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Application of the Galerkin method in the case of the optimization of the time required to grill potato slices

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ABSTRACT
this article presents the solution by numerical simulations of the amount of heat required for roasting a thin piece of potato 5mm thick and cut from a medium tuber of the Spunta variety, the approach to this problem requires a finite element study, we have the Galerkin method which allows the discretization of the spatial domain and provides an approximate solution to the heat conduction equation between the piece and the metal pellet. By increasing the number of basis functions, the accuracy of the solution can be improved. However, it is important to note that the Galerkin method is a numerical approximation technique and can introduce errors depending on the choice of the basis functions and the discretization scheme, we seek to apply this method to prepare a type of chips, since of these products that taste good and also the potato rich as a food that requires study to optimize time and energy, the results show that after 4 minutes in total on both sides; and to avoid burning the cooking it is necessary to alternate the conduction phases, place on a plate heated to more than 450°K; then this application these conditions is available either at home or in the industry; we can say that the fries are well cooked and in a simple way.

Keywords: Potato slice, Galerkin method, Numerical simulation, heat transfer, thermal conduction

I. INTRODUCTION

The finite element method divides a complex domain into a set of smaller and simpler subdomains called "finite elements". Together, these elements form a network that spans the gradient [1]. The main idea is to approximate a PDE solution by creating an approximation function for each particular element and then aggregating these local approximations into a global solution for the entire domain. Among the approaches used, Galerkin's method is a powerful technique for simulating and solving partial differential equations (PDEs) using numerical approximations. It is widely used in various scientific fields First of all, formulate the problem the physical problem to be simulated is formulated as a set of PDEs, these equations describe the behavior and evolution of the system studied the PDEs are generally derived from the basic principles and laws of physics [2,3,4].The Galerkin method starts with the weak formulation of PDEs, the goal is to multiply PDEs with modified test functions and integrate them into the simulation domain [5]. This leads to a system of integral equations known as the weak form of PDEs, fundamental functions and finite element discrimination In the Galerkin method, the simulation domain is divided into smaller subdomains called finite elements. In each finite element, unknown fields (such as temperature) and test functions are approximated using basis functions [6,7]. These basic functions are specified to be defined multiple times in each element and persisted across the element's interfaces. Galerkin's approximation, Galerkin's method uses the concept of reducing the residual, which is the difference between the actual PDEs and their estimates. By minimizing the residual over the finite element space, a set of algebraic equations is derived [8, 9]. It involves applying basis functions and test functions to the weak form of the PDEs and merging each specific element, grouping and solving, the individual contributions of each

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determinant are combined to form an overall system of algebraic equations to study the types of temperature transfer in potatoes in this problem. It involves grouping element-by-element contributions into matrices and vectors, commonly referred to as stiffness matrix and load vector, respectively [10, 11, 12, 13]. The stiffness matrix represents the coefficients of the unknowns, while the load vector takes into account the contributions of the source conditions and the boundary conditions [14].In this context we are going to study a very important phenomenon, it is the transfer of heat, we prepare a potato slip in the form of chips by grilling it, especially since this type of product is accessible to all [15].

II. MATERIALS AND METHODS

2 . 1 THEORETICAL METHODS

Assuming isotropic material properties, low mass transfer, low dissipation, negligible viscosity, and linear dependence of heat flux on temperature gradient (Fourier's law), the energy balance of each subdomain Ωi is governed by the classical energy balance equation[13, 14,15] .

$$\frac{\partial T}{\partial t} = \alpha \nabla^2 T - q(x,t) \qquad (1)$$

where T denotes the temperature, the enthalpy (per unit volume), κ = κ(T) the thermal conductivity of the material, assumed to be isotropic, and q = q(x,t) is the heat input by volume during firing,$\dot{T}$the celerity the temperature distribution in the solid as a function of time t, α is the thermal diffusivity and $\nabla^2 T$est the Laplacian operator of T and ρ is the density of the material, cp is the specific heat capacity, xi is the spatial coordinate., the three-dimensional heat transfer equation becomes[16,17,18]:

$$\rho c \frac{\partial T}{\partial t} = \frac{\partial}{\partial}\left(k \frac{\partial T}{\partial x_i^2}\right) - Q(x,t) \qquad (x=(x_1, x_2, x_3) \in \Omega \qquad (2)$$

The boundary conditions define the outer domain boundary conditions on heat conduction ∂Ω and determine the imposed temperature and heat flux fields $\partial\Omega_T$, $\partial\Omega_q$ and $\partial\Omega_c$ being non-overlapping parts of the body boundary ∂Ω, with a prescribed temperature, can be cited as follows

Dirichlet Conditions: Direct Specification of Temperature T at an edge of the domain Ω_T.

