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Applications of "Emad-Sara Transform" for the Solution of System of Differential Equations

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ABSTRACT
In this paper we use Emad-Sara transform to solve the system of ordinary differential equations of first order and first degree keywords ; system of differential equations , Integral transforms , Emad-sara transform .

Key Words: System of differential equations, Emad Sara Transform, Integral transforms.

INTRODUCTION

Recently, Integral transforms are one of the mostly used simple mathematical technique to obtain the solutions of advance problems of space, science, technology, engineering , commerce and economics . The important feature of these integral transform is to provide exact solution of of problem without lengthy calculations.

Due to this important feature of the integral transforms many researchers are attracted to this field and are engaged in introducing various integral transforms. Recently, in September 2021, Kushare and Patil [1] introduced Kushare transform for facilitating the process of solving differential equations in time domain. Further in October 2021 Khakale and Patil [2] introduced Soham transform. As researchers are introducing the new integral transforms at the same time they are also interested in applying the transforms to various fields, various equations in different domain. In January 2022, Sanap and Patil [3] used Kushare transform to solve the problems on Newton's law of Cooloing . In April 2022 D.P. Patil ,etal [4] used Kushare transform for solving the problems on growth and decay . In October 2021, D.P. Patil [5] used Sawi transform in Bessel functions. Further, Patil [6] used Sawi transform of error functions to evaluate improper integrals. Laplace transform and Shehu transforms are used to Patil [7] in chemical sciences . Patil [8] solved wave equation by using Sawi transform and its convolution theorem using Mahgoubtransform , parabolic boundary value problems are solved by D .P. Patil [9]. Solution of wave equation is obtained by using double Laplace and double Sumudu transforms by D .P. Patil [10]. Dr. Patil [11] also obtained dualities between double integral transforms. Laplace, Elzaki and Mahgoub transforms are compared and used for solving system of first order and first degree by Kushare and Patil [12] .D.P.Patil [13] used Aboodhand Mahgoub transform for solving boundary value problems of the system of ordinary differential equations. Double Mahgoub transformed is used by Patil [14] to solve parabolic boundary value problems.

Laplac, Sumudu ,Aboodh , Elazki and Mahagoub transform and used it for solving boundary value problems. Patil et al [16] used Emad-Sara transform for solving Volterra Integral equations of first kind. Futher Patil with Tile and Shinde [17] used transform for solving Volterra integral equations for first kind. Vispute, Jadhav and Patil [18] used Emad Sara transform for solving telegraph equation. Kandalkar, Zankar and Patil[19] used general integral transform of error function for evaluating improper integrals. Dinkar Patil, PreranaThakare.

Prajakta Patil used double general integral transform for the solution of parabolic boundary value problems [20]. Patil used EmadFalih transform for solving problems based on Newton's law of cooling [21]. D. P. Patil et al [22] used Soham transform in Newton's law of cooling. Dinkar Patil et al [23] used HY integral transform for handling growth and Decay problems, D. P. Patil et al used HY transform for Newton's law of cooling, D. P. Patil [26] used Emad-Falih transform for general solution of telegraph equation.Dinkar Patil et al [27] introduced double kushare transform. Recently, D. P. Patil et al [28] solved population growth and decay problems by using Emad Sara transform. Alenzi transform is used in population growth and decay problems by Patil et al [29]. Raundal and Patil D. P.[30] used double general integral transform for solving boundary value problems. Further Patil et al [31] used Sohamtransform to solve problems in Chemical Sciences.

In this paper, we use Emad-Sara transform to obtain the solution of the system of first order and first degree differential equations.

1. Preliminaries:

In this section we state some basic requirements. Now we state some required definitions.

Emad-Sara transform:[25] The Emad-Sara transformation is defined for an exponential order function:

$B = \{f(t): \exists k, m_1, m_2 > 0, |f(t)| < ke^{m_j|t|}$ if $t \in (-1)^j X[0, \infty)\}$……….(1)

Where f(t) is a function in the B set, K is a finite constant number, m_1 and m_2 may or may not be finite.

The kernel function of Emad-Sara integral transformation symbolized by ES (**.**) is defined by the integral equation:

$ES\{f(t)\} = T(\alpha) = \frac{1}{\alpha^2}\int_0^\infty e^{-\alpha t} f(t)dt$…..(2)

Where $t \geq 0$, $m_1 \leq \propto \leq m_2$ and $\propto$ is a variablethat is used as a factor to the variable t in the function f .

EMAD-SARA Transform of the Elementary Functions: [25]

For any function f(t) , We assume that the integral equation exist. The sufficient conditions for the existence of Emad-Sara transformation are that for $t \geq 0$ the function f(t) be piecewise Emad-Sara transformation may or may not exists.

In this section we state Emad-Sara transformation of some elementary functions.

Sr.No.	function	Emad-Sara Transformation
1	1	$\frac{1}{\alpha^3}$
2	K	$\frac{K}{\alpha^3}$
3	t	$\frac{1}{\alpha^4}$
4	e^{at}	$\frac{1}{\alpha^2(\alpha - a)}$
5	ES[sinat]	$\frac{a}{\alpha^2(\alpha^2 + a^2)}$
6	ES[cosat]	$\frac{1}{\alpha(\alpha^2 + a^2)}$
7	ES[sinhat]	$\frac{a}{\alpha^2(\alpha^2 - a^2)}$
8	ES[coshat]	$\frac{1}{\alpha(\alpha^2 - a^2)}$

Emad Sara Transform of derivative of function:[25] Let $T(\alpha)$ is the Emad Sara integral transform, where

$$ES\{f(t)\} = T(\alpha)$$

then

- $ES\{f'(t)\} = \frac{-f(0)}{\alpha^2} + \alpha T(\alpha)$
- $ES\{f''(t)\} = \frac{-f'(0)}{\alpha^2} + \alpha ES\{f'(t)\}$
- $ES\{f^{(n)}(t)\} = \frac{-f^{(n-1)}(0)}{\alpha^2} + \alpha ES\{f^{(n-1)}(t)\}$

2. Applications:

In this section we used Emad-Sara transform to solve the system of differential equation.

