon_0\psi_2 + \varepsilon_1\psi_1 + \varepsilon_2\psi_0$ (7)
n^{th} order-λ^n :$H_0\psi_n + v\psi_{n-1} = \varepsilon_0\psi_n + \varepsilon_{n-1}\psi_{n-1} + \varepsilon_n\psi_0$
Here we deal with first and second order perturbation only. Equation (6) and (7) enable us to calculate the first and second order perturbation.
We demand that ψ and ψ both normalize to unity.
$<\psi/\psi> = 1 = (\{\psi_0 + \lambda\psi_1 + \lambda^2\psi_2 \dots\}) (\{\psi_0 + \lambda\psi1 + \lambda2\psi2 \})=1$
By expanding the bracket
$(\psi_0, \psi_0) + (\psi_0, \lambda\psi_1) + (\psi_0, \lambda^2\psi_2) + (\lambda\psi_1, \psi_0) + (\lambda\psi_1, \lambda\psi_1) + (\lambda\psi_1, \lambda^2\psi_2) + (\lambda^2\psi_2, \psi_0) + (\lambda^2\psi_2, \lambda\psi_1) + (\lambda^2\psi_2, \lambda^2\psi_2)=1$
$(\psi_0, \psi_0) + \lambda\{(\psi_0, \psi_1 + (\psi_1, \psi_1)\} + \lambda^2\{(\psi_0, \psi_2) + (\psi_1, \psi_2) + (\psi_2, \psi_0) = 1$ (8)
$\therefore (\psi_0, \psi_1) + (\psi_1, \psi_0) = 0, \lambda \neq 0$

          (9)
and
$(\psi_0, \psi_2) + (\psi_1, \psi_2) + (\psi_2, \psi_0) = 0$

$\lambda \neq 0$

Our perturbation theory will be apply to bound state wave function which are real.

$(\psi_0, \psi_1) = (\psi_1, \psi_0)^* = (\psi_1, \psi_0)$

$\therefore (\psi_0, \psi_1) = (\psi_1, \psi_0$

Hence equation (9) implies that

$(\psi_0, \psi_1 = 0)$ (10)

The first order correction ψ_0 is orthogonal to the unperturbed function ψ_0. we cannot say

Anything about (ψ_1, ψ_1). Only that the ψ_0's ψ_0's

Take the scalar product of $(H_0\psi_1)(v\psi_0)$ with ψ_0

$(\psi_0, H_0\psi_1)(\psi_0, v\psi_0) = \varepsilon_0(\psi_0, \psi_1) + \varepsilon_1(\psi_0, \psi_0)$ (11)

Since $(\psi_0, \psi_1) = 0$

And $(\psi_0, \psi_0) = 1$

Equation (11) becomes

$(\psi_0, H\psi_1)(\psi_0, v\psi_0) = \varepsilon_1$

Assume H_0 is Hermitian

$H_0 = H_0^+$

$(H_0\psi_0, \psi_1 = (\psi_0, H_0^+\psi_1) = (\psi_0, H_0\psi_1)$ (12)

Using equation (12)

$(H_0\psi_0, \psi_1) + (\psi_0, v\psi_0) = \varepsilon_1$

Or

$\varepsilon_0(\psi_0, \psi_1) + (\psi_0, v\psi_o) = \varepsilon_1$

$\varepsilon_1 = (\psi_0, v\psi_0)$

Equation (12) is the first order correction by perturbation theory.

To obtain the second order correction to the energy, take the product of equation (10) with ψ_0 on the left.

$(\psi_0, H_0\psi_2) + (\psi_0, v\psi_1) = \varepsilon_0(\psi_0, \psi_2) + \varepsilon_1(\psi_0, \psi_1) + \varepsilon_2(\psi_0, \psi_0)$

$\varepsilon_0(\psi_0, \psi_2) + \varepsilon_2$

Since H is Hermitian

$(H_0\psi_0, \psi_2) = (\psi_0, H_0\psi_2)$

$\therefore (H_0\psi_0, \psi_2) + (\psi_0, v\psi_1)$
$= \varepsilon_2(\psi_0, \psi_2) + \varepsilon_2$

Or

$\varepsilon_0(\psi_0, \psi_2) + (\psi_0, v\psi_1) = \varepsilon_0(\psi_0, \psi_2) + \varepsilon_2$

$\therefore \varepsilon_2 = (\psi_0, v\psi_1)$ (13)

Equation 3.23 is the second order energy correction.

