

# Bayesian Optimization Model in Multi-purpose/Multi-objective Projects in Anambra – Imo River Basin Resources Project Utilization, Nigeria.

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## ABSTRACT

The work is aimed at investigating optimization of river basin resources utilization in Anambra – Imo river basin to mitigate climate variability using Bayesian theory model. The objective is to determine optimal benefits under various net benefits (objectives) is a multi-purpose/multi-objective capital projects to develop sustainability in the river basin. The methodology involves the use of Bayesian Decision Theory (BDT) model based on the data generated from the Bill of Engineering Measurement and Evaluation (BEME). The result shows that the optimal solution from the Bayesian model analysis of the Maximum Expected Monetary Value (EMV\*) was N68.72b on third iteration. When the amount of N12.504 billion released to Anambra – Imo river basin for the period were appropriated to various purpose/objective was deducted from the revenue generated from Bayesian EMV\* (N68.72billion), N56.22 billion emerged as profit margin from the investment. The world recommended that since there are much uncertainties in climate change projection which impacts on the environment, optimal strategies should incorporate delivery benefits irrespective of climate variability which Bayesian optimal strategies would mitigate.

**Keywords:** Optimization, multi-purpose/multi-objective, River basin, Bayesian theory.

## I. INTRODUCTION

The multi-purpose/multi-objective river basin development project planning and management will help to determine levels of

development to be apportioned to various purposes for water resources projects. The planning and management of these projects are multi-disciplinary and may involve a lot of complex situations. Barrow (1998) opined that River basin development and planning is the process of identifying the best way in which a river and its tributaries may be used to meet competing demands while maintaining river health. It includes the allocation of scarce water resources between different users and purposes, choosing between environmental objectives and competing human needs and choosing between competing food risk management requirements (Molle, 2006). The increasing complexity of many of the river basins occasioned by increasing development and population pressure, have resulted many serious crisis related to floods, degradation of water quality, acute water shortage and degradation of ecological health. The various approaches to river basin planning is ultimately playing significant roles to the adaptation of the local circumstances. The consideration of economic efficiency, federal economic redistribution, regional economic redistribution, state economic redistribution, local economic redistribution, social well-being, environmental quality improvement, youth employment, gender equality and security are becoming more relevant due to some political, ecological and health concern of the people. Ezenweani (2017) identified inability of management of river basin to control the whole basin and lack of baseline data with inadequate monitoring are some of the problems that hinders

River basin development planning and management. Klare (2001) also said that politics to determine who is to be employed, what is on the agenda and how river basin development planning and management proceeds also affects them. The required decisions will need to be made by concerned stakeholders in the government and river basin development authority for adequate benefits to be derived from the resources development and utilization.

The Bayesian Decision Theory (BDT) model is a Dynamic programming techniques concerned with the method of computing posterior probabilities from prior probabilities using Bayes' theorem. An initial probability statement to evaluate expected payoff is called a prior probability distribution. The one which has been revised in the light of new information is called posterior probability distribution. What is a posterior to one sequence of state of nature becomes the prior on others which are yet to happen. A further analysis of problems using these probabilities with respect to new expected payoffs with additional information is called prior-posterior analysis.

## II. AIM AND OBJECTIVES

The aim is to investigate optimization of river basin resources utilization in Anambra – Imo river basin to mitigate climate variability using Bayesian theory model. The objective is to firstuse the net benefits generated from Bill of Engineering Measurement and Evaluation (BEME) to determine optimal benefits under the various objectives in a multi-purpose/multi-objective projects to champion the course of green revolution in the river basin management planning and development.

Secondly, to determine the magnitude of differences between alternative courses of action with the degree of association indicators available for decision making under the situation of certainty and uncertainty in the river basin.

## III. METHODOLOGY

This involve the use of Bayesian Decision Theory (BDT) model from the data generated from the Bill of Engineering Measurement and Evaluation (BEME). The policy iteration algorithm were used to determine the optimal benefits under the various objectives in the river basin.

## IV. ANALYSIS AND DISCUSSION OF RESULTS

### 4.1 Net Benefits Multi-Purpose under Various Multi-Objectives

Table 1: Summary of Net Benefits for all the Objectives against the Purposes in Billion Naira

S/N	Purpose	B1	B2	B3	B4	B5	B6	B7	B8	B9	B10
(1)	Irrigated Agriculture	3.65	4.84	6.36	3.60	3.44	4.37	4.05	4.22	1.12	8.73
(2)	Hydro-electric power generation	13.38	7.55	9.60	9.68	9.29	5.46	6.05	6.39	1.37	10.95
(3)	Water supply	4.54	4.34	6.04	3.78	3.52	4.56	4.22	4.37	1.13	9.13
(4)	Navigation	8.30	5.83	10.46	8.19	8.24	11.39	10.96	12.20	3.33	25.77
(5)	Drainage/ Dredging	17.21	6.01	12.26	3.68	6.08	8.96	11.51	10.83	3.00	21.96
(6)	Flood control	19.43	5.58	10.20	3.39	1.55	8.68	10.32	11.35	2.90	22.12
(7)	Recreation / Tourism	16.93	3.94	10.36	3.42	3.33	10.57	11.33	12.25	3.33	25.94
(8)	Erosion control	13.91	3.01	10.27	3.15	3.26	9.56	7.13	8.72	2.21	16.78
(9)	Plantation / Forestry	14.01	6.83	8.08	6.40	6.59	8.96	7.66	8.40	2.26	18.08
(10)	Reservoir/	82.72	5.66	12.16	3.36	3.48	19.99	20.54	20.71	5.77	41.23

	Gullies									
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- B<sub>1</sub> = Economic efficiency,
- B<sub>2</sub> = Federal Economic Redistribution,
- B<sub>3</sub> = Regional Economic Redistribution,
- B<sub>4</sub> = State Economic Redistribution,
- B<sub>5</sub> = Local Economic Redistribution,
- B<sub>6</sub> = Social Well-being,
- B<sub>7</sub> = Youth Empowerment,
- B<sub>8</sub> = Environmental Quality Improvement,
- B<sub>9</sub> = Gender Equality,
- B<sub>10</sub> = Security

**Discussion of Results in Table 1:**

The Table 1 explained the summary results calculation of Net benefits from Bill of Engineering Measurement and Evaluation (BEME) in billions of naira.

- (i) Under Irrigation Agriculture the highest benefits of ₦8.73 billion from Security while the least amount of benefit was ₦1.12 billion from Gender Equality
- (ii) On Hydro-electric Power Generation, Economic Efficiency has the highest value of ₦13.38 billion while lowest value of ₦1.37 billion was on Gender Equality
- (iii) Under the purpose of Reservoir and Gullies, the highest benefit of ₦82.72 billion was from objective of Economic Efficiency and the lowest was ₦3.36 billion on State Economic Redistribution
- (iv) In other purposes the Net benefits has the highest from objectives on Security with the following values; ₦9.13 billion from Water

Supply; ₦25.77 billion from Navigation; ₦21.96 billion from Drainage/Dredging; ₦22.12 billion from Flood Control; ₦25.94 billion from Recreation/Tourism; ₦16.78 billion from Erosion Control and ₦18.08 billion from Plantation/Forestry. Except for Hydro-electric power generation and Reservoir/Gullies, other purpose have the highest net benefits under the objective of security improvement.

- (v) Except for state economic redistribution under reservoir/gullies that has the lowest net benefits of ₦3.36 billion, the rest of the lowest net benefits under all other purpose were under the objective of Gender Equality. These are ₦1.12 billion from irrigation agriculture, ₦1.37 billion from hydro-electric power generation, ₦1.13 billion from Water Supply; ₦3.33 billion from Navigation; ₦3.00 billion from Drainage/Dredging; ₦2.90 billion from Flood Control; ₦3.33 billion from Recreation/Tourism ; ₦2.21 billion from Erosion Control and ₦2.26 billion from Plantation/Forestry.

**4.2 Bayesian Decision Model Simulation Based on Courses of Action**

Using the Bayesian Decision Analysis, the prior probability was derived from the benefits and used in the analysis for previous prediction i.e. states of nature probabilities; N<sub>1</sub> = 0.02, N<sub>2</sub> = 0.07, N<sub>3</sub> = 0.03, N<sub>4</sub> = 0.04, N<sub>5</sub> = 0.09, N<sub>6</sub> = 0.10, N<sub>7</sub> = 0.09, N<sub>8</sub> = 0.07, N<sub>9</sub> = 0.08, N<sub>10</sub> = 0.41.

**4.2.1 Calculation of Likelihood Forecast of Probabilities**

Table 2: The Likelihood Forecast of Probability Estimated from the Various Courses of Action for Net Benefits.

States of Nature	Courses of Action									
	B <sub>1</sub>	B <sub>2</sub>	B <sub>3</sub>	B <sub>4</sub>	B <sub>5</sub>	B <sub>6</sub>	B <sub>7</sub>	B <sub>8</sub>	B <sub>9</sub>	B <sub>10</sub>
N <sub>1</sub>	0.08	0.11	0.14	0.08	0.08	0.10	0.09	0.10	0.02	0.20
N <sub>2</sub>	0.17	0.09	0.12	0.12	0.12	0.07	0.07	0.08	0.02	0.14
N <sub>3</sub>	0.10	0.10	0.13	0.08	0.08	0.10	0.09	0.10	0.02	0.20
N <sub>4</sub>	0.08	0.05	0.10	0.08	0.08	0.11	0.10	0.12	0.03	0.25
N <sub>5</sub>	0.17	0.06	0.12	0.03	0.06	0.09	0.11	0.11	0.03	0.22
N <sub>6</sub>	0.20	0.06	0.11	0.03	0.02	0.09	0.11	0.12	0.03	0.23
N <sub>7</sub>	0.17	0.04	0.10	0.04	0.03	0.10	0.11	0.12	0.03	0.26
N <sub>8</sub>	0.18	0.04	0.13	0.04	0.04	0.12	0.09	0.11	0.03	0.22
N <sub>9</sub>	0.16	0.08	0.09	0.07	0.07	0.10	0.09	0.10	0.03	0.21
N <sub>10</sub>	0.38	0.02	0.06	0.01	0.02	0.09	0.10	0.10	0.03	0.19

Where the courses of action are;  
 $N_1$  = Irrigation Agriculture,

$N_2$  = Hydro-electric Power Generation,  
 $N_3$  = Water Supply,

$N_4$  = Navigation/Water Transport,  
 $N_5$  = Drainage/Dredging,  
 $N_6$  = Flood Control,  
 $N_7$  = Recreation/Tourism,  
 $N_8$  = Erosion Control,  
 $N_9$  = Plantation/Forestry,  
 $N_{10}$  = Reservoir/Gullies

Where the states of nature are;  
 $B_1$  = Economic efficiency,

$B_2$  = Federal Economic Redistribution,  
 $B_3$  = Regional Economic Redistribution,  
 $B_4$  = State Economic Redistribution,

$B_5$  = Local Economic Redistribution,  
 $B_6$  = Social Well-being,  
 $B_7$  = Youth Empowerment,  
 $B_8$  = Environmental Quality Improvement,

$B_9$  = Gender Equality,  
 $B_{10}$  = Security

**Discussion of Results in Table 2:**

Table 2 shows the likelihood forecast probabilities from various courses of action for the purposes. These probabilities were used in calculating the Joint probability outcomes on first iteration in order to determine the Marginal probability outcomes.

The next step is to calculate the Expected Monetary Values (EMVs) using the Prior Probabilities for the States of Nature.