T(x,y,z,t) = T0, where T0 is the prescribed temperature Ω_T.

Neumann conditions: Specify the heat flow (q, often related to the temperature gradient) at one edge of the domain Ω_q.

∂T/∂n = qn, where qn is the prescribed normal heat flux

Mixed conditions: A combination of Dirichlet and Neumann conditions.

To apply Galerkin's method to this equation, one starts by choosing a set of basis functions, typically polynomials, to approximate the solution u. These basis functions must satisfy the boundary conditions of the problem. Denote the basis functions as φi(x), where i is the index of the basis function and x represents the spatial coordinates.

Galerkin's method consists of multiplying the heat conduction equation by each basis function φi and integrating over the domain of interest. To solve this equation using Galerkin's method, we start by defining a test function that satisfies the boundary conditions of the problem. The basis function is usually chosen as a polynomial or a set of functions. The temperature field T is then expressed as a linear combination of these functions Ti such that[19;20]:

$$T(x,t) \approx \Sigma\, \varphi_i(x) T_i(t) \qquad (3)$$

where $\varphi_i(x)$ represents the test functions and $T_i(t)$ are the coefficients to be determined. Substituting this approximation into the heat transfer equation, on

$$\rho c\, \Sigma\, \varphi_i \frac{\partial T_i}{\partial t} = \frac{\partial}{\partial x}\left(k\, \Sigma\, \varphi_i \frac{\partial T_i}{\partial x}\right) \qquad (4)$$

By multiplying both sides of the equation by a function φ□ and integrating over the domain Ω, over

$$\int \left(\rho c\, \varphi\text{□}\ \Sigma\, \varphi_i \frac{\partial T_i}{\partial t}\right) dxi = \int \left(\varphi\text{□} \frac{\partial}{\partial x}\left(k\, \Sigma\, \varphi_i \frac{\partial T_i}{\partial xi}\right)\right) dxi \quad (5)$$

Application of integration by parts to the right side of the equation and consideration of boundary conditions. The system of equations can be written in the following matrix form:

$$[K]\{V\} = \{F\}$$

where [K] is a stiffness matrix defined as the integral of the product of the spatial derivatives of two specific basis functions (or moduli) over the domain as a function of a(Ni(x),Nj(x)) at each node, {V} is a vector containing the approximations to the solution, F is the load vector, which represents any applied heat source or boundary conditions. Overall, the Galerkin method is a powerful tool for solving heat transfer problems by approaching the solution using a set of test functions. Finally, we can solve the resulting system of ordinary differential equations using numerical methods, such as Euler's method or Runge-Kutta methods, to determine the time evolution of the $T_i(t)$ coefficients and obtain the approximate solution for the temperature field.La forme faible ou variable de l'équation d'équilibre, qui est donnée par les conditions aux limites de

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Eqs, est dérivée, en utilisant la méthode des résidus. La sélection appropriée des fonctions de pondération ainsi que l'application de la théorie du transfert permettent l'annulation des conditions résultant des conditions d'interface. De plus, en utilisant la définition, une forme faible basée sur la température de l'équation directrice est obtenue

$$\int_{\Omega} w\rho c\frac{\partial T}{\partial t}dV+\frac{\partial}{\partial t}\int_{\Omega}W\rho Li dV+\int_{\Omega T}k\nabla W.\nabla T dV+\int_{\Omega q}w\dot{q}dS=0 \quad (6)$$

Where W is the weighting function.

2 . 2 NUMUNICAL METHOD

When simulating a heat transfer problem, the first step in solving a finite element is to create a numerical model to solve and define the geometric boundary conditions (Figure 1-a,1-b), give the thermomechanical properties of the potato which are presented in table N°1.