Scrhodinger's Time Independent Wave Equation (STIWE)

For a plane rigid rotator with a uniform electric field applied in the plane is

$-\frac{\hbar^2}{2I}\frac{d^2\psi}{d\psi^2} - (\varepsilon + \ell E_e \cos\varphi) = 0$ (14)

Where I = moment of inertia

φ = angle of rotation

e = electric dipole moment

E_e = electric field intensity.

Regarding the term $\ell E_e \cos\varphi$ as a perturbation, show that the energy correction to second order is given as

$\varepsilon_m = \frac{m^2\hbar^2}{2I} + \frac{I\rho^2 E^2_e}{\hbar^2 (4m^2-1)}$

where m=0, $\pm 1, \pm 2$ ...

$-\frac{\hbar^2}{2I}\frac{d^2\psi}{d\varphi^2} - (\varepsilon + \rho E_e \cos\varphi)\psi = 0$

For the unperturbed rigid rotator STIWE is

$-\frac{\hbar^2}{2I}\frac{d^2\psi}{d\varphi^2} = \varepsilon\psi$

$\frac{\hbar^2}{2I}\frac{d^2\psi}{d\varphi^2} + \varepsilon\psi = 0$

where $H_o = \frac{\hbar^2}{2I}\frac{d^2}{d\varphi^2}$

$H' = \rho E_e \cos\varphi$.

$\varepsilon_m^0 = \frac{m^2\hbar^2}{2I}$ (15)

where m=0, $\pm 1, \pm 2$.

$\psi_m^0 = \frac{1}{\sqrt{2\pi}}e^{im\varphi}$

$\varepsilon_m^{(1)} = H_{mn}^1 = <\psi_m^0|H'|\psi_m^0>$

First order correction.

$= \int_0^{2\pi}(\psi_m^0)^* \rho E_e \cos\varphi \psi_m^0 d\varphi$

$= \frac{1}{2\pi}\int_0^{2\pi} e^{-im\varphi} \rho E_e \cos\varphi e^{im\varphi} d\varphi$

$= \frac{\rho E_e}{2\pi}\int_0^{2\pi} \cos\varphi \, d\varphi = 0.$

$\varepsilon_m^{(2)} = \sum_{m=1}^{\infty} \frac{|H'mm|^2}{\varepsilon_n^0 - \varepsilon_m^0}$

$H_{nm}^{/} = <\psi_m|H'|\psi_m>$

$= \frac{1}{2\pi}\int_0^{2\pi} e^{+in\varphi} \rho E_e \cos\varphi e^{-im\varphi} d\varphi$

$$= \frac{\rho E_e}{2\pi} \int_0^{2\pi} e^{i(n-m)\varphi} \cos \varphi \, d\varphi$$

But $\cos \varphi = \frac{1}{2}(e^{i\varphi} + e^{-i\varphi})$

$$H'_{nm} = \frac{\rho E_e}{4\pi} \int_0^{2\pi} e^{i(n-m)\varphi} . \{e^{i\varphi} + e^{-i\varphi}\} \, d\varphi$$

$$= \frac{\rho E_e}{4\pi} \{\int_0^{2\pi} e^{i(n-m+1)\varphi} \, d\varphi + \int_0^{2\pi} e^{i(n-m-1)\varphi} \, d\varphi\}.$$

$$\int_0^{2\pi} e^{in\varphi} \, d\varphi = 0$$

where r=0
∴integrals vanishes unless either $\{\begin{smallmatrix} n-m+1=0 \\ n-m-1=0 \end{smallmatrix}$

$$= n - m = \pm 1$$

$H'_{mn} = 0$ unless   n = m+1

n = m − m

$$H'_{m,n+1} = \frac{\rho E_e}{4\pi} . 2\pi = \frac{\rho E_e}{2}$$

$$\therefore \varepsilon^{(2)}_{m \, n} = \sum \frac{|H'_{mn}|^2}{\varepsilon^0_m - \varepsilon^0_n} \qquad (16)$$

$$= \frac{|H_{m,m-1}|^2}{\varepsilon^0_{m-1} - \varepsilon^0_n} + \frac{|H_{m,m+1}|^2}{\varepsilon^0_{m+1} - \varepsilon^0_m}$$