**4.2.2 Determination of Expected Monetary Value (EMVs) at First Iteration**

The Expected Monetary Values (EMVs) or Expected Utility explains criteria for various courses of action (alternatives) under risk. The EMV is the weighted sum of possible payoffs from each alternative. It is obtained by adding up the payoffs of each course of action multiplied by the probabilities associated with each state of nature. This was calculated and shown on Table 3 below as follows;

**Table 3: Calculation of Expected Monetary Values (EMVs) at First Iteration**

States of Nature	Prior Probability	Conditional Net Benefits Course of Action in Billions of Naira										Expected Net Benefits in Billions of Naira Course of Action									
		$S_1$	$S_2$	$S_3$	$S_4$	$S_5$	$S_6$	$S_7$	$S_8$	$S_9$	$S_{10}$	$S_1$	$S_2$	$S_3$	$S_4$	$S_5$	$S_6$	$S_7$	$S_8$	$S_9$	$S_{10}$
$N_1$	0.02	3.65	4.84	6.36	3.60	3.44	4.37	4.05	4.22	1.12	8.73	0.073	0.0968	0.1272	0.072	0.0688	0.0874	0.081	0.0844	0.0224	0.1746
$N_2$	0.07	13.38	7.55	9.60	9.68	9.29	3.46	6.05	6.39	1.37	10.95	0.9366	0.5285	0.672	0.6776	0.6503	0.3822	0.4235	0.4473	0.0959	0.7665
$N_3$	0.03	4.34	4.34	6.04	3.78	3.32	4.56	4.22	4.37	1.13	9.13	0.1362	0.1302	0.1812	0.1134	0.1056	0.1368	0.1266	0.1311	0.0339	0.2739
$N_4$	0.04	8.30	5.83	10.46	8.19	8.24	11.39	10.96	12.20	3.33	25.77	0.332	0.2332	0.4184	0.3276	0.3296	0.4556	0.4384	0.488	0.1332	1.0308
$N_5$	0.09	17.21	6.01	12.26	3.68	6.08	8.96	11.31	10.83	3.00	21.96	1.5489	0.5409	1.1034	0.3312	0.5472	0.8064	1.0359	0.9747	0.27	1.9764
$N_6$	0.10	19.43	5.58	10.20	3.39	1.55	8.68	10.32	11.35	2.90	22.12	1.943	0.558	1.02	0.339	0.155	0.868	1.032	1.135	0.290	2.212
$N_7$	0.09	16.93	3.94	10.36	3.42	3.33	10.57	11.33	12.25	3.33	25.94	1.5237	0.3546	0.9324	0.3078	0.2997	0.9513	1.0197	1.1025	0.2997	2.3346
$N_8$	0.07	13.91	3.01	10.27	3.15	3.26	9.56	7.13	8.72	2.21	16.78	0.9737	0.2107	0.7189	0.2205	0.2282	0.6692	0.4991	0.6104	0.1547	1.1746
$N_9$	0.08	14.01	6.83	8.08	6.40	6.39	8.96	7.66	8.40	2.26	18.08	1.1208	0.5464	0.6464	0.512	0.5272	0.7168	0.6128	0.672	0.1808	1.4464
$N_{10}$	0.41	82.72	5.66	12.16	3.36	3.48	19.99	20.54	20.71	5.77	41.23	33.9972	2.3206	0.8856	1.3776	1.4268	8.1959	8.4214	8.4911	2.3657	16.9043
		<b>Expected monetary values (EMVs)</b>										<b>42.5851</b>	<b>5.5199</b>	<b>6.7055</b>	<b>4.2787</b>	<b>4.3364</b>	<b>13.2496</b>	<b>13.6904</b>	<b>14.1365</b>	<b>3.8463</b>	<b>28.2941</b>

The Maximum Expected Monetary Value (EMV\*) = ~~N~~42.5851 Billionon Economic Efficiency is the optimal course of action with other optimal course of action with other objectives to be considered for maximum benefits.

Where the states of nature are;

$S_1$  = Economic efficiency,  
 $S_2$  = Federal Economic Redistribution,  
 $S_3$  = Regional Economic Redistribution,  
 $S_4$  = State Economic Redistribution,

S<sub>5</sub> = Local Economic Redistribution,  
S<sub>6</sub> = Social Well-being,  
S<sub>7</sub> = Youth Empowerment,  
S<sub>8</sub> = Environmental Quality Improvement,  
S<sub>9</sub> = Gender Equality,  
S<sub>10</sub> = Security

**Discussion of Result in Table 3:**

(i). The prior probabilities are multiplied by the Conditional Net benefits courses of action to get the Expected Monetary Values (EMVs). The Maximum Expected Monetary Value (EMV\*) is S<sub>1</sub> = ₦42.59 billion, S<sub>2</sub> = ₦5.52 billion, S<sub>3</sub> = ₦6.71 billion, S<sub>4</sub> = ₦4.28 billion, S<sub>5</sub> = ₦4.34 billion, S<sub>6</sub> = ₦13.25 billion, S<sub>7</sub> = ₦13.69 billion, S<sub>8</sub> = ₦14.14 billion, S<sub>9</sub> = ₦3.85 billion, S<sub>10</sub> = ₦28.29 billion.

- (ii). The Maximum Expected Benefit for each states of nature was ₦42.5851 billion.
- (iii). Expected Profit with Perfect Information (EPPI) = 0.02(8.73) + 0.07(13.38) + 0.03(9.13) + 0.04 (25.77) + 0.09 (21.96) + 0.10 (22.12) + 0.09 (25.94) + 0.07 (16.78) + 0.08 (18.08) + 0.41 (82.92) = ₦45.5571 billion.
- (iv). The Expected Value of Perfect Information (EVPI) = EPPI – EMV\* = ₦45.5571 - ₦42.5851 = ₦2.972 billion.

**4.2.3 Determination of Forecast Likelihood of Probabilities**

The Table 4 shows the Forecast Likelihood of Probabilities determined from the table of net benefits on Tables 1 and 2 respectively.

Table 4: Forecast Likelihood of Probabilities

States of Nature	Forecast Likelihood									
	B <sub>1</sub> /N	B <sub>2</sub> /N	B <sub>3</sub> /N	B <sub>4</sub> /N	B <sub>5</sub> /N	B <sub>6</sub> /N	B <sub>7</sub> /N	B <sub>8</sub> /N	B <sub>9</sub> /N	B <sub>10</sub> /N
N <sub>1</sub>	0.08	0.11	0.14	0.08	0.08	0.10	0.09	0.10	0.02	0.20
N <sub>2</sub>	0.17	0.09	0.12	0.12	0.12	0.07	0.07	0.08	0.02	0.14
N <sub>3</sub>	0.10	0.10	0.13	0.08	0.08	0.10	0.09	0.10	0.02	0.20
N <sub>4</sub>	0.08	0.05	0.10	0.08	0.08	0.11	0.10	0.12	0.03	0.25
N <sub>5</sub>	0.17	0.06	0.12	0.03	0.06	0.09	0.11	0.11	0.03	0.22
N <sub>6</sub>	0.21	0.06	0.11	0.03	0.02	0.09	0.11	0.12	0.03	0.23
N <sub>7</sub>	0.17	0.04	0.10	0.04	0.03	0.10	0.11	0.12	0.03	0.26
N <sub>8</sub>	0.18	0.04	0.13	0.04	0.04	0.12	0.09	0.11	0.03	0.22
N <sub>9</sub>	0.16	0.08	0.09	0.07	0.07	0.10	0.09	0.10	0.03	0.21
N <sub>10</sub>	0.38	0.02	0.06	0.01	0.02	0.09	0.10	0.10	0.03	0.19

**Discussion of Results in Table 4:**

- (i). The Forecast Likelihood Probabilities in Table 4 was calculated by dividing each value of the summary of Net benefits in Table 1 by the total value in each row of the table.
- (ii). The Expected Profit with Perfect Information (EPPI) was obtained by multiplying the each prior probability by their respective highest net benefit on each row and adding up the values which is ₦45.5571 billion.
- (iii). The Expected Value of Perfect Information (EVPI) was obtained by subtracting the value of Maximum Expected Monetary Value (EMV\*) ₦42.5851 from the amount of Expected Profit with Perfect Information (EPPI) ₦45.5571 billion which gives a balance of ₦2.972 billion.
- (iv). For each of the forecast result, the prior and posterior probabilities are calculated in Tables 5 and 6 respectively.

**4.2.4 Determination of Joint Probabilities Outcomes on First Iteration**

The determination of Joint Probabilities Outcomes on First (1st) Iteration was obtained by multiplying the states of nature (prior) probabilities P(N<sub>i</sub>) with each of the conditional probabilities (B<sub>i</sub>/N). These were calculated for each course of action outcomes (B<sub>i</sub>) as shown in Table 5.

It should be noted that in these tables the prior probabilities of states of Nature P(N<sub>i</sub>) for i = 1, 2, 3, 4, 5, 6, 7, 8, 9 and 10 are multiplied by each of the conditional probabilities outcomes P(B<sub>i</sub>/N<sub>i</sub>) to get the joint values probabilities outcomes i.e. P(B<sub>i</sub>∩N<sub>i</sub>) = P(N<sub>i</sub>) P(B<sub>i</sub>/N<sub>i</sub>) as shown below. i.e. (P(N<sub>i</sub>) for (i=1, 2, 3, 4, 5, 6, 7, 8, 9, 10) × P(B<sub>i</sub>/N<sub>i</sub>) for i = 1, 2, 3, 4, 5, 6, 7, 8, 9, 10). For Example, purpose B<sub>i</sub> (irrigation agriculture) for states of nature N<sub>i</sub>, Joint Probability = P(N<sub>i</sub>)×P(B<sub>i</sub> /N<sub>i</sub>)

For  $B_1 = 0.02 \times 0.08 = 0.0016$   
 For  $B_2 = 0.02 \times 0.11 = 0.0022$   
 For  $B_3 = 0.02 \times 0.14 = 0.0028$  etc.  
 The Marginal Probability for each benefits  $B_i$  for ( $i = 1, 2, 3, 4, 5, 6, 7, 8, 9, 10$ )  
 for  $B_i = \sum \{P(N_i) \times P(B_i/N_i)\}$

for  $B_1 = \sum \{N_1B_1 + N_2B_1 + N_3B_1 + N_4B_1 + N_5B_1 + N_6B_1 + N_7B_1 + N_8B_1 + N_9B_1 + N_{10}B_1\}$   
 for  $B_2 = \sum \{N_1B_2 + N_2B_2 + N_3B_2 + N_4B_2 + N_5B_2 + N_6B_2 + N_7B_2 + N_8B_2 + N_9B_2 + N_{10}B_2\}$   
 ... etc to  $N_iB_{10}$  for  $i = \dots, 3, 4, 5, 6, 7, 8, 9, 10$ .

Table 5: Joint Probabilities Outcomes at First Iteration

States of Nature ( $N_i$ )	Prior Probability $P(N_i)$	Outcomes ( $B_i$ )	Conditional Probability $P(B_i/N_i)$	Joint Probability $P(B_i \cap N_i) = P(N_i) P(B_i/N_i)$																	
				$B_1$	$B_2$	$B_3$	$B_4$	$B_5$	$B_6$	$B_7$	$B_8$	$B_9$	$B_{10}$								
$N_1$	0.02	$B_1$	0.08	0.0016																	
		$B_2$	0.11		0.0022																
		$B_3$	0.14				0.0028														
		$B_4$	0.08					0.0016													
		$B_5$	0.08							0.0016											
		$B_6$	0.10								0.0020										
		$B_7$	0.09									0.0018									
		$B_8$	0.10										0.0020								
		$B_9$	0.02											0.0004							
		$B_{10}$	0.20																		0.00040
$N_2$	0.07	$B_1$	0.17	0.00119																	
		$B_2$	0.09		0.0063																
		$B_3$	0.12				0.0084														
		$B_4$	0.12					0.0084													
		$B_5$	0.12						0.0084												
		$B_6$	0.07							0.0049											
		$B_7$	0.07								0.0049										
		$B_8$	0.08										0.0056								
		$B_9$	0.02											0.0014							
		$B_{10}$	0.14																		0.0098
$N_3$	0.03	$B_1$	0.10	0.003																	
		$B_2$	0.10		0.003																
		$B_3$	0.13				0.0039														
		$B_4$	0.08					0.0024													
		$B_5$	0.08						0.0024												
		$B_6$	0.10							0.003											
		$B_7$	0.09								0.0027										
		$B_8$	0.10									0.003									
		$B_9$	0.02											0.0006							
		$B_{10}$	0.20																		0.006

Note: This Table 5 continued on the next page from states of nature  $N_4$

Table 5: Joint Probabilities Outcomes at First Iteration Continued

States of Nature (N <sub>i</sub> )	Prior Probability P(N <sub>i</sub> )	Outcomes (B <sub>j</sub> )	Conditional Probability P(B <sub>j</sub> /N <sub>i</sub> )	Joint Probability P(B <sub>i</sub> ∩ N <sub>j</sub> ) = P(N <sub>i</sub> ) P(B <sub>j</sub> /N <sub>i</sub> )																	
				B <sub>1</sub>	B <sub>2</sub>	B <sub>3</sub>	B <sub>4</sub>	B <sub>5</sub>	B <sub>6</sub>	B <sub>7</sub>	B <sub>8</sub>	B <sub>9</sub>	B <sub>10</sub>								
N <sub>4</sub>	0.04	B <sub>1</sub>	0.06	0.032																	
		B <sub>2</sub>	0.05		0.002																
		B <sub>3</sub>	0.10			0.004															
		B <sub>4</sub>	0.08				0.0032														
		B <sub>5</sub>	0.08					0.0032													
		B <sub>6</sub>	0.11							0.0044											
		B <sub>7</sub>	0.10									0.0040									
		B <sub>8</sub>	0.12											0.0012							
		B <sub>9</sub>	0.03												0.0012						
		B <sub>10</sub>	0.25																		0.0100
N <sub>5</sub>	0.09	B <sub>1</sub>	0.17	0.0153																	
		B <sub>2</sub>	0.06		0.0054																
		B <sub>3</sub>	0.12			0.0108															
		B <sub>4</sub>	0.03				0.0027														
		B <sub>5</sub>	0.06					0.0054													
		B <sub>6</sub>	0.09							0.0081											
		B <sub>7</sub>	0.11									0.0099									
		B <sub>8</sub>	0.11											0.0099							
		B <sub>9</sub>	0.03												0.0027						
		B <sub>10</sub>	0.22																		0.0198
N <sub>6</sub>	0.10	B <sub>1</sub>	0.20	0.0200																	
		B <sub>2</sub>	0.06		0.0060																
		B <sub>3</sub>	0.11			0.0110															
		B <sub>4</sub>	0.03				0.003														
		B <sub>5</sub>	0.02					0.002													
		B <sub>6</sub>	0.09							0.009											
		B <sub>7</sub>	0.11									0.0110									
		B <sub>8</sub>	0.12											0.012							
		B <sub>9</sub>	0.03												0.003						
		B <sub>10</sub>	0.23																		0.023