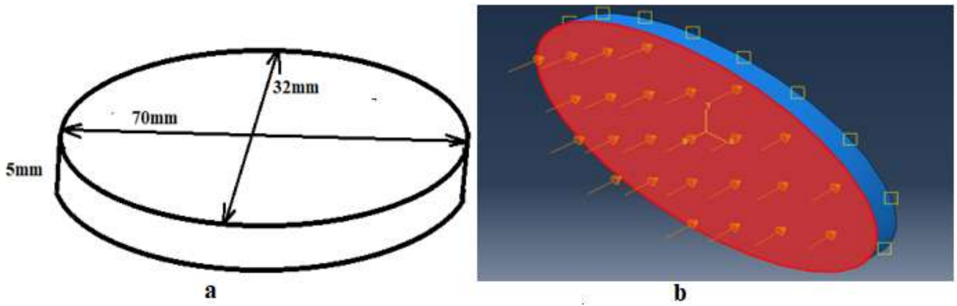


Figure1. the geometry of the potato like a piece of French fries, from below placed against the plate to be charged with a heat flux

Table1.Properties of raw potatoes (Variety spunta) ;[21,22]

Properties of raw potatoes	
Density (tonne/mm3) 9,87e-10 ≪ **ρ** ≪1,092e-9	Facet temperature =473.13°k
the heat transfer coefficient h=7,81w .m^{-2} k^{-1}	Young Module E=4.4MPa
Thermal conductivity k=0.56 w.$m^{-1}k^{-1}$	Poisson's ratio μ (-) 0.42
thermal diffusivity (α) = 0.89 ± 0.01 × 10 −6 m2.s^{-1}	

Define the boundary conditions recommended for the heat transfer problem, such as imposed temperatures, heat fluxes or heat exchange coefficients (convection conditions).Meshing refers to the process of discretizing a geometric model into finite elements. This allows complex geometry to be broken down into smaller elements, which facilitates the numerical analysis of structures and physical phenomena (figure.2).

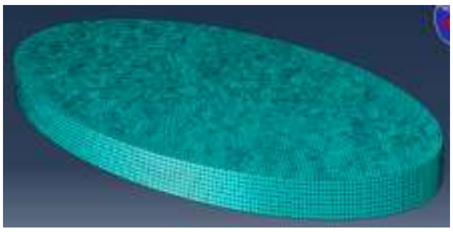
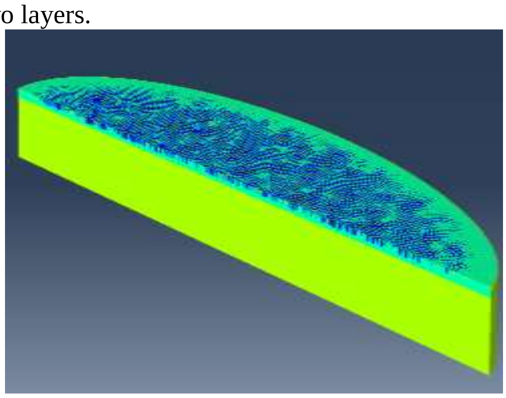


Figure.2 Mesh creation is essential for accurate results from finite element analysis

III. ANALYSIS OF RESULTS:

Analyze and interpret simulation results including temperature distributions, heat fluxes, thermal gradients, low conductivity of potato shows thermal separation between the part in direct contact with the temperature source and the part in contact with the open air as shown in figure 3 in two layers.



Fgure.3under the heat, the superposition of the layers appears

The figureshows that after 2.30 min the lower part exceeded 460¨°k but the upper part has 300°k (Figur4), these results are explained by the morphology of the potato which slows the flow of heat.