$$\varepsilon^0_m = \frac{m^2 \hbar^2}{2I},$$
$$\varepsilon^0_{m-1} = \frac{(m-1)^2 \hbar^2}{2I} \qquad (17)$$
$$\varepsilon^0_{m+1} = \frac{(m+1)^2 \hbar^2}{2I}$$

$$\varepsilon^{(2)}_m = \frac{\rho^2 E_e^2}{4}\left[\frac{1}{\frac{(m-1)^2 \hbar^2}{2I} - \frac{m^2 \hbar^2}{2I}} + \frac{1}{\frac{(m+1)^2 \hbar^2}{2I} - \frac{m^2 \hbar^2}{2I}}\right]$$

$$= \frac{\rho^2 E_e^2}{4} . \frac{2I}{\hbar^2} \left[\frac{1}{(m-1)^2 - m^2} + \frac{1}{(m+1)^2 - m^2}\right]$$

$$= \frac{\rho^2 E_e^2 I}{2\hbar^2} \left[\frac{1}{-(2m-1)} + \frac{1}{(2m+1)}\right]$$

$$= \frac{\rho^2 E_e^2 I}{2\hbar^2} \left[\frac{(2m-1) - (2m+1)}{(4m^2 - 1)}\right]$$

$$= -\frac{\rho^2 E_e^2 I}{\hbar^2(4m^2 - 1)}$$

$$\therefore \varepsilon_m = \varepsilon^0_n + \varepsilon^2_m$$

$$\varepsilon_m = \frac{m^2 \hbar^2}{2I} - \frac{\rho^2 E_e^2 I}{(4m^2 - 1)\hbar^2}.$$

The separated model correspond to functions V(r) of the form

$$V(r) = -\frac{3}{2}\frac{Ze^2}{R} + \frac{Ze^2}{2R^3}r^2 (r < R)$$

$$V(r) = -\frac{Ze^2}{R^2}r^2 (r < R)$$

which can be recalled for r < R in terms of dimensionless quantities as
$$\frac{V(r)}{R} + \frac{Ze^2}{2R^3}r^2 (r < R)$$

$$V(r) = -\frac{Ze^2}{R}(r < R)$$

Which can be rescaled for r < R in terms of dimensionless quantities as
$$\frac{V(r)}{mc^2} = -(b' - \xi r'^2)$$

$$\frac{V(r)}{mc^2} = \frac{\alpha Z}{R'} \qquad (18)$$

Where
$$r' = \left(\frac{mc}{\hbar}\right)r$$
$$R' = \left(\frac{mc}{\hbar}\right)R$$
$$b' = \frac{3}{2}\frac{\alpha Z}{R} \qquad (19)$$
$$\xi = \frac{3}{2}\frac{\alpha Z}{R'^3}$$

It is sufficient to construct a series solution of radial equation for r < R (Likta, 2018). For this purpose, it is useful to assume the forms.
$$g(r') = Ar'^{v-1} \sum_{n=0}^{\infty} a_n r^m$$
$$f(r') = Br'^{v-1} \sum_{n=0}^{\infty} b_n r^m \qquad (20)$$

Substitution of equation 3.59 results in relations for the coefficients of a_n and b_n that have non trivial solutions only under either of the two conditions
V = - k, b_0 = 0 k< 0   (21)
V = + k, a_n = 0 k > 0   (22)
The two choices result in the separate solutions
$$g(r') = Ar'^{|K|-1}(1 + a_2 r'^2 + a_4 r'^4 + \cdots), f(r') = Ar'^{|K|-1}(b_1 + b_3 r'^2 + b_5 r'^4 + \cdots) K < 0 \quad (23)$$
$$g(r') = Br'^{|K|-1}(a_1 + a_3 r'^2 + a_5 r'^4 \dots), f(r') = Br'^{|K|-1}(1 + b_2 r'^2 + b_4 r'^4 + \cdots) K > 0 (24)$$

For V(r) in the form (24), the coefficients of the leading terms have the forms
$$a_1 = \frac{Q_+}{2|k|+1}, a_2 = \frac{Q_+}{2(2|k|+1)}, a_3 = -\frac{1}{2|k|+3}\left(\frac{Q_+ Q_- Q_+}{2(|k|+1)} + \xi\right),$$
$$a_4 = \frac{Q_+ Q_-}{8(2|k|+3)(2|k|+1)} + \frac{\xi}{4}\left[\frac{Q_+}{2|k|+3} + \frac{Q_-}{2|k|+1}\right],$$
$$a_4 = \frac{Q_+}{2|k|+5}(Q_+ b_+ + \xi b_2)$$