Note: This Table 5 continued on the next page from states of nature N<sub>7</sub>

Table 5: Joint Probabilities Outcomes at First Iteration Continued

States of Nature (N <sub>i</sub> )	Prior Probability P(N <sub>i</sub> )	Outcomes (B <sub>j</sub> )	Conditional Probability P(B <sub>j</sub> /N <sub>i</sub> )	Joint Probability P(B <sub>i</sub> ∩ N <sub>j</sub> ) = P(N <sub>i</sub> ) P(B <sub>j</sub> /N <sub>i</sub> )																		
				B <sub>1</sub>	B <sub>2</sub>	B <sub>3</sub>	B <sub>4</sub>	B <sub>5</sub>	B <sub>6</sub>	B <sub>7</sub>	B <sub>8</sub>	B <sub>9</sub>	B <sub>10</sub>									
N <sub>7</sub>	0.09	B <sub>1</sub>	0.17	0.0153																		
		B <sub>2</sub>	0.04		0.0036																	
		B <sub>3</sub>	0.10			0.009																
		B <sub>4</sub>	0.04				0.0036															
		B <sub>5</sub>	0.03					0.0027														
		B <sub>6</sub>	0.10							0.009												
		B <sub>7</sub>	0.11									0.0099										
		B <sub>8</sub>	0.12											0.0108								
		B <sub>9</sub>	0.03												0.0027							
		B <sub>10</sub>	0.26																			0.0234
N <sub>8</sub>	0.07	B <sub>1</sub>	0.18	0.0126																		
		B <sub>2</sub>	0.04		0.0028																	
		B <sub>3</sub>	0.13			0.0091																
		B <sub>4</sub>	0.04				0.0028															
		B <sub>5</sub>	0.04					0.0028														
		B <sub>6</sub>	0.12							0.0084												
		B <sub>7</sub>	0.09									0.0063										
		B <sub>8</sub>	0.11											0.0077								
		B <sub>9</sub>	0.03												0.0021							
		B <sub>10</sub>	0.22																			0.0154

Note: This Table 5 continued on the next page from states of nature N<sub>9</sub>

Table 5: Joint Probabilities Outcomes at First Iteration Continued

States of Nature (N <sub>i</sub> )	Prior Probability P(N <sub>i</sub> )	Outcomes (B <sub>j</sub> )	Conditional Probability P(B <sub>j</sub> /N <sub>i</sub> )	Joint Probability P(B <sub>i</sub> ∩N <sub>i</sub> ) = P(N <sub>i</sub> ) P(B <sub>j</sub> /N <sub>i</sub> )																	
				B <sub>1</sub>	B <sub>2</sub>	B <sub>3</sub>	B <sub>4</sub>	B <sub>5</sub>	B <sub>6</sub>	B <sub>7</sub>	B <sub>8</sub>	B <sub>9</sub>	B <sub>10</sub>								
N <sub>9</sub>	0.08	B <sub>1</sub>	0.16	0.0128																	
		B <sub>2</sub>	0.03		0.0064																
		B <sub>3</sub>	0.09			0.0072															
		B <sub>4</sub>	0.07				0.0056														
		B <sub>5</sub>	0.07					0.0056													
		B <sub>6</sub>	0.10						0.0088												
		B <sub>7</sub>	0.09							0.0072											
		B <sub>8</sub>	0.10									0.0080									
		B <sub>9</sub>	0.03											0.0024							
		B <sub>10</sub>	0.21																		0.0168
N <sub>10</sub>		B <sub>1</sub>	0.38	0.1558																	
		B <sub>2</sub>	0.02		0.0082																
		B <sub>3</sub>	0.06			0.0246															
		B <sub>4</sub>	0.01				0.0041														
		B <sub>5</sub>	0.02					0.0082													
		B <sub>6</sub>	0.09						0.0369												
		B <sub>7</sub>	0.10							0.041											
		B <sub>8</sub>	0.10								0.041										
		B <sub>9</sub>	0.03											0.0123							
		B <sub>10</sub>	0.19																		0.0779
<b>Marginal Probability</b>				<b>0.2515</b>	<b>0.0599</b>	<b>0.0908</b>	<b>0.0374</b>	<b>0.0423</b>	<b>0.0941</b>	<b>0.0987</b>	<b>0.1048</b>	<b>0.0288</b>	<b>0.2061</b>								

**Discussion of Results in Table 5:**

(i). The joint values probabilities outcomes were calculated by multiplying prior probability of each states of nature by the conditional probability outcomes and adding of the result of each of them to obtain the marginal probability values as shown on Table 5.

(ii) The values of the marginal probabilities are 0.2515 for economic efficiency; 0.0599 for federal economic redistribution; 0.0908 for regional economic redistribution; 0.0374 for State economic redistribution; 0.0423 for local economic redistribution; 0.0941 for social well-being; 0.0987 for youth empowerment; 0.1048 for environmental quality improvement; 0.0288 for gender equality and 0.2061 for security.

**4.2.5 Determination of Posterior Probability Outcomes at First Iteration**

The Posterior Probability  $P(N_i/B_i) = P(N_i \cap B_i)/P(B_i)$  where  $P(B_i)$  is the values of the marginal probabilities which is the total sum of each values of the joint probabilities outcomes  $P(N_i \cap B_i)$ .

The Posterior Probability Outcomes at first iteration on Table 6 is computed by dividing each states of nature (N<sub>i</sub>) for i = 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 by each values of marginal probability outcomes  $P(B_i)$  for i = 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 under each group values. These results are shown on Table 6 below.

Table 6: Posterior Probability Outcomes at First Iteration

Outcomes B <sub>i</sub>	Probability P(B <sub>i</sub> )	States of Nature N <sub>i</sub>	Joint Probability P(B <sub>i</sub> ∩N <sub>i</sub> ) = P(N <sub>i</sub> ) P(B <sub>i</sub> /N <sub>i</sub> )	Posterior Probability P(N <sub>i</sub> /B <sub>i</sub> ) = P(N <sub>i</sub> ∩B <sub>i</sub> )/P(B <sub>i</sub> )
B <sub>1</sub>	0.2515	N <sub>1</sub>	0.0016	0.0016/0.2515 = 0.0064
		N <sub>2</sub>	0.0119	0.0119/0.2515 = 0.0473
		N <sub>3</sub>	0.003	0.003/0.2515 = 0.0119
		N <sub>4</sub>	0.0032	0.0032/0.2515 = 0.0127
		N <sub>5</sub>	0.0153	0.0153/0.2515 = 0.0608
		N <sub>6</sub>	0.020	0.020/0.2515 = 0.0795
		N <sub>7</sub>	0.0153	0.0153/0.2515 = 0.0608
		N <sub>8</sub>	0.0126	0.0126/0.2515 = 0.0501



B <sub>2</sub>	0.0599	N <sub>9</sub>	0.0128	0.0128/0.2515 = 0.0509
		N <sub>10</sub>	0.1558	0.1558/0.2515 = 0.6195
		N <sub>1</sub>	0.0022	0.0022/0.0599 = 0.0367
		N <sub>2</sub>	0.0063	0.0063/0.0599 = 0.1052
		N <sub>3</sub>	0.003	0.003/0.0599 = 0.0501
		N <sub>4</sub>	0.002	0.002/0.0599 = 0.0334
		N <sub>5</sub>	0.0054	0.0054/0.0599 = 0.0902
		N <sub>6</sub>	0.006	0.006/0.0599 = 0.1002
		N <sub>7</sub>	0.0036	0.0036/0.0599 = 0.0601
		N <sub>8</sub>	0.0028	0.0028/0.0599 = 0.0467
		N <sub>9</sub>	0.0064	0.0064/0.0599 = 0.1068
B <sub>3</sub>	0.0908	N <sub>10</sub>	0.0082	0.0082/0.0599 = 0.1369
		N <sub>1</sub>	0.0028	0.0028/0.0908 = 0.0308
		N <sub>2</sub>	0.0084	0.0084/0.0908 = 0.0925
		N <sub>3</sub>	0.0039	0.0039/0.0908 = 0.0430
		N <sub>4</sub>	0.0040	0.0040/0.0908 = 0.0441
		N <sub>5</sub>	0.0108	0.0108/0.0908 = 0.1189
		N <sub>6</sub>	0.0110	0.0110/0.0908 = 0.1211
		N <sub>7</sub>	0.009	0.009/0.0908 = 0.0991
		N <sub>8</sub>	0.0091	0.0091/0.0908 = 0.1002
		N <sub>9</sub>	0.0072	0.0072/0.0908 = 0.0793
		N <sub>10</sub>	0.0246	0.0246/0.0908 = 0.2709
B <sub>4</sub>	0.0374	N <sub>1</sub>	0.0016	0.0016/0.0374 = 0.0428
		N <sub>2</sub>	0.0084	0.0084/0.0374 = 0.2246
		N <sub>3</sub>	0.0024	0.0024/0.0374 = 0.0642
		N <sub>4</sub>	0.0032	0.0032/0.0374 = 0.0856
		N <sub>5</sub>	0.0027	0.0027/0.0374 = 0.0722
		N <sub>6</sub>	0.003	0.003/0.0374 = 0.0802
		N <sub>7</sub>	0.0036	0.0036/0.0374 = 0.0963
		N <sub>8</sub>	0.0028	0.0028/0.0374 = 0.0749
		N <sub>9</sub>	0.0056	0.0056/0.0374 = 0.1497
		N <sub>10</sub>	0.0041	0.0041/0.0374 = 0.1096

Table 6: Posterior Probability Outcomes at First Iteration Continued

Outcomes B <sub>i</sub>	Probability P(B <sub>ii</sub> )	States Nature N <sub>i</sub>	of Joint Probability P(B <sub>i</sub> ∩N <sub>i</sub> ) = P(N <sub>i</sub> ) P(B <sub>i</sub> /N <sub>i</sub> )	Posterior Probability P(N <sub>i</sub> /B <sub>i</sub> ) = P(N <sub>i</sub> ∩B <sub>i</sub> ) / P(B <sub>i</sub> )
B <sub>5</sub>	0.0423	N <sub>1</sub>	0.0016	0.0016/0.0423 = 0.0378
		N <sub>2</sub>	0.0084	0.0084/0.0423 = 0.1986
		N <sub>3</sub>	0.0024	0.0024/0.0423 = 0.0567
		N <sub>4</sub>	0.0032	0.0032/0.0423 = 0.0757
		N <sub>5</sub>	0.0054	0.0054/0.0423 = 0.1277
		N <sub>6</sub>	0.002	0.002/0.0423 = 0.0473
		N <sub>7</sub>	0.0027	0.0027/0.0423 = 0.0638
		N <sub>8</sub>	0.0028	0.0028/0.0423 = 0.0662
		N <sub>9</sub>	0.0056	0.0056/0.0423 = 0.1324
		N <sub>10</sub>	0.0082	0.0082/0.0423 = 0.1939
B <sub>6</sub>	0.0941	N <sub>1</sub>	0.0020	0.0020/0.0941 = 0.0213
		N <sub>2</sub>	0.0049	0.0049/0.0941 = 0.0521
		N <sub>3</sub>	0.003	0.003/0.0941 = 0.0319
		N <sub>4</sub>	0.0044	0.0044/0.0941 = 0.0468
		N <sub>5</sub>	0.0081	0.0081/0.0941 = 0.0861