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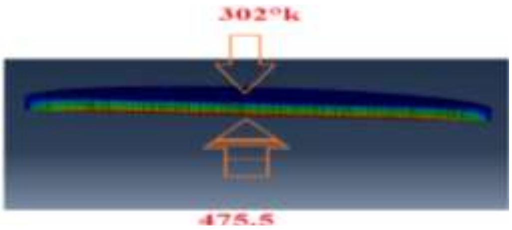

Figure.4 the temperature value and the idea of heat flow out after more than two minutes

The results show a superposition of thermally isothermal layers, in the direction X1 and X2 the temperature evolves slightly with time at each point of this plane at each level along X3, the curve shows a rapid evolution then almost stationary (Figure 5), this who later explains why she takes in total over four minutes to find a 5mm piece of potato chips ready to feed, i.e. time depends on thickness of course.

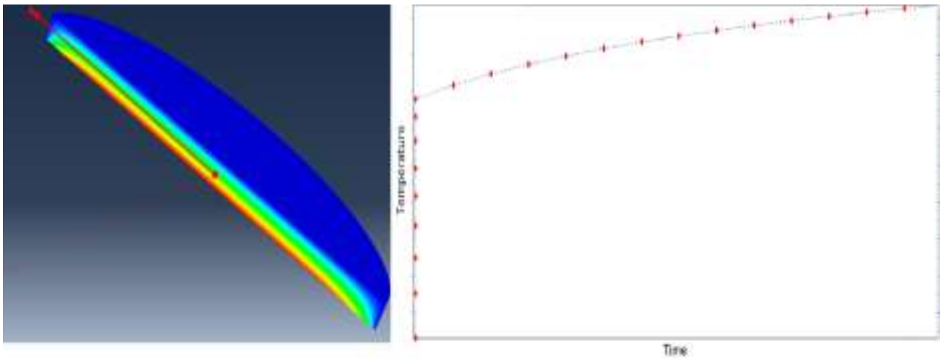

Figure.5 the thermal evolution of the toasted slide (Spunta) often the horizontal plane X1X2

Figure 6 shows the temperature increase in the X3 direction well, but also the results show that as soon as the distance increases, the flows become unable to cross the material, the simulation shows but even if the four minutes are exceeded, the other side not be heated. on the other hand, the face in front of the source heats up enormously.

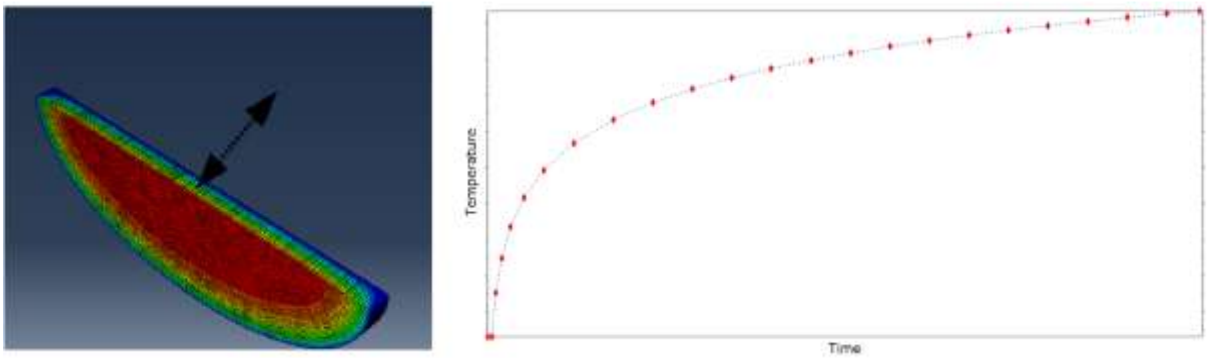

Figure .6 the thermal evolution of the grilled blade often the vertical direction X3

IV. CONCLUSION :

Finite element models have been used to analyze the thermal phenomena observed in chip firing processes. The disclosed methods provide an efficient method for optimizing process parameters. The model can be used to predict and minimize distortions due to sequence changes. A mathematical model was applied to properly represent the intensity distribution of an external heat source. The results show that after four minutes the product will be well prepared with an average of two minutes on each side, in reality it takes a good distribution of time on each side. transients The boundary conditions and a function of the simplified form used, solve this kind of propagation of heat in the direction X1 and X2 there is allowed temperature but the (evolution according to X3 especially that the potato has a low conductivity and rich in quantity of water, this method can be applied to other more complex problems.

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