$b_1 = -\frac{Q_-}{2|k|+1}$,

$b_2 = -\frac{Q_+Q_-}{2(2|k|+1)}$, $b_3 = -\frac{1}{2(2|k|+1)}(\frac{Q_-Q_+Q_-}{2(2|k|+1)} + \xi)$,

$b_4 = \frac{(Q_+Q_-)^2}{8(2|k|+3)} + \frac{\xi}{4}[\frac{Q_-}{2|k|+3} + \frac{Q_+}{2|k|+1}]$, $b_5 =$

$-\frac{1}{2|k|+3}(Q_-Q_+ + \xi Q)$ (25)

where

$Q_\pm \equiv E' + b' \pm 1$ (26)

Where $b' = \frac{\alpha Z}{R'}$

$V(r) = -e\int_r^R R\,dr' - \frac{ze^2}{R}$ (27)

$= \frac{ze^2}{R^3}\int_r^R r'\,dr' - \frac{ze^2}{R}$

$= -\frac{ze^2 r^2}{2R^3}\Big|_r^R - \frac{ze^2}{2R^3}$

$= \frac{ze^2 r^2}{2R^3} - \frac{ze^2}{2R}$

So we see that

$V(r) = -\frac{ze^2}{r}$ r > R (28)

$V(r) = \frac{ze^2 r^2}{2R^3} - \frac{ze^2}{2R}$ r ≤ R

$H = H^O + H'$

$H' = 0$ r > R

$H' = \frac{ze^2 r^2}{2R^3} - \frac{ze^2}{2R} + \frac{ze}{r}$ r ≤ R

$E' = \langle nLm|H'|nLm\rangle = R_{nl}^*\int_r^R(\frac{ze^2 r^2}{2R^3} - \frac{ze^2}{2R} + zer)Rnlr2\,dr$ (29)

Using equation (29) to obtain the 2nd order Perturbation theory (Likta, 2018).

II. RESULTS

Equation (29) will be use to obtain the 2nd order perturbation theory below

$E^2 = \langle nLm|H'|nLm\rangle = \int_0^R R_{nl}^*(\frac{ze^2 r^2}{2R^3} - \frac{ze^2}{2R} + \frac{ze}{r})R_{nl}r^2\,dr$

$= \int_0^R R_{nl}^*(\frac{ze^2 r^2}{2R^3} - \frac{ze^2}{2R} + \frac{ze}{r})R_{nl}r^2\,dr$

$= \frac{1}{2}[\frac{ze^2 r^2}{2R^3} - \frac{ze^2}{2R} + \frac{ze}{r}]_0^R$ (30)

$= \frac{1}{2} \times \frac{Ze^2}{2\pi}(\frac{a_0^3}{2R^3} - \frac{a_0 R^2}{2R} + \frac{R^3}{a_0} - \frac{a_0 Z^{\frac{2R}{a_0}}}{a_0})$

$= \frac{Ze^2}{4\pi}(\frac{3a_0^3 - 3a_0 R^2 a_0^2 + R^2 3R^3 - 3a_0 Z^{-\frac{2R}{a_0}}}{2R^3 a_0})$

$= \frac{Ze^2}{4\pi}(\frac{3a_0^3 - 3a_0 R^2 + 3R^3 - 3a_0 Z^{-\frac{2R}{a_0}}(a_0 + R)^2}{2R^3 a_0})$

(31)

Equation (31) is the 2nd order perturbation theory where

$a_o = \frac{4\pi\epsilon_o \, h^2}{me^2}$

$R = r_0 A^{\frac{1}{3}}$

$r_0 = 1.2 \times 10^{-15} m$

$R = 1.2 \times 10^{-15} A^{\frac{1}{3}}$

for α particle
A = (A + 4)

$\therefore R = 1.2 \times 10^{-15}(A + 4)^{\frac{1}{3}}$

for β and γ
A = (A + 0)

$R = 1.2 \times 10^{-15} A^{\frac{1}{3}}$

Equation (31) was used to calculate energy (E) of the second order correction of the atomic spectral according to table 1 to 3

Table 1 Heavy Nuclei of Atomic spectral for second order

S/N	Nucleus	A	Z	M	R	a_o	E (Exptal)	E (Cal.)