		N <sub>6</sub>	0.009	0.009/0.0941 = 0.0956
		N <sub>7</sub>	0.009	0.009/0.0941 = 0.0956
		N <sub>8</sub>	0.0084	0.0084/0.0941 = 0.0893
		N <sub>9</sub>	0.0088	0.0088/0.0941 = 0.0935
		N <sub>10</sub>	0.00369	0.00369/0.0941 = 0.3921
B <sub>7</sub>	0.0987	N <sub>1</sub>	0.0018	0.0018/0.0987 = 0.0182
		N <sub>2</sub>	0.0049	0.0049/0.0987 = 0.0496
		N <sub>3</sub>	0.0027	0.0027/0.0987 = 0.0274
		N <sub>4</sub>	0.004	0.004/0.0987 = 0.0405
		N <sub>5</sub>	0.0099	0.0099/0.0987 = 0.1003
		N <sub>6</sub>	0.012	0.012/0.0987 = 0.1216
		N <sub>7</sub>	0.0099	0.0099/0.0987 = 0.1003
		N <sub>8</sub>	0.0063	0.0063/0.0987 = 0.0638
		N <sub>9</sub>	0.0072	0.0072/0.0987 = 0.0729
		N <sub>10</sub>	0.041	0.041/0.0987 = 0.1418
B <sub>8</sub>	0.1048	N <sub>1</sub>	0.0020	0.0020/0.1048 = 0.0191
		N <sub>2</sub>	0.0056	0.0056/0.1048 = 0.0534
		N <sub>3</sub>	0.003	0.003/0.1048 = 0.0286
		N <sub>4</sub>	0.0048	0.0048/0.1048 = 0.0458
		N <sub>5</sub>	0.0099	0.0099/0.1048 = 0.0859
		N <sub>6</sub>	0.012	0.012/0.1048 = 0.1145
		N <sub>7</sub>	0.0108	0.0108/0.1048 = 0.1031
		N <sub>8</sub>	0.0077	0.0077/0.1048 = 0.0735
		N <sub>9</sub>	0.008	0.008/0.1048 = 0.0763
		N <sub>10</sub>	0.041	0.041/0.1048 = 0.3912
B <sub>9</sub>	0.0288	N <sub>1</sub>	0.0004	0.0004/0.0288 = 0.1389
		N <sub>2</sub>	0.0014	0.0014/0.0288 = 0.0486
		N <sub>3</sub>	0.0006	0.0006/0.0288 = 0.2083
		N <sub>4</sub>	0.0012	0.0012/0.0288 = 0.0417
		N <sub>5</sub>	0.0027	0.0027/0.0288 = 0.0938
		N <sub>6</sub>	0.003	0.003/0.0288 = 0.1042
		N <sub>7</sub>	0.0027	0.0027/0.0288 = 0.0938
		N <sub>8</sub>	0.0021	0.0021/0.0288 = 0.0729
		N <sub>9</sub>	0.0024	0.0024/0.0288 = 0.0833
		N <sub>10</sub>	0.0123	0.0123/0.0288 = 0.4271

Table 6: Posterior Probability Outcomes at First Iteration Continued

Outcomes B <sub>i</sub>	Probability P(B <sub>i</sub> )	States of Nature N <sub>i</sub>	Joint Probability P(B <sub>i</sub> ∩ N <sub>i</sub> ) = P(N <sub>i</sub> ) P(B <sub>i</sub> /N <sub>i</sub> )	Posterior Probability P(N <sub>i</sub> /B <sub>i</sub> ) = P(N <sub>i</sub> ∩ B <sub>i</sub> ) / P(B <sub>i</sub> )
B <sub>10</sub>	0.2061	N <sub>1</sub>	0.004	0.004/0.2061 = 0.0194
		N <sub>2</sub>	0.0098	0.0098/0.2061 = 0.0475
		N <sub>3</sub>	0.006	0.006/0.2061 = 0.0291
		N <sub>4</sub>	0.010	0.010/0.2061 = 0.0485
		N <sub>5</sub>	0.0198	0.0198/0.2061 = 0.0961
		N <sub>6</sub>	0.023	0.023/0.2061 = 0.1116
		N <sub>7</sub>	0.0234	0.0234/0.2061 = 0.1135
		N <sub>8</sub>	0.0154	0.0154/0.2061 = 0.0747
		N <sub>9</sub>	0.0168	0.0168/0.2061 = 0.8151
		N <sub>10</sub>	0.0779	0.0779/0.2061 = 0.3780

**Discussion of Result in Table 6:**

(i). The Posterior Probability was computed by dividing each states of Nature total joint probabilities (referred to as marginal probabilities) by probability values of each outcomes for each of the course of action ( $B_i$ ) for  $B_1, B_2, B_3, B_4, B_5, B_6, B_7, B_8, B_9$  and  $B_{10}$  as stated before for all  $N_1, N_2, N_3, N_4, N_5, N_6, N_7, N_8, N_9$  and  $N_{10}$  for each set of  $B_i$ .

**4.2.6 Determination of Forecast Outcomes for the Objectives/Benefits at First Iteration (Posterior Expected Opportunity Loss)**

These are determined by calculating the forecast outcomes for the objectives/benefits which is the sum of the multiplication of each respective value of the posterior probability results with the

conditional opportunity loss of each of the states of nature to get the Expected Opportunity Loss (EOL). The sum totals of each set of values are referred to as the Posterior Expected Opportunity Loss for each of the objectives/benefits. The Conditional Opportunity Loss is obtained for each states of nature by subtracting each objectives/benefits ( $B_i$ ) from the highest benefits of each group. For example,  $B_1$  (economic efficiency); the COL for  $N_1 = 8.73 - 3.65 = 5.08$ ; the COL for  $N_2 = 8.73 - 4.84 = 3.89$ ; the COL for  $N_3 = 8.73 - 6.36 = 2.37$ ; the COL for  $N_4 = 8.73 - 3.6 = 5.13$ ; the COL for  $N_5 = 8.73 - 3.44 = 5.29$ ; the COL for  $N_6 = 8.73 - 4.37 = 4.36$  etc. the COL for  $N_7 = 8.73 - 4.05 = 4.68$ ; the COL for  $N_8 = 8.73 - 4.22 = 4.51$ ; the COL for  $N_9 = 8.73 - 1.12 = 7.61$ ; etc. These details are shown on Table 7 below.

Table 7: Forecast outcomes for objectives/Benefit at first iteration (Posterior Expected Opportunity Loss)

States of Nature	$B_1$ (Economic Efficiency)			$B_2$ (Federal Economic Redistribution)			$B_3$ (Regional Economic Redistribution)			$B_4$ (State Economic Redistribution)			$B_5$ (Local Economic Redistribution)			
	Prob.	COL	EOL	Prob.	COL	EOL	Prob.	COL	EOL	Prob.	COL	EOL	Prob.	COL	EOL	
$N_1$	0.0064	5.08	0.0325	0.0367	0	0	0.0308	4.59	0.1414	0.0428	17.49	0.7477	0.0378	4.95	0.1796	
$N_2$	0.0473	3.89	0.1840	0.1052	5.83	0.6133	0.0925	4.79	0.4431	0.2246	19.94	4.4785	0.1986	15.95	3.1677	
$N_3$	0.0119	2.37	0.0282	0.0501	3.78	0.1894	0.0430	3.09	0.1329	0.0642	15.31	0.9829	0.0567	9.70	0.5500	
$N_4$	0.0127	5.13	0.0652	0.0334	3.70	0.1236	0.0441	5.35	0.2359	0.0856	17.58	1.5048	0.0757	18.28	1.3838	
$N_5$	0.0608	5.29	0.3216	0.0902	4.09	0.3689	0.1189	5.61	0.6670	0.0722	17.53	0.2657	0.1277	15.88	2.0279	
$N_6$	0.0795	4.36	0.3466	0.1002	7.92	0.7936	0.1211	4.57	0.5534	0.0802	14.38	0.1533	0.0473	13.00	0.6149	
$N_7$	0.0608	4.68	0.2845	0.0601	7.33	0.4405	0.0991	4.91	0.4866	0.0963	14.81	1.4262	0.0638	10.45	0.6667	
$N_8$	0.0501	4.51	0.2260	0.0467	6.99	0.3264	0.1002	4.76	0.4770	0.0749	13.57	1.0164	0.0662	11.13	0.7369	
$N_9$	0.0509	7.61	0.3873	0.1068	12.01	1.2827	0.0793	8.0	0.6344	0.1497	22.44	3.3593	0.1324	18.96	2.5103	
$N_{10}$	0.6195	0	0	0.1369	2.43	0.3327	0.2709	0	0	0.1096	0	0	0.1939	0	0	
Posterior EOL			1.8959				4.4711				3.7717				15.9348	11.8377
States of Nature	$B_6$ (Social Well-being)			$B_7$ (Youth Empowerment)			$B_8$ (Environmental Quality Improvement)			$B_9$ (Gender Equality)			$B_{10}$ (Security)			
	Prob.	COL	EOL	Prob.	COL	EOL	Prob.	COL	EOL	Prob.	COL	EOL	Prob.	COL	EOL	
$N_1$	0.0213	2.69	0.0573	0.0182	9.01	0.1640	0.0191	2.87	0.0548	0.1389	4.07	0.5653	0.0194	0	0	
$N_2$	0.0521	16.54	0.8617	0.0496	22.00	1.0912	0.0534	13.77	0.7353	0.0486	11.25	0.5468	0.0475	77.26	3.6699	
$N_3$	0.0319	11.92	0.3802	0.0274	15.58	0.4269	0.0286	6.51	0.1862	0.2083	10.00	2.0830	0.0291	80.76	2.3501	
$N_4$	0.0468	18.73	0.8766	0.0405	22.52	0.9121	0.0458	13.63	0.6243	0.0417	11.68	0.4891	0.0485	79.56	3.8587	
$N_5$	0.0861	20.57	1.7711	0.1003	22.61	2.2678	0.0859	13.52	1.1614	0.0938	11.49	1.778	0.0961	79.44	7.6342	
$N_6$	0.0956	13.44	1.2849	0.1216	15.37	1.8690	0.1145	7.22	0.8267	0.1042	9.12	0.9503	0.1116	62.93	7.0230	
$N_7$	0.0956	11.80	1.1281	0.1003	14.61	1.4654	0.1031	9.65	0.9949	0.0938	10.42	0.9774	0.1135	62.38	7.0801	
$N_8$	0.0893	10.77	0.9618	0.0638	13.69	0.8734	0.0735	8.06	0.5924	0.0729	9.68	0.9057	0.0747	62.21	4.6471	
$N_9$	0.0935	19.22	1.7971	0.0729	22.61	1.6483	0.0763	5.7	1.1117	0.0833	15.82	1.3178	0.8151	77.15	62.8850	
$N_{10}$	0.3921	0	0	0.1418	0	0	0.3912	0	0	0.4271	0	0	0.3780	41.69	15.7588	
Posterior EOL			9.1188				10.7181				6.2877				8.7112	114.9069

**Discussion of Results in Table 7:** (i). The forecast outcomes for the objectives/benefits (Posterior Expected Opportunity Loss) was obtained by multiplying each of the posterior probabilities for each state of nature by the Conditional Opportunity Loss (COL) and adding up the results. (ii) The total Posterior Expected Opportunity Loss (EOL) for the objectives are; ₦1.8959 billion for economic efficiency; ₦4.4711 billion for federal economic redistribution; ₦3.7717 billion for regional

economic redistribution; ₦15.9348 billion for state economic redistribution; ₦11.8377 billion for local economic redistribution; ₦9.1188 billion for social well-being; ₦10.81 billion for youth empowerment; ₦6.2877 billion for environmental quality improvement; ₦8.7112 billion for gender equality; and ₦114.9069 billion for security.

**4.2.7 Determination of the Expected Value of Sample Information at First Iteration**

These are calculated based on the information from Marginal probabilities multiplied by values of Expected Opportunity Loss (E.O.L.). The expected value of sample information was

obtained by multiplying Posterior Expected Opportunity Loss (EOL) with the Marginal probabilities of various outcomes as shown on Table 8.

Table 8: The Expected Value of Sample Information at First Iteration

Outcomes	Marginal probability P(B <sub>i</sub> )	Expected Opportunity Loss (E.O.L.)	Expected Value of Sample Information
B <sub>1</sub>	0.2515	1.8759	0.4718
B <sub>2</sub>	0.0599	4.4711	0.2678
B <sub>3</sub>	0.0908	3.7717	0.3425
B <sub>4</sub>	0.0374	15.9348	0.5960
B <sub>5</sub>	0.0423	11.8377	0.5001
B <sub>6</sub>	0.0941	9.1188	0.8581
B <sub>7</sub>	0.0987	10.7181	1.0579
B <sub>8</sub>	0.1048	6.2877	0.6590
B <sub>9</sub>	0.0288	8.7112	0.2509
B <sub>10</sub>	0.2061	114.9069	23.6823
<b>EVSI</b>	<b>TOTAL</b>		<b>28.6864</b>

#### Discussion of Results in Table 8

(i) The Expected Value of Sample Information (EVSI) for each of the objectives/benefits is obtained by multiplying the marginal probabilities of each objectives by the Expected Opportunity Loss of each objectives. The values are ₦0.4718 billion for economic efficiency; ₦0.278 billion for federal economic redistribution; ₦0.3425 billion for regional economic redistribution; ₦0.5960 billion for state economic redistribution; ₦0.5001 billion for local economic redistribution; ₦0.8581 billion for social well-being; ₦1.0579 billion for youth empowerment; ₦0.659 billion for environmental quality improvement; ₦0.2509 billion for gender equality; and ₦23.6823 billion for security.