Example: (1) Consider the system of equations

$\frac{dx}{dt} - 2y = \cos 2t$ ……………. (1)

$\frac{dy}{dt} + 2x = \sin 2t$ ……………. (2)

with initial conditions, x(0) = 1 and y(0) = 0

Solution: Applying the Emad-Sara transform to both sides of equations

$ES\left(\frac{dx}{dt}\right) - 2ES(y) = ES(\cos 2t)$

$$ES\left(\frac{dy}{dt}\right) + 2ES(x) = ES(\sin 2t)$$

Let, $ES[X(t)] = T_1(\alpha)$ and $ES[y(t)] = T_2(\alpha)$

$\frac{-X(0)}{\alpha^2} + \alpha T_1(\alpha) - 2T_2(\alpha) = \frac{1}{\alpha(\alpha^2+4)}$

$\alpha T_1(\alpha) - 2T_2(\alpha) = \frac{1}{\alpha(\alpha^2+4)} + \frac{1}{\alpha^2}$ ……….(3)

And $(-y(0))/\alpha\hat{}2 + \alpha T_2(\alpha) + 2T_1(\alpha) = \frac{2}{\alpha^2(\alpha^2+4)}$

$2T_1(\alpha) + \alpha T2(\alpha) = \frac{2}{\alpha^2(\alpha^2+4)}$ …………(4)

Multiplying equation (3) by α & (4) by 2 and adding the obtained equations, we get,

$\alpha^2 T_1(\alpha) + 4T_1(\alpha) = \frac{1}{\alpha^2} + \frac{1}{\alpha}$

$T_1(\alpha) = \frac{1}{\alpha^2(\alpha^2+4)} + \frac{1}{\alpha(\alpha^2+4)}$.........(5)

Substituting value of $T_1(\alpha)$ in equation (4), we get

$T_2(\alpha) = \frac{-2}{\alpha^2(\alpha^2+4)}$...........(6)

Applying inverse Emad-Sara transform to equations (5) & (6)

$ES^{-1}T_1(\alpha) = \frac{1}{2}ES^{-1}\left[\frac{2}{\alpha^2(\alpha^2+4)}\right] + ES^{-1}\left[\frac{1}{\alpha(\alpha^2+4)}\right]$

$X(t) = \frac{1}{2}\sin 2t + \cos 2t$

and $ES^{-1}T_2(\alpha) = -ES^{-1}\left[\frac{2}{\alpha^2(\alpha^2+4)}\right]$

$\therefore Y(t) = -\sin 2t$

Thus, the required solution of given system of differential equations is

$x(t) = \frac{1}{2}\sin 2t + \cos 2t$ and $Y(t) = -\sin 2t$

Example: (2) Consider the system of equations

$\frac{dx}{dt} + y = \sin t$ ......(1)

$\frac{dy}{dt} + x = \cos t$ ......(2)

with initial conditions $x(0) = 0$ and $y(0) = 2$

Solution :

Applying the Emad-Sara intigral transform of both sides of equations (1) & (2)

$ES(\frac{dx}{dt}) + ES(y) = ES(\sin t)$

$ES(\frac{dy}{dt}) + ES(x) = ES(\cos t)$

Let

$ES[x(t)] = T_1(\alpha)$ and $ES[y(t)] = T_2(\alpha)$

$\frac{-X(0)}{\alpha^2} + \alpha T_1(\alpha) + T_2(\alpha) = \frac{1}{\alpha^2(\alpha^2+1)}$

$\alpha T_1(\alpha) + T_2(\alpha) = \frac{1}{\alpha^2(\alpha^2+1)}$.......(3)

And

$\frac{-Y(0)}{\alpha^2} + \alpha T_2(\alpha) + T_1(\alpha) = \frac{1}{\alpha(\alpha^2+1)}$

$T_1(\alpha) + \alpha T_2(\alpha) = \frac{1}{\alpha(\alpha^2+1)} + \frac{2}{\alpha^2}$......(4)

Multiplying equation (3) by α & subtracting (3) & (4) we get,

$\alpha^2 T_1(\alpha) - T_1(\alpha) = \frac{-2}{\alpha^2}$

$T_1(\alpha) = \frac{-2}{\alpha^2(\alpha^2-1)}$.......(5)

Substituting value of in equation (3), we get,

$T_2(\alpha) = \frac{1}{\alpha^2(\alpha^2+1)} + \frac{2}{\alpha(\alpha^2-1)}$........(6)

Applying inverse Soham transform to equations (5) & (6) , we get,

$ES^{-1}(T_1(\alpha)) = -2ES^{-1}\left[\frac{1}{\alpha(\alpha^2-1)}\right]$

$X(t) = -2\sinh t$

and

$ES^{-1}(T_2(\alpha)) = ES^{-1}\left[\frac{1}{\alpha^2(\alpha^2+1)}\right] + 2ES^{-1}\left[\frac{1}{\alpha(\alpha^2-1)}\right]$

$Y(t) = \sin t + 2\cosh t$

Thus, the required solution of given system of differential equations is

$X(t) = -2\sinh at$

and

$Y(t) = \sin t + 2\cosh t$

Conclusion: We have applied Emad-Sara transform for obtaining the solution of the system of first order and first degree differential equations.

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