1	Te	108	52	4.20	6.87E-15	1.15E-41	-18387.20	3.87E+22
2	I	110	53	3.44	6.95E-15	1.41E-41	-19101.20	2.67E+22
3	Ce	133	58	97.1	7.43E-15	4.98E-43	-22875.2	2.38E+25
4	Nd	129	60	1.83	7.46E-15	2.64E-41	-24480.0	9.02E+21
5	Pm	128	61	2.24	7.47E-15	2.15E-41	-25302.80	1.40E+22
6	Sm	129	62	4.31	7.53E-15	1.12E-41	-26139.20	5.29E+22
7	Gd	135	64	2.14	7.56E-15	2.26E-41	-27852.80	1.38E+22
8	Dy	141	66	2.87	7.59E-15	1.68E-41	-29620.80	2.64E+22
9	Dy	144	66	4.21	7.64E-15	1.15E-41	-29620.82	5.64E+22
10	Er	171	68	1.95	7.69E-15	2.48E-41	-31443.20	1.28E+22
11	Tm	154	69	5.10	7.58E-15	9.49E-42	-32378.80	9.10E+22
12	Yb	157	70	2.32	7.77E-15	2.08E-41	-33320.00	1.89E+22
13	Lu	155	71	5.57	7.62E-15	8.68E-42	-34278.80	1.14E+23
14	Hf	174	72	2.50	7.84E-15	1.93E-41	-35251.20	2.31E+22
15	Ta	158	73	5.97	7.64E-15	8.10E-42	-36237.20	1.39E+23
16	W	160	74	1.80	7.96E-15	2.68E-41	-37236.80	1.25E+22
17	Re	162	75	6.09	7.69E-15	7.94E-42	-38250.00	1.51E+23
18	Os	186	76	2.76	7.98E-15	1.75E-41	-39276.80	3.07E+22
19	Ir	166	77	6.56	7.74E-15	7.37E-42	-40317.20	1.84E+23
20	Pt	190	78	3.18	8.03E-15	1.52E-41	-41371.20	4.28E+22
21	Au	173	79	6.73	7.83E-15	7.18E-42	-42438.80	2.02E+23
22	Hg	172	80	7.35	7.82E-15	6.58E-42	-43520.00	2.47E+23
23	Ti	177	81	6.90	7.86E-15	7.01E-42	-44614.80	2.22E+23
24	Pb	181	82	7.02	8.18E-15	6.89E-42	-45723.20	2.26E+23
25	Bi	185	83	4.91	8.23E-15	9.85E-42	-46845.20	1.12E+23
26	Fr	214	87	8.48	8.28E-15	5.70E-42	-51469.20	3.66E+23

Table 2 Medium Nuclei of Atomic spectral for second order

S/N	Nucleus	A	Z	M	R	a_o	E (Exptal)	E (Cal.)
1	Kr	71	36	09.10	4.5549E-15	5.31E-42	-8812.80	1.31E+23
2	Rb	76	37	04.10	4.6594E-15	1.18E-41	-9309.20	2.76E+22
3	Rb	91	37	05.85	4.9477E-15	8.26E-42	-9309.24	5.28E+22
4	Sr	77	38	05.60	4.6798E-15	8.63E-42	-9819.20	5.40E+22
5	Sr	79	38	04.10	4.7199E-15	1.18E-41	-9819.22	2.87E+22
6	Sr	95	38	06.09	5.0192E-15	7.94E-42	-9819.23	5.95E+22
7	Y	80	39	05.50	4.7398E-15	8.79E-42	-10342.80	5.42E+22