(ii) The total Expected Value of Sample Information (EVSI) of ₦28, 6564 billion indicates the money which can be paid for hiring the services of consultants for the River Basin operation yield for all purposes which include; Irrigation Agriculture, Hydroelectric Power Generation, Water Supply, Navigation, Drainage/Dredging, Flood Control, Recreation/Tourism, Erosion Control, Plantation / Forestry, Reservoir/Gullies if all the objectives as stated are to be achieved for optimization of resources utilization in Anambra-Imo River Basin Development Authority covering the five (5) Eastern states of Nigeria.

#### 4.3 Second Bayesian Decision Model Iteration Process

The Bayesian theory can be subjective but its subjectivity can be employed as a powerful attribute which considers experts' unbiased opinion as input into the policy iteration algorithm to

produce an optimum solution or decision. It describes the magnitude of difference between the alternative actions and provides a variety of estimates for consideration. The decision problem involving prior probabilities are called "data problems" which the second and third iterations tend to achieve.

It should be noted that the Bayesian Decision Model or Payoff Matrix involves the policy algorithm which can handle number of "state of nature" and alternative course of action infinitely. This has justified the need for the second iteration process of the Bayesian Decision Model to improve on the results on the first iteration process.

#### 4.3.1 Determination of Expected Monetary Values (EMVs) on Second (2nd) Iteration of Bayesian analysis with the Expected Profit with Perfect Information (EPPI) and Expected Value of Sample Information.

The optimal Bayes strategy is generally referred to as one which maximizes the expected monetary value. The expected (or mean) value is the long run average value that would result if the decision were repeated a large number of times.

The Posterior probability of the course of action having the maximum Expected Monetary Value (EMV\*) in the first iteration is used in the second iteration process as prior probability. The revised probabilities will be used to recalculate the Expected Monetary Value (EMV) which was generated based on perfect information. This can be referred to as value with data and are stated on Table 9 as shown.

In this case, the benefits that has the maximum Expected Monetary Value (EMV\*) is on B<sub>1</sub> with the values in the Posterior probabilities as in (B<sub>1</sub>)

which are; N<sub>1</sub> = 0.0064, N<sub>2</sub> = 0.0473, N<sub>3</sub> = 0.0119, N<sub>4</sub> = 0.0127, N<sub>5</sub> = 0.0608, N<sub>6</sub> = 0.0795, N<sub>7</sub> = 0.0608, N<sub>8</sub> = 0.0501, N<sub>9</sub> = 0.0509, N<sub>10</sub> = 0.6195.

**Table 9: Calculation of Expected Monetary Values (EMVs) at Second (2nd) Iteration**

Stat es of Nat ure	Prior Probabil ity P(N <sub>i</sub> )	Conditional Net Benefits Course of Action in Billions of Naira										Expected Net Benefits in Billions of Naira Course of Action											
		S <sub>1</sub>	S <sub>2</sub>	S <sub>3</sub>	S <sub>4</sub>	S <sub>5</sub>	S <sub>6</sub>	S <sub>7</sub>	S <sub>8</sub>	S <sub>9</sub>	S <sub>10</sub>	S <sub>1</sub>	S <sub>2</sub>	S <sub>3</sub>	S <sub>4</sub>	S <sub>5</sub>	S <sub>6</sub>	S <sub>7</sub>	S <sub>8</sub>	S <sub>9</sub>	S <sub>10</sub>		
N <sub>1</sub>	0.0064	3.65	4.84	6.36	3.60	3.44	4.37	4.05	4.22	1.12	8.73	0.0234	0.0310	0.0407	0.0230	0.0220	0.0280	0.0259	0.0270	0.0072	0.0559		
N <sub>2</sub>	0.0473	13.38	7.55	9.60	9.68	9.29	5.46	6.05	6.39	1.37	10.95	0.6329	0.3571	0.4541	0.4579	0.4394	0.2583	0.2862	0.3022	0.0648	0.5179		
N <sub>3</sub>	0.0119	4.54	4.34	6.04	3.78	3.52	4.56	4.22	4.37	1.13	9.13	0.0540	0.5165	0.0719	0.4500	0.0419	0.0543	0.0502	0.0520	0.0134	0.1086		
N <sub>4</sub>	0.0127	8.30	5.83	10.46	8.19	8.24	11.39	10.96	12.20	3.33	25.77	0.1054	0.0740	0.1328	0.1040	0.1046	0.1447	0.1392	0.1549	0.0423	1.3283		
N <sub>5</sub>	0.0608	17.21	6.01	12.26	3.68	6.08	8.96	11.51	10.83	3.00	21.96	1.0464	0.3654	0.7454	0.2237	0.3697	0.5448	0.6998	0.6585	0.1824	1.3352		
N <sub>6</sub>	0.0795	19.43	5.58	10.20	3.39	1.55	8.68	10.32	11.35	2.90	22.12	1.5447	0.4436	0.8109	0.2695	0.1232	0.6901	0.8204	0.9023	0.2301	1.7585		
N <sub>7</sub>	0.0608	16.93	3.94	10.36	3.42	3.33	10.57	11.33	12.25	3.33	25.94	1.0293	0.2396	0.6299	0.2079	0.2025	0.6427	0.6889	0.7448	0.2025	1.5772		
N <sub>8</sub>	0.0501	13.91	3.01	10.27	3.15	3.26	9.56	7.13	8.72	2.21	16.78	0.6969	0.1508	0.5145	0.1578	0.1633	0.4790	0.3572	0.4369	0.1107	0.8407		
N <sub>9</sub>	0.0509	14.01	6.83	8.08	6.40	6.59	8.96	7.66	8.40	2.26	13.88	0.7131	0.3476	0.4113	0.3258	0.3354	0.4561	0.3899	0.4276	0.1150	0.9203		
N <sub>10</sub>	0.6195	82.72	5.66	2.16	3.36	3.48	19.99	20.54	20.71	5.77	41.23	51.3689	3.5064	1.3381	2.0815	2.1559	12.383	12.7245	12.8298	3.5745	25.5420		
		Expected Monetary Values (EMVs) =										57.215	6.032	5.1496	4.3011	3.9579	15.681	16.1822	16.536	4.5429	32.9836		

**Discussion of Results in Table 9**

The information on Table 4.38 shows that the expected monetary values of each of the objectives for the second iteration are: ₦57.215 billion for economic efficiency; ₦6.032 billion on federal economic redistribution; ₦5.1496 billion for regional economic redistribution; ₦4.3011 billion for state economic redistribution; ₦3.95 billion for local economic redistribution; ₦15.6818 billion for fsocial well-being; ₦16.1822 billion youth empowerment; ₦16.563 billion for environmental quality improvement; ₦4.5429 billion for gender equality; and ₦32.9836 billion for security. The policy algorithm of Bayesian Model at 2nd iteration of EMVs is an improvement from the first iteration. The maximum Expected Monetary Value (EMV\*) = ₦57.215 billion on economic efficiency. This shows that with information provided by expert or consultant the maximum Expected Monetary Value (EMV\*) increased. Referring to the data on Table 4.38, the maximum benefit for each state of nature is used to calculate the Expected Profit with Perfect Information (EPPI) i.e. EPPI = 0.0064 (8.73) +

0.0473(13.38) + 0.0119 (9.13) + 0.0127(25.77) + 0.0608 (21.96) + 0.0795(22.12) + 0.0608 (25.94) + 0.0501(16.78) + 0.0509 (18.08) + 0.6195 (82.92) = ₦58.9254 billion. The Expected Value of Perfect Information (EVPI) = EPPI – EMV\* = ₦58.9254 - ₦57.215 billion = ₦1.7104 billion. For each of the forecast result the prior and posterior probabilities are calculated in Tables 4.39 and 4.40 respectively.

**4.3.2 Determination of Joint Probabilities Outcomes at Second (2nd) Iteration**

The determination of Joint Probabilities Outcomes at Second (2nd) Iteration was obtained by multiplying the revised State of nature (Prior) probabilities P(N<sub>i</sub>) from posterior probability outcomes of the first iteration with the conditional probability outcomes P(B<sub>i</sub>/N). These are calculated for each course of action outcomes (B<sub>i</sub>) as shown on Table 10 below.

The Joint Probabilities are calculated by multiplying value of prior probability by conditional probability which will be totaled to get the marginal probability.

Table .10: Joint Probabilities Outcomes at Second (2nd) Iteration

States of Nature $(N_i)$	Prior Probability $P(N_i)$	Outcomes $(B_j)$	Conditional Probability $P(B_j/N_i)$	Joint Probability $P(B_j \cap N_i) = P(N_i) P(B_j/N_i)$										
				$B_1$	$B_2$	$B_3$	$B_4$	$B_5$	$B_6$	$B_7$	$B_8$	$B_9$	$B_{10}$	
$N_1$	0.0064	$B_1$	0.08	0.0005										
		$B_2$	0.11		0.0007									
		$B_3$	0.14			0.0009								
		$B_4$	0.08				0.0005							
		$B_5$	0.08					0.0005						
		$B_6$	0.10						0.0006					
		$B_7$	0.09								0.0006			
		$B_8$	0.10									0.0006		
		$B_9$	0.02										0.0001	
		$B_{10}$	0.20											0.0013
$N_2$	0.0473	$B_1$	0.17	0.0080										
		$B_2$	0.09		0.0043									
		$B_3$	0.12			0.0057								
		$B_4$	0.12				0.0057							
		$B_5$	0.12					0.0057						
		$B_6$	0.07						0.0033					
		$B_7$	0.07								0.0033			
		$B_8$	0.08									0.0038		
		$B_9$	0.02										0.0009	
		$B_{10}$	0.14											0.0066
$N_3$	0.0119	$B_1$	0.10	0.0012										
		$B_2$	0.10		0.0012									
		$B_3$	0.13			0.0015								
		$B_4$	0.08				0.0010							
		$B_5$	0.08					0.001						
		$B_6$	0.10						0.0012					
		$B_7$	0.09								0.0011			
		$B_8$	0.10									0.0012		
		$B_9$	0.02										0.0002	
		$B_{10}$	0.20											0.0024

Table 10: Joint Probabilities Outcomes at Second (2nd) Iteration Continued

States of Nature $(N_i)$	Prior Probability $P(N_i)$	Outcomes $(B_j)$	Conditional Probability $P(B_j/N_i)$	Joint Probability $P(B_j \cap N_i) = P(N_i) P(B_j/N_i)$										
				$B_1$	$B_2$	$B_3$	$B_4$	$B_5$	$B_6$	$B_7$	$B_8$	$B_9$	$B_{10}$	
$N_4$	0.0127	$B_1$	0.08	0.0010										
		$B_2$	0.05		0.0006									
		$B_3$	0.10			0.0013								
		$B_4$	0.08				0.001							
		$B_5$	0.08					0.001						
		$B_6$	0.11						0.0014					
		$B_7$	0.10								0.0013			
		$B_8$	0.10									0.0013		
		$B_9$	0.03										0.0004	
		$B_{10}$	0.25											0.0032
$N_5$	0.0608	$B_1$	0.17	0.0003										
		$B_2$	0.06		0.0036									
		$B_3$	0.12			0.0073								
		$B_4$	0.03				0.0018							
		$B_5$	0.06					0.0036						
		$B_6$	0.09						0.0055					
		$B_7$	0.11								0.0067			
		$B_8$	0.11									0.0067		
		$B_9$	0.03										0.0018	
		$B_{10}$	0.22											0.0134
$N_6$	0.0795	$B_1$	0.20	0.0159										
		$B_2$	0.06		0.0048									
		$B_3$	0.11			0.0087								
		$B_4$	0.03				0.0024							
		$B_5$	0.02					0.0016						
		$B_6$	0.09						0.0072					
		$B_7$	0.11								0.0087			
		$B_8$	0.12									0.0095		
		$B_9$	0.03										0.0024	
		$B_{10}$	0.23											0.00183