8	Y	92	39	03.64	4.9658E-15	1.33E-41	-10342.82	2.26E+22
9	Y	94	39	04.92	5.0015E-15	9.82E-42	-10342.84	4.11E+22
10	Zr	81	40	06.10	4.7594E-15	7.92E-42	-10880.00	6.98E+22
11	Zr	95	40	03.68	5.0192E-15	1.31E-43	-10880.01	2.41E+26
12	Zr	99	40	03.54	5.0887E-15	1.37E-41	-10880.02	2.20E+22
13	Nb	96	41	0748	5.0367E-15	6.46E-44	-11430.80	1.04E+27
14	Nb	98	41	01.99	5.0715E-15	2.43E-41	-11430.82	7.32E+21
15	Nb	101	41	43.00	5.1227E-15	1.12E-42	-11430.83	3.39E+24
16	Md	86	42	03.90	4.8554E-15	1.24E-41	-11995.20	3.08E+22
17	Mo	86	42	05.30	4.8554E-15	9.12E-42	-11995.22	5.69E+22
18	Mo	87	42	1090	4.8742E-15	4.46E-44	-11995.23	2.38E+27
19	Te	90	43	05.20	4.9295E-15	9.30E-42	-12573.2	5.66E+22
20	Te	91	43	02.20	4.9477E-15	2.20E-41	-12573.22	1.01E+22
21	Te	103	43	04.60	5.1563E-15	1.05E-41	-12573.24	4.23E+22
22	Ru	107	44	05.30	5.2222E-15	9.12E-42	-13164.80	5.81E+22
23	Ru	93	44	1187	4.9837E-15	4.07E-44	-13164.83	3.05E+27
24	Ru	105	44	02.30	5.1895E-15	2.10E-41	-13164.84	1.10E+22
25	Rh	109	45	92.00	5.2545E-15	5.25E-43	-13770.00	1.82E+25
26	Rh	106	45	02.60	5.2059E-15	1.86E-41	-13770.01	1.47E+22
27	Pd	110	46	01.03	5.2706E-15	4.70E-44	-14388.80	2.37E+27
28	Pd	109	46	02.80	5.2545E-15	1.73E-42	-14388.81	1.76E+24
29	Ag	112	47	05.30	5.3023E-15	9.12E-44	-15021.20	6.53E+26
30	Ag	112	47	03.94	5.3023E-15	1.23E-41	-15021.23	3.61E+22
31	Cd	101	48	04.50	5.1227E-15	1.07E-41	-15667.20	5.08E+22
32	Cd	115	48	01.62	5.3492E-15	2.98E-41	-15667.23	6.31E+21
33	In	121	49	03.70	5.4407E-15	1.31E-41	-16326.80	3.37E+22
34	In	123	49	03.70	5.4705E-15	1.31E-41	-16326.83	3.37E+22

Table 3 Light Nuclei of Atomic spectral for second order

S/N	Nucleus	A	Z	M	R	a_o	E (Exptal)	E (Cal.)
1	C	16	06	0120	4.40E-15	4.01E-43	-244.8	1.32348E+24
2	N	17	07	0871	4.12E-15	5.55E-44	-333.2	1.00536E+26
3	O	20	08	1060	4.92E-15	4.57E-44	.-435.2	1.62135E+26

|								
4	F	23	09	1700	5.28E-15	2.84E-44	-550.8	4.95971E+26
5	Ne	24	10	0472	5.39E-15	1.02E-43	-680.0	4.61902E+25
6	Na	31	11	1480	6.02E-15	3.26E-44	-822.8	4.92869E+26
7	Mg	22	12	0582	5.16E-15	8.31E-44	-979.2	1.05491E+26
8	Al	34	13	0929	6.41E-15	5.20E-44	-1149.2	2.53748E+26
9	Si	33	14	1850	6.32E-15	2.62E-44	-1332.8	1.18151E+27
10	P	28	15	1780	5.92E-15	2.72E-44	-1530.1	1.34139E+27
11	P	35	15	1570	6.51E-15	3.07E-44	-1740.82	9.53761E+26
12	S	30	16	0677	6.02E-15	7.14E-44	-1965.20	2.17439E+26
13	S	31	16	1270	6.12E-15	3.82E-44	-1965.21	7.47804E+26
14	S	40	16	0212	6.96E-15	2.28E-43	-1956.22	1.84545E+25
15	CI	33	17	0841	6.32E-15	5.75E-44	-2203.02	3.60871E+26
16	CI	39	17	1270	6.87E-15	3.81E-44	-2202.22	7.53836E+26
17	CI	41	17	0167	7.04E-15	2.89E-43	-2458.8	1.27692E+25
18	Ar	34	18	0667	6.41E-15	7.25E-44	-2720.00	2.50398E+26
19	Ar	44	18	0183	7.30E-15	2.65E-43	-2720.01	1.65211E+25
20	K	43	19	0373	7.21E-15	1.30E-43	-2998.8	7.76148E+25
21	Ca	38	20	1570	6.78E-15	3.08E-44	-3291.20	1.61837E+27
22	Ca	39	20	2520	6.87E-15	1.92E-44	-3291.22	4.13275E+27
23	Sc	51	21	1440	7.86E-15	3.36E-44	-3597.21	1.29356E+27
24	Ti	42	22	0611	7.13E-15	7.91E-44	-3916.80	2.83015E+26
25	Ti	43	22	2290	7.21E-15	2.11E-44	-3291.22	3.92064E+27
26	V	45	23	40.1	7.38E-15	1.21E-42	-3597.21	1.28635E+24
27	Cr	56	24	90.6	7.70E-15	4.80E+25	-3916.80	-2.3663E-06
28	Cr	58	24	0683	8.38E-15	2.54E+25	-3916.83	-9.7494E-07
29	Mn	57	25	0122	8.30E-15	3.20E+26	-4250.0	-1.3667E-05
30	Mn	61	25	0629	8.59E-15	1.11E+26	-4250.4	-4.2628E-06
31	Fe	52	26	0930	7.93E-15	1.17E+26	-4596.8	-6.1906E-06
32	Co	55	27	0931	8.16E-15	1.68E+26	-4957.2	-8.8209E-06
33	Ni	56	28	0158	8.23E-15	1.34E+27	-5331.2	-7.3884E-05
34	Cu	67	29	0185	9.00E-15	1.51E+27	-5718.8	-6.7873E-05
35	Zn	58	30	0203	8.38E-15	1.73E+27	-6120.0	-0.00010379
36	Ga	61	31	0088	8.59E-15	4.94E+27	-6120.0	-0.00029262
37	Ge	66	32	43.8	8.94E-15	1.20E+28	-6534.8	-0.00067355
38	As	67	33	0123	9.00E-15	5.10E+27	-6963.2	-0.00029751
39	Se	70	34	49.5	9.20E-15	1.48E+28	-7860.8	-0.00086031
40	Br	73	35	64.9	9.40E-15	1.31E+28	-8330.0	-0.00075723
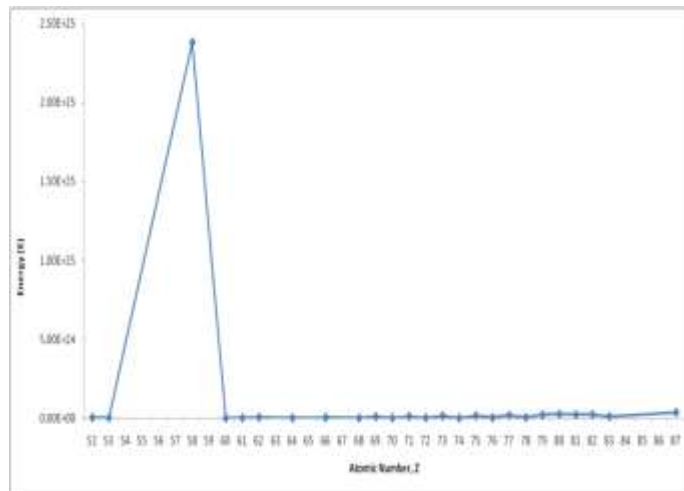
Fig. 1   Energy versus Z for the second order Heavy Nuclei