**Table 10: Joint Probabilities Outcomes at Second (2nd) Iteration Continued**

States of Nature (N <sub>i</sub> )	Prior Probability P(N <sub>i</sub> )	Outcomes (B <sub>j</sub> )	Conditional Probability P(B <sub>j</sub> /N <sub>i</sub> )	Joint Probability P(B <sub>j</sub> ∩N <sub>i</sub> ) = P(N <sub>i</sub> ) P(B <sub>j</sub> /N <sub>i</sub> )										
				B <sub>1</sub>	B <sub>2</sub>	B <sub>3</sub>	B <sub>4</sub>	B <sub>5</sub>	B <sub>6</sub>	B <sub>7</sub>	B <sub>8</sub>	B <sub>9</sub>	B <sub>10</sub>	
N <sub>7</sub>	0.0608	B <sub>1</sub>	0.17	0.0103										
		B <sub>2</sub>	0.04		0.0024									
		B <sub>3</sub>	0.10			0.0061								
		B <sub>4</sub>	0.04				0.0024							
		B <sub>5</sub>	0.03					0.0018						
		B <sub>6</sub>	0.10						0.0061					
		B <sub>7</sub>	0.11							0.0067				
		B <sub>8</sub>	0.12								0.0073			
		B <sub>9</sub>	0.03									0.0018		
		B <sub>10</sub>	0.26											0.0158
N <sub>8</sub>	0.0501	B <sub>1</sub>	0.18	0.009										
		B <sub>2</sub>	0.04		0.002									
		B <sub>3</sub>	0.13			0.0065								
		B <sub>4</sub>	0.04				0.002							
		B <sub>5</sub>	0.04					0.002						
		B <sub>6</sub>	0.12						0.006					
		B <sub>7</sub>	0.09							0.0045				
		B <sub>8</sub>	0.11								0.0055			
		B <sub>9</sub>	0.03									0.0015		
		B <sub>10</sub>	0.22											0.011
N <sub>9</sub>	0.0509	B <sub>1</sub>	0.16	0.0081										
		B <sub>2</sub>	0.08		0.0041									
		B <sub>3</sub>	0.09			0.0046								
		B <sub>4</sub>	0.07				0.0036							
		B <sub>5</sub>	0.07					0.0036						
		B <sub>6</sub>	0.10						0.0051					
		B <sub>7</sub>	0.09							0.0046				
		B <sub>8</sub>	0.10								0.0051			
		B <sub>9</sub>	0.03									0.0015		
		B <sub>10</sub>	0.03											0.0015

**Table 10: Joint Probabilities Outcomes at Second (2nd) Iteration Continued**

States of Nature (N <sub>i</sub> )	Prior Probability P(N <sub>i</sub> )	Outcomes (B <sub>j</sub> )	Conditional Probability P(B <sub>j</sub> /N <sub>i</sub> )	Joint Probability P(B <sub>j</sub> ∩N <sub>i</sub> ) = P(N <sub>i</sub> ) P(B <sub>j</sub> /N <sub>i</sub> )										
				B <sub>1</sub>	B <sub>2</sub>	B <sub>3</sub>	B <sub>4</sub>	B <sub>5</sub>	B <sub>6</sub>	B <sub>7</sub>	B <sub>8</sub>	B <sub>9</sub>	B <sub>10</sub>	
N <sub>10</sub>	0.6195	B <sub>1</sub>	0.38	0.2354										
		B <sub>2</sub>	0.02		0.0124									
		B <sub>3</sub>	0.06			0.0372								
		B <sub>4</sub>	0.01				0.0062							
		B <sub>5</sub>	0.02					0.0124						
		B <sub>6</sub>	0.09						0.0558					
		B <sub>7</sub>	0.10							0.0620				
		B <sub>8</sub>	0.10								0.0620			
		B <sub>9</sub>	0.03									0.0186		
		B <sub>10</sub>	0.19											0.1177
<b>Marginal Probability =</b>				<b>0.2987</b>	<b>0.0361</b>	<b>0.0798</b>	<b>0.0266</b>	<b>0.026</b>	<b>0.0922</b>	<b>0.0955</b>	<b>0.1030</b>	<b>0.0292</b>	<b>0.2004</b>	

**Discussion of Results in Table 10:**

(i). The joint probabilities outcomes were calculated by multiplying prior probability of each states of nature by the conditional probability outcomes and adding of the result of each of them to obtain the marginal probability values as shown on Table 10.

(ii) The marginal probabilities values are: 0.2987 for economic efficiency; 0.0361 for federal economic redistribution; 0.0798 for regional economic redistribution; 0.0266 for state economic redistribution; 0.026 for local economic

redistribution; 0.0922 for social well-being; 0.0955 for youth empowerment; 0.1030 for environmental quality improvement; 0.0292 for gender equality and 0.2004 for security.

(iii) Comparing the second iteration results with the results obtained from first iteration, there was an increase in joint probability for B<sub>1</sub> (economic efficiency) = 0.2987 and B<sub>9</sub> (gender equality) = 0.0292 while other show a reduction of the values.

**4.3.3 Determination of Posterior Probability Outcomes at Second Iteration**

The Posterior Probability  $P(N_i/B_i) = P(N_i \cap B_i)/P(B_i)$  where  $P(B_i)$  is the values of the marginal probabilities which is the total sum of each values of the joint probabilities outcomes  $P(N_i \cap B_i)$ .

The Posterior Probability Outcomes at first iteration is computed by dividing each states of nature ( $N_i$ ) for  $i = 1, 2, 3, 4, 5, 6, 7, 8, 9, 10$  by each values of marginal probability outcomes  $P(B_i)$  for  $i = 1, 2, 3, 4, 5, 6, 7, 8, 9, 10$  under each group values. These results are shown on Table 11 below.

Table 11: Posterior Probability Outcomes at Second Iteration

Outcome s $B_i$	Probability $P(B_i)$	States of Nature ( $N_i$ )	Joint Probability $P(B_i \cap N_i) = P(N_i) P(B_i/N_i)$	Posterior Probability $P(N_i/B_i) = P(N_i \cap B_i)/P(B_i)$
$B_1$	0.2987	$N_1$	0.0005	$0.0005/0.2987 = 0.0017$
		$N_2$	0.008	$0.0080/0.2987 = 0.0024$
		$N_3$	0.0012	$0.0012/0.2987 = 0.0040$
		$N_4$	0.001	$0.0010/0.2987 = 0.0043$
		$N_5$	0.0103	$0.0103/0.2987 = 0.0345$
		$N_6$	0.0159	$0.0159/0.2987 = 0.0532$
		$N_7$	0.0103	$0.0103/0.2987 = 0.0345$
		$N_8$	0.009	$0.0090/0.2987 = 0.0276$
		$N_9$	0.0081	$0.0081/0.2987 = 0.0281$
		$N_{10}$	0.2354	$0.2354/0.2987 = 0.7881$
$B_2$	0.0361	$N_1$	0.0007	$0.0007/0.0361 = 0.0194$
		$N_2$	0.0043	$0.0043/0.0361 = 0.1191$
		$N_3$	0.0012	$0.0012/0.0361 = 0.0332$
		$N_4$	0.0006	$0.0006/0.0361 = 0.0166$
		$N_5$	0.0036	$0.0036/0.0361 = 0.0997$
		$N_6$	0.0048	$0.0048/0.0361 = 0.1330$
		$N_7$	0.0024	$0.0024/0.0361 = 0.0665$
		$N_8$	0.002	$0.002/0.0361 = 0.0554$
		$N_9$	0.0081	$0.0081/0.0361 = 0.2244$
		$N_{10}$	0.0124	$0.0124/0.0361 = 0.3435$
$B_3$	0.0798	$N_1$	0.0009	$0.0009/0.0798 = 0.0113$
		$N_2$	0.0057	$0.0057/0.0798 = 0.0714$
		$N_3$	0.0015	$0.0015/0.0798 = 0.0188$
		$N_4$	0.0013	$0.0013/0.0798 = 0.0163$
		$N_5$	0.0073	$0.0073/0.0798 = 0.0915$
		$N_6$	0.0087	$0.0087/0.0798 = 0.1090$
		$N_7$	0.0061	$0.0061/0.0798 = 0.0764$
		$N_8$	0.0065	$0.0065/0.0798 = 0.0815$
		$N_9$	0.0046	$0.0046/0.0798 = 0.0576$
		$N_{10}$	0.0372	$0.0372/0.0798 = 0.4662$
$B_4$	0.0266	$N_1$	0.0005	$0.0005/0.0266 = 0.0188$
		$N_2$	0.0057	$0.0057/0.0266 = 0.2143$
		$N_3$	0.001	$0.0010/0.0266 = 0.0376$
		$N_4$	0.001	$0.0010/0.0266 = 0.0376$
		$N_5$	0.0018	$0.0018/0.0266 = 0.0677$
		$N_6$	0.0024	$0.0024/0.0266 = 0.0902$
		$N_7$	0.0024	$0.0024/0.0266 = 0.0902$
		$N_8$	0.002	$0.002/0.0266 = 0.0752$
		$N_9$	0.0036	$0.0036/0.0266 = 0.1353$
		$N_{10}$	0.0062	$0.0062/0.0266 = 0.2331$



Table 11: Posterior Probability Outcomes at Second Iteration continued

Outcomes $B_i$	Probability $P(B_{ii})$	States of Nature ( $N_i$ )	Joint Probability $P(B_i \cap N_i) = P(N_i) \cdot P(B_i/N_i)$	Posterior Probability $P(N_i/B_i) = P(N_i \cap B_i) / P(B_i)$
B <sub>5</sub>	0.026	N <sub>1</sub>	0.0005	0.0005/0.026 = 0.0192
		N <sub>2</sub>	0.0057	0.0057/0.026 = 0.2192
		N <sub>3</sub>	0.001	0.0010/0.026 = 0.0385
		N <sub>4</sub>	0.001	0.0010/0.026 = 0.0385
		N <sub>5</sub>	0.0036	0.0036/0.026 = 0.1385
		N <sub>6</sub>	0.0016	0.0016/0.026 = 0.0615
		N <sub>7</sub>	0.0018	0.0018/0.026 = 0.0692
		N <sub>8</sub>	0.002	0.0020/0.026 = 0.0769
		N <sub>9</sub>	0.0036	0.0036/0.026 = 0.1385
		N <sub>10</sub>	0.0124	0.0124/0.026 = 0.4769
B <sub>6</sub>	0.0922	N <sub>1</sub>	0.0006	0.0006/0.0922 = 0.0065
		N <sub>2</sub>	0.0033	0.0033/0.0922 = 0.0358
		N <sub>3</sub>	0.0012	0.0012/0.0922 = 0.0130
		N <sub>4</sub>	0.0014	0.0014/0.0922 = 0.0152
		N <sub>5</sub>	0.0055	0.0055/0.0922 = 0.0542
		N <sub>6</sub>	0.0072	0.0072/0.0922 = 0.0781
		N <sub>7</sub>	0.0061	0.0061/0.0922 = 0.0662
		N <sub>8</sub>	0.006	0.006/0.0922 = 0.0651
		N <sub>9</sub>	0.0051	0.0051/0.0922 = 0.0553
		N <sub>10</sub>	0.0558	0.0558/0.0922 = 0.6052
B <sub>7</sub>	0.0995	N <sub>1</sub>	0.0006	0.0006/0.0995 = 0.0060
		N <sub>2</sub>	0.0033	0.0033/0.0995 = 0.0332
		N <sub>3</sub>	0.0011	0.0011/0.0995 = 0.0111
		N <sub>4</sub>	0.0013	0.0013/0.0995 = 0.0131
		N <sub>5</sub>	0.0067	0.0067/0.0995 = 0.0673
		N <sub>6</sub>	0.0087	0.0087/0.0995 = 0.0874
		N <sub>7</sub>	0.0067	0.0067/0.0995 = 0.0673
		N <sub>8</sub>	0.0045	0.0045/0.0995 = 0.0452
		N <sub>9</sub>	0.0046	0.0046/0.0995 = 0.0462
		N <sub>10</sub>	0.0620	0.0620/0.0995 = 0.0623
B <sub>8</sub>	0.1030	N <sub>1</sub>	0.0006	0.0006/0.1030 = 0.0058
		N <sub>2</sub>	0.0038	0.0038/0.1030 = 0.0369
		N <sub>3</sub>	0.0012	0.0012/0.1030 = 0.0117
		N <sub>4</sub>	0.0013	0.0013/0.1030 = 0.0126
		N <sub>5</sub>	0.0067	0.0067/0.1030 = 0.0650
		N <sub>6</sub>	0.0095	0.0095/0.1030 = 0.0922
		N <sub>7</sub>	0.0073	0.0073/0.1030 = 0.0709
		N <sub>8</sub>	0.0055	0.0055/0.1030 = 0.0534
		N <sub>9</sub>	0.0051	0.0051/0.1030 = 0.0495
		N <sub>10</sub>	0.0620	0.0620/0.1030 = 0.6019
B <sub>9</sub>	0.0292	N <sub>1</sub>	0.0001	0.0001/0.0292 = 0.0034
		N <sub>2</sub>	0.0009	0.0009/0.0292 = 0.0308
		N <sub>3</sub>	0.0002	0.0002/0.0292 = 0.0685
		N <sub>4</sub>	0.0004	0.0004/0.0292 = 0.0137
		N <sub>5</sub>	0.0018	0.0018/0.0292 = 0.0616
		N <sub>6</sub>	0.0024	0.0024/0.0292 = 0.0822
		N <sub>7</sub>	0.0018	0.0018/0.0292 = 0.0616
		N <sub>8</sub>	0.0015	0.0015/0.0292 = 0.0514

		N <sub>9</sub>	0.0015	0.0015/0.0292 = 0.0514
		N <sub>10</sub>	0.0186	0.0186/0.0292 = 0.6370