Figure 4.1 shows the plots of energy (E) verses Atomic number (Z) for the second order Heavy nuclei. The graph shows that the Heavy Nucleus Cerium (Ce) with atomic number 58, have the highest energy (E) which corresponds to the value 2.38×10^{25} J while the Nucleus Tungsten (w) with atomic number (Z=74) has the lowest value of energy corresponding to the value 1.25×10^{22} J. This is attributed to the atomic mass (M) Nuclei, because the atomic mass of Ce is the greatest according to table 4.1 while the atomic mass of W is the lowest. This shows that the grater the atomic number of the Heavy nuclei the greater the energy (E) and vice versa.
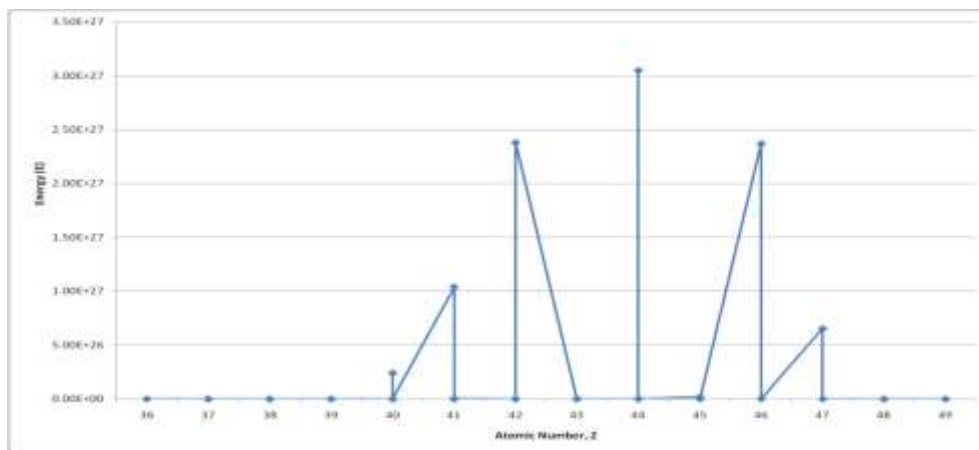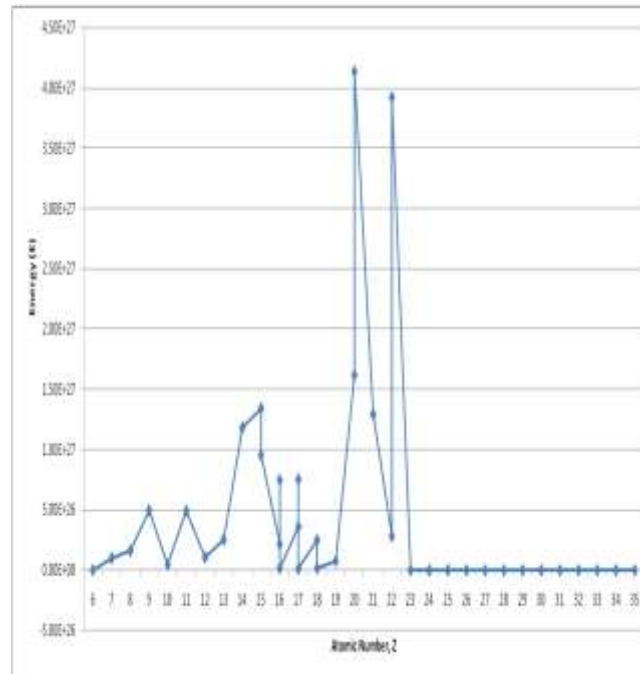

Fig. 2   Energy versus Z for the second order Medium Nuclei

Figure 4.2 shows the plots of energy (E) verses Atomic number (Z) for the second order Medium nuclei. The graph shows that the Medium Nucleus Ruthenium (Ru) with atomic number Z (44), have the highest energy (E) which corresponds to the value 3.05×10^{29} J while the Nucleus Cadmium (Cd) with atomic number (Z=48) has the lowest value of energy corresponding to the value 6.31×10^{21} J. This ii because the larger the atomic mass the greater the energy of the second order Medium correction of the atomic spectral and vice versa. This shows that the grater the atomic number of the Heavy nuclei the greater the energy (E) and vice versa. Ru is a solid at room temperature while Cd is a soft silvery white metal.

Fig. 3   Energy versus A for the second order Light Nuclei

Figure 4.3 shows the plots of energy (E) verses Atomic number (Z) for the second order Light nuclei. The graph shows that the Light Nucleus Calcium (Ca) with atomic number 39, have the highest energy (E) which corresponds to the value 4.1328×10^{27} J while the Nucleus Vanadium (v) with atomic number (Z=45) has the lowest value of energy corresponding to the value 1.286×10^{24} J. This shows that the grater the atomic number of the Light nuclei the greater the energy (E) and vice versa.

III.   SUMMARY
The equation 4.5 above is the second order perturbation theory, where a_0, R, r_0, A, β and γ Have their usual meaning. The dependence correction of energy level of the atomic spectral is in the form of the potential energy level. The size of the nuclear is very small where Z = 1 and have higher values, the effect will be very large and hence can be measured easily.

IV.   CONCLUSION
The function of the potential that has separated the dependence variable on the radial coordinate for r greater or less that a given value R have been achieved for the energy level of the second order correction of the atomic spectral. This method has advantage of allowing 2nd order correction of the atomic spectral to the ground state and the method is easier with accuracy higher than the other perturbation theory.

V.   RECOMMENDATION
Based on this research work, I recommend that similar research work should be carried out with different mathematical operations such that modified equations can be computer with the experimental data.

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