Table 11: Posterior Probability Outcomes at Second Iteration continued

Outcomes B <sub>i</sub>	Probability P(B <sub>ii</sub> )	States of Nature (N <sub>i</sub> )	Joint Probability P(B <sub>i</sub> ∩N <sub>i</sub> )=P(N <sub>i</sub> ) P(B <sub>i</sub> /N <sub>i</sub> )	Posterior Probability P(N <sub>i</sub> /B <sub>i</sub> ) = P(N <sub>i</sub> ∩B <sub>i</sub> )/P(B <sub>i</sub> )
B <sub>10</sub>	0.2004	N <sub>1</sub>	0.0013	0.0013/0.2004 = 0.0065
		N <sub>2</sub>	0.0066	0.0066/0.2004 = 0.0329
		N <sub>3</sub>	0.0024	0.0024/0.2004 = 0.0120
		N <sub>4</sub>	0.0032	0.0032/0.2004 = 0.0160
		N <sub>5</sub>	0.0134	0.0134/0.2004 = 0.0669
		N <sub>6</sub>	0.0183	0.0183/0.2004 = 0.0913
		N <sub>7</sub>	0.0158	0.0158/0.2004 = 0.0788
		N <sub>8</sub>	0.011	0.011/0.2004 = 0.0549
		N <sub>9</sub>	0.0107	0.0107/0.2004 = 0.0534
		N <sub>10</sub>	0.1177	0.1177/0.2004 = 0.5873

**Discussion of Result in Table 11**

(i). The Posterior Probability was computed by dividing each states of Nature total joint probabilities (referred to as marginal probabilities) by probability values of each outcomes for each of the objectives (B<sub>i</sub>) for B<sub>1</sub>, B<sub>2</sub>, B<sub>3</sub>, B<sub>4</sub>, B<sub>5</sub>, B<sub>6</sub>, B<sub>7</sub>, B<sub>8</sub>, B<sub>9</sub> and B<sub>10</sub> as stated before for all N<sub>1</sub>, N<sub>2</sub>, N<sub>3</sub>, N<sub>4</sub>, N<sub>5</sub>, N<sub>6</sub>, N<sub>7</sub>, N<sub>8</sub>, N<sub>9</sub> and N<sub>10</sub> for each set of B<sub>i</sub>.

(ii) For example, the benefits B<sub>1</sub> (economic efficiency and the values of posterior probabilities under it were: N<sub>1</sub> (irrigated agriculture) = 0.0017, N<sub>2</sub> = 0.0024, N<sub>3</sub> = 0.0040, N<sub>4</sub> = 0.0043, N<sub>5</sub> = 0.0345, N<sub>6</sub> = 0.0532, N<sub>7</sub> = 0.0345, N<sub>8</sub> = 0.0276, N<sub>9</sub> = 0.0281 and N<sub>10</sub> = 0.7881. It follows the same pattern for the benefits B<sub>2</sub> (Federal economic redistribution), B<sub>3</sub> (Regional economic redistribution), B<sub>4</sub>(Stateeconomic redistribution), B<sub>5</sub> (Local economic redistribution), B<sub>6</sub> (Social well-being), B<sub>7</sub> (Youth empowerment), B<sub>8</sub>(Environmental quality improvement), B<sub>9</sub> (Gender equality) and B<sub>10</sub> (Security). These are shown on Table 11.

(iii) Comparing the results from the first iterations, there are reductions in the posterior probability outcomes while only on purpose of reservoir and gullies, the value increased from 0.6915 to 0.7881.

**4.3.4 Determination of Forecast Outcomes for Benefits at Second (2nd) Iteration (Posterior Expected Opportunity Loss)**

This is determined by calculating the forecast outcomes for the benefits which is the sum of the multiplication of each respective value of the posterior probability results with the conditional opportunity loss of each of the states of nature to get the Expected Opportunity Loss (EOL). The sum totals of each set of values are referred to as the Posterior Expected Opportunity Loss for each of the objectives/benefits. The Conditional Opportunity Loss is obtained for each states of nature by subtracting each net benefit (B<sub>i</sub>) from the highest benefits of each group. For example, B<sub>1</sub> (Economic efficiency); the COL for N<sub>1</sub> = 8.73 – 3.65 = 5.08; the COL for N<sub>2</sub> = 8.73 – 4.84 = 3.89; the COL for N<sub>3</sub> = 8.73 – 6.36 = 2.37; the COL for N<sub>4</sub> = 8.73 – 3.6 = 5.13; the COL for N<sub>5</sub> = 8.73 – 3.44 = 5.29; the COL for N<sub>6</sub> = 8.73 – 4.37 = 4.36 etc. the COL for N<sub>7</sub> = 8.73 – 4.05 = 4.68; the COL for N<sub>8</sub> = 8.73 – 4.22 = 4.51; the COL for N<sub>9</sub> = 8.73 – 1.12 = 7.61; etc. This is calculated by multiplying the individual posterior probabilities with the conditional Opportunity Loss as shown on Table 12.

Table 12 Forecast Outcomes for Objectives/Benefits at Second (2nd) Iteration (Posterior Expected Opportunity Loss)

States of Nature	B <sub>1</sub> (Economic Efficiency)			B <sub>2</sub> (Federal Economic Redistribution)			B <sub>3</sub> (Regional Economic Redistribution)			B <sub>4</sub> (State Economic Redistribution)			B <sub>5</sub> (Local Economic Redistribution)		
	Prob.	COL	EOL	Prob.	COL	EOL	Prob.	COL	EOL	Prob.	COL	EOL	Prob.	COL	EOL
N <sub>1</sub>	0.0017	5.08	0.0086	0.0194	0	0	0.0113	4.59	0.0519	0.0188	17.49	0.3284	0.0192	4.95	0.0912
N <sub>2</sub>	0.0024	3.89	0.0934	0.1191	5.83	0.6944	0.0714	4.79	0.3420	0.2143	19.94	4.2731	0.2192	15.95	3.4962
N <sub>3</sub>	0.0040	2.37	0.0095	0.0332	3.78	0.1255	0.0188	3.09	0.0581	0.0376	15.31	0.5757	0.0385	9.70	0.3735
N <sub>4</sub>	0.0043	5.13	0.0221	0.0166	3.70	0.0614	0.0163	5.35	0.0872	0.0376	17.58	0.6610	0.0385	18.28	0.7038
N <sub>5</sub>	0.0345	5.29	0.1825	0.0997	4.09	0.4078	0.0915	5.61	0.5133	0.0677	17.53	1.1868	0.1385	15.88	2.1994
N <sub>6</sub>	0.0532	4.36	0.2320	0.1330	7.92	1.0534	0.1090	4.57	0.4981	0.0902	14.38	1.2971	0.0615	13.00	0.7995
N <sub>7</sub>	0.0345	4.68	0.1615	0.0665	7.33	0.4874	0.0764	4.91	0.3751	0.0902	14.81	1.3359	0.0692	10.45	0.7231
N <sub>8</sub>	0.0276	4.51	0.1245	0.0554	6.99	0.3872	0.0815	4.76	0.3879	0.0752	13.57	1.0205	0.0769	11.13	0.8559
N <sub>9</sub>	0.0281	7.61	0.2138	0.2244	12.01	2.6950	0.0576	8.0	0.4608	0.1353	22.44	3.0361	0.1385	18.96	2.6260
N <sub>10</sub>	0.7881	0	0	0.3435	2.43	0.8396	0.4662	0	0	0.2331	0	0	0.4769	0	0
Posterior EOL	1.0479			6.7517			2.7744			13.7146			11.0983		
States of Nature	B <sub>6</sub> (Social Well-being)			B <sub>7</sub> (Youth Empowerment)			B <sub>8</sub> (Environmental Quality Improvement)			B <sub>9</sub> (Gender Equality)			B <sub>10</sub> (Security)		
	Prob.	COL	EOL	Prob.	COL	EOL	Prob.	COL	EOL	Prob.	COL	EOL	Prob.	COL	EOL
N <sub>1</sub>	0.0065	2.69	0.0175	0.0060	9.01	0.0541	0.0058	2.87	0.0166	0.0034	4.07	0.0138	0.0065	0	0
N <sub>2</sub>	0.0358	16.54	0.5921	0.0332	22.00	0.7304	0.0369	13.77	0.5081	0.0308	11.25	0.3465	0.0329	77.26	2.5419
N <sub>3</sub>	0.0130	11.92	0.1550	0.0111	15.58	0.1729	0.0117	6.51	0.0762	0.0685	10.00	0.6850	0.0120	80.76	0.9691
N <sub>4</sub>	0.0152	18.73	0.2847	0.0131	22.52	0.2950	0.0126	13.63	0.1717	0.0137	11.68	0.1600	0.0160	79.56	1.2730
N <sub>5</sub>	0.0542	20.57	1.1149	0.0673	22.61	1.5217	0.0650	13.52	0.8788	0.0616	11.49	0.7078	0.0669	79.44	5.3145
N <sub>6</sub>	0.0781	13.44	1.0497	0.0874	15.37	1.3433	0.0922	7.22	0.6657	0.0822	9.12	0.7497	0.0913	62.93	5.7455
N <sub>7</sub>	0.0662	11.80	0.7812	0.0673	14.61	0.9833	0.0709	9.65	0.6842	0.0616	10.42	0.6419	0.0788	62.38	4.9155
N <sub>8</sub>	0.0651	10.77	0.7011	0.0452	13.69	0.6188	0.0534	8.06	0.4304	0.0514	9.68	0.4976	0.0549	62.21	3.4153
N <sub>9</sub>	0.0553	19.22	1.0629	0.0462	22.61	1.0446	0.0495	14.	0.7212	0.0514	15.82	1.8131	0.0534	77.15	4.1198
N <sub>10</sub>	0.6052	0	0	0.0623	0	0	0.6019	0	0	0.6370	0	0	0.5873	41.69	24.4845
Posterior EOL	5.7591			6.7641			4.1529			4.6154			52.7791		

**Discussion of Results in Table 12:** (i) The forecast outcomes for the objectives/benefits (Posterior Expected Opportunity Loss) was obtained by multiplying each of the posterior probabilities for each state of nature by the Conditional Opportunity Loss (COL) and adding up the results. (ii) The total Posterior Expected Opportunity Loss (EOL) for the objectives are; ₦1.0479 billion for economic efficiency; ₦6.7517 billion for federal economic redistribution; ₦2.7744 billion for regional economic redistribution; ₦13.7146 billion for state economic redistribution; ₦ 11.0983 billion for local economic redistribution; ₦ 5.7591 billion for social

well-being; ₦ 6.7641 billion for youth empowerment; ₦ 4.1529 billion for environmental quality improvement; ₦4.6154 billion for gender equality; and ₦52.7791 billion for security.

### 3.5 Determination of Expected Value of Sample Information Outcomes at Second (2nd) Iteration

The Expected Value of Sample Information (EVSI) is calculated by multiplying each value of the Marginal probabilities with the Expected Opportunity Loss Values (EOL) as shown on Table 13.

Table 13: The Expected Value of Sample Information Outcomes at Second (2nd) Iteration

Outcomes B <sub>i</sub>	Marginal probability P(B <sub>i</sub> )	Expected Opportunity Loss (EOL)	Expected Value of Sample Information
B <sub>1</sub>	0.2987	1.0479	0.3130
B <sub>2</sub>	0.0361	6.7517	0.2437
B <sub>3</sub>	0.0798	2.7744	0.2214
B <sub>4</sub>	0.0266	13.7146	0.3648
B <sub>5</sub>	0.0260	11.0983	0.2886
B <sub>6</sub>	0.0922	5.7591	0.5310
B <sub>7</sub>	0.0995	6.7641	0.6730
B <sub>8</sub>	0.1030	4.1529	0.4277
B <sub>9</sub>	0.0292	4.6154	0.1348
B <sub>10</sub>	0.2004	52.7791	10.5769
<b>TOTAL (EVSI) =</b>			<b>13.7749 billion</b>

**Discussion of Results in Table 13:**

(i) The Expected Value of Sample Information (EVSI) for each of the objectives are obtained by multiplying the marginal probabilities of each objectives by the Expected Opportunity Loss of each the objective. The values are: ₦0.3130 billion for economic efficiency; ₦0.2437 billion for federal economic redistribution; ₦0.2214 billion for regional economic redistribution; ₦0.3648 billion for state economic redistribution; ₦0.2886 billion for local economic redistribution; ₦0.5310 billion for social well-being; ₦0.6730 billion for youth empowerment; ₦0.4277 billion for environmental quality improvement; ₦0.1348 billion for gender equality; and ₦10.5769 billion for security.

(ii) The total expected Value of Sample Information (EVSI) as calculated in Table 13 is ₦13.7749 billion which indicates the money which can be paid for hiring the services of consultants for the River Basin operation yield for all the ten (10) purposes of irrigation agriculture, hydroelectric power generation, water supply, navigation, drainage/dredging, flood control, recreation/tourism, erosion control, plantation/forestry, reservoir/gullies etc. respectively.

**4.4 Third (3rd) Bayesian Decision Model Iteration Process**

The Posterior productivity of the course of action having the maximum Expected Monetary Value (EMV\*) in the second iteration process is used in the third iteration process. The revised probabilities will be used to recalculate the Expected Monetary Value (EMV) which was generated based on perfect information. This can be referred to as when more data were provided based on the performance of the previous data. It should be noted that the Bayesian Decision Model or Payoff Matrix involves the policy

algorithm which can handle number of “state of nature” and alternative course of action infinitely. This has justified the need for the third iteration process of the Bayesian Decision Model to improve on the results on the second iteration process.

**4.4.1 Determination of Expected Monetary Values (EMVs) on Third Iteration**

The optimal Bayes strategy is generally referred to as one which maximizes the expected monetary value. The expected (or mean) value is the long run average value that would result if the decision were repeated a large number of times.

The Posterior probability of the course of action having the maximum Expected Monetary Value (EMV\*) in the second iteration is used in the third iteration process as prior probability. The revised probabilities will be used to recalculate the Expected Monetary Value (EMV) which was generated based on perfect information. This can be referred to as value with additional data as shown.

In this case, the objective that has the maximum Expected Monetary Value (EMV\*) is on B<sub>1</sub> which is economic efficiency with the values in the Posterior probabilities as in (B<sub>1</sub>) which are; N<sub>1</sub> = 0.0017, N<sub>2</sub> = 0.0024, N<sub>3</sub> = 0.0040, N<sub>4</sub> = 0.0043, N<sub>5</sub> = 0.0345, N<sub>6</sub> = 0.0532, N<sub>7</sub> = 0.0345, N<sub>8</sub> = 0.0276, N<sub>9</sub> = 0.0281, N<sub>10</sub> = 0.7881.

The Expected Monetary Values (EMVs) are calculated by multiplying each values of the conditional net benefits course of action by each value of the corresponding value of probability. Subsequently, the expected profit with perfect information is calculated as the sum of each value of the total maximum expected monetary values of each expected net benefit course of action as shown on Table 14 and the Expected Value of Perfect Information (EVPI).

Table 14: Calculation of Expected Monetary Values (EMVs) at Third (3rd) Iteration

States of Nature	Prior Probability P(N)	Conditional Net Benefits Course of Action in Billions of Naira										Expected Net Benefits in Billions of Naira Course of Action									
		S <sub>1</sub>	S <sub>2</sub>	S <sub>3</sub>	S <sub>4</sub>	S <sub>5</sub>	S <sub>6</sub>	S <sub>7</sub>	S <sub>8</sub>	S <sub>9</sub>	S <sub>10</sub>	S <sub>1</sub>	S <sub>2</sub>	S <sub>3</sub>	S <sub>4</sub>	S <sub>5</sub>	S <sub>6</sub>	S <sub>7</sub>	S <sub>8</sub>	S <sub>9</sub>	S <sub>10</sub>
N <sub>1</sub>	0.0017	3.65	4.84	6.36	3.60	3.44	4.37	4.05	4.22	1.12	8.73	0.0062	0.0082	0.0108	0.0061	0.0058	0.0074	0.0069	0.0072	0.0019	0.0148
N <sub>2</sub>	0.0024	13.38	7.55	9.60	9.68	9.29	5.46	6.05	6.39	1.37	10.95	0.3211	0.1812	0.2304	0.2323	0.2230	0.1310	0.1452	0.1534	0.0329	0.2628
N <sub>3</sub>	0.0040	4.54	4.34	6.04	3.78	3.52	4.56	4.22	4.37	1.13	9.13	0.0182	0.0174	0.0242	0.0151	0.0141	0.0182	0.0502	0.0175	0.0045	0.0365
N <sub>4</sub>	0.0043	8.30	5.83	10.46	8.19	8.24	11.39	10.96	12.20	3.33	25.77	0.0357	0.0251	0.0450	0.0352	0.0354	0.0490	0.0471	0.0525	0.0143	0.1108
N <sub>5</sub>	0.0345	17.21	6.01	12.26	3.68	6.08	8.96	11.51	10.83	3.00	21.96	0.5937	0.2073	0.4230	0.1270	0.2098	0.3091	0.3971	0.03736	0.1035	0.7576
N <sub>6</sub>	0.0532	19.43	5.58	10.20	3.39	1.55	8.68	10.32	11.35	2.90	22.12	1.0337	0.2969	0.5426	0.1803	0.0825	0.4618	0.5490	0.6038	0.1543	1.1768
N <sub>7</sub>	0.0345	16.93	3.94	10.36	3.42	3.33	10.57	11.33	12.25	3.33	25.94	0.5841	0.1359	0.3574	0.1180	0.1149	0.3647	0.3909	0.4226	0.1149	0.8949
N <sub>8</sub>	0.0276	13.91	3.01	10.27	3.15	3.26	9.56	7.13	8.72	2.21	16.78	0.3839	0.0831	0.2835	0.0869	0.0900	0.2639	0.1968	0.2407	0.0610	0.4631
N <sub>9</sub>	0.0281	14.01	6.83	8.08	6.40	6.59	8.96	7.66	8.40	2.26	18.08	0.3937	0.1919	0.2270	0.1798	0.1852	0.2518	0.2152	0.2360	0.0635	0.5080
N <sub>10</sub>	0.7881	82.72	5.66	2.16	3.36	3.48	19.99	20.54	20.71	5.77	41.23	63.3493	4.4606	1.7023	2.6480	2.7426	15.7541	16.1876	16.3216	4.5473	32.4934
		Expected Monetary Values (EMVs) =										68.7196	5.6076	3.8462	3.6287	3.7033	17.611	18.1527	18.4289	5.0981	36.7187

**Discussion of Results in Table 14:**

(i) The information on Table 14 shows that the expected monetary values of each of the objectives for the third iteration are: ₦68.7196 billion for economic efficiency; ₦5.6076 billion for federal economic redistribution; ₦3.8462 billion for regional economic redistribution; ₦3.6287 billion for state economic redistribution; ₦3.7033 billion for local economic redistribution; ₦17.611 billion for social well-being; ₦18.1527 billion for youth empowerment; ₦18.4289 billion for environmental quality improvement; ₦5.0981 billion for gender equality; and ₦36.7187 billion for security. (ii) The policy algorithm of Bayesian Model at third iteration of EMVs is an improvement from the second iteration. (iii). The maximum Expected Monetary Value (EMV\*) = ₦68.7196 billion for economic efficiency. (iv) This shows that with information provided by expert or consultant the maximum Expected Monetary Value (EMV\*) increased. Referring to the data on Table 14, the maximum benefit for each states of nature is used to calculate the Expected Profit with Perfect Information (EPPI)= 0.0017(8.73) + 0.024(13.38)

$$+ 0.004 (9.13) + 0.0043 (25.77) + 0.0345 (21.96) + 0.0532 (22.12) + 0.0345(25.94) + 0.0276 (16.78) + 0.0281(18.08) + 0.7881 (82.92) = ₦69.633$$

The Expected Value of Perfect Information (EVPI) = EPPI – EMV = ₦69.633 - ₦68.7196 billion= ₦0.9134 billion

For each of the forecast result, the Prior and Posterior probabilities are calculated in Tables 15 and 16 respectively.

**4.4.2 Determination of Joint Probabilities Outcomes on Third (3<sup>rd</sup>) Iteration**

The determination of Joint Probabilities Outcomes at third (3rd) Iteration was obtained by multiplying the revised State of nature (Prior) probabilities P(N<sub>i</sub>) from posterior probability outcomes of the second iteration with the conditional probability outcomes P(B<sub>j</sub>/N). These are calculated for each courses of action outcomes (B<sub>j</sub>) as shown on Table 15 below. The Joint Probabilities are calculated by multiplying value of prior probability by conditional probability which will be totaled to get the marginal probability

Table 15: Joint Probabilities Outcomes at Third (3rd) Iteration

States of Nature (N <sub>i</sub> )	Prior Probability P(N <sub>i</sub> )	Outcomes (B <sub>j</sub> )	Conditional Probability P(B <sub>j</sub> /N <sub>i</sub> )	Joint Probability P(B <sub>j</sub> ∩N <sub>i</sub> ) = P(N <sub>i</sub> ) P(B <sub>j</sub> /N <sub>i</sub> )																	
				B <sub>1</sub>	B <sub>2</sub>	B <sub>3</sub>	B <sub>4</sub>	B <sub>5</sub>	B <sub>6</sub>	B <sub>7</sub>	B <sub>8</sub>	B <sub>9</sub>	B <sub>10</sub>								
N <sub>1</sub>	0.0017	B <sub>1</sub>	0.08	0.0001																	
		B <sub>2</sub>	0.11		0.0002																
		B <sub>3</sub>	0.14			0.0002															
		B <sub>4</sub>	0.08				0.0001														
		B <sub>5</sub>	0.08					0.0001													
		B <sub>6</sub>	0.10						0.0002												
		B <sub>7</sub>	0.09							0.0002											
		B <sub>8</sub>	0.10									0.0002									
		B <sub>9</sub>	0.02											0.00003							
		B <sub>10</sub>	0.20																		0.0003
N <sub>2</sub>	0.024	B <sub>1</sub>	0.17	0.0041																	
		B <sub>2</sub>	0.09		0.0022																
		B <sub>3</sub>	0.12			0.0029															
		B <sub>4</sub>	0.12				0.0029														
		B <sub>5</sub>	0.12					0.0029													
		B <sub>6</sub>	0.07							0.0017											
		B <sub>7</sub>	0.07								0.0017										
		B <sub>8</sub>	0.08										0.0019								
		B <sub>9</sub>	0.02												0.0005						
		B <sub>10</sub>	0.14																		0.0034

Table 15: Joint Probabilities Outcomes at Third (3rd) Iteration Continued

States of Nature (N <sub>i</sub> )	Prior Probability P(N <sub>i</sub> )	Outcomes (B <sub>j</sub> )	Conditional Probability P(B <sub>j</sub> /N <sub>i</sub> )	Joint Probability P(B <sub>j</sub> ∩N <sub>i</sub> ) = P(N <sub>i</sub> ) P(B <sub>j</sub> /N <sub>i</sub> )										
				B <sub>1</sub>	B <sub>2</sub>	B <sub>3</sub>	B <sub>4</sub>	B <sub>5</sub>	B <sub>6</sub>	B <sub>7</sub>	B <sub>8</sub>	B <sub>9</sub>	B <sub>10</sub>	
N <sub>3</sub>	0.004	B <sub>1</sub>	0.10	0.0004										
		B <sub>2</sub>	0.10		0.0004									
		B <sub>3</sub>	0.13			0.0005								
		B <sub>4</sub>	0.08				0.0003							
		B <sub>5</sub>	0.08					0.0003						
		B <sub>6</sub>	0.10						0.0004					
		B <sub>7</sub>	0.09							0.0004				
		B <sub>8</sub>	0.10								0.0004			
		B <sub>9</sub>	0.02									0.00008		
		B <sub>10</sub>	0.20											0.0008
N <sub>4</sub>	0.0043	B <sub>1</sub>	0.08	0.0003										
		B <sub>2</sub>	0.05		0.0002									
		B <sub>3</sub>	0.10			0.0004								
		B <sub>4</sub>	0.08				0.0003							
		B <sub>5</sub>	0.08					0.0003						
		B <sub>6</sub>	0.11						0.0005					
		B <sub>7</sub>	0.10							0.0004				
		B <sub>8</sub>	0.12								0.0005			
		B <sub>9</sub>	0.03									0.0001		
		B <sub>10</sub>	0.25											0.0011
N <sub>5</sub>	0.0345	B <sub>1</sub>	0.17	0.0059										
		B <sub>2</sub>	0.06		0.0021									
		B <sub>3</sub>	0.12			0.0041								
		B <sub>4</sub>	0.03				0.0010							
		B <sub>5</sub>	0.06					0.0021						
		B <sub>6</sub>	0.09						0.0031					
		B <sub>7</sub>	0.11							0.0038				
		B <sub>8</sub>	0.11								0.0038			
		B <sub>9</sub>	0.03									0.0010		
		B <sub>10</sub>	0.22											0.0076

Table 15: Joint Probabilities Outcomes at Third (3rd) Iteration Continued

States of Nature (N <sub>i</sub> )	Prior Probability P(N <sub>i</sub> )	Outcomes (B <sub>j</sub> )	Conditional Probability P(B <sub>j</sub> /N <sub>i</sub> )	Joint Probability P(B <sub>j</sub> ∩N <sub>i</sub> ) = P(N <sub>i</sub> ) P(B <sub>j</sub> /N <sub>i</sub> )										
				B <sub>1</sub>	B <sub>2</sub>	B <sub>3</sub>	B <sub>4</sub>	B <sub>5</sub>	B <sub>6</sub>	B <sub>7</sub>	B <sub>8</sub>	B <sub>9</sub>	B <sub>10</sub>	
N <sub>6</sub>	0.0532	B <sub>1</sub>	0.20	0.0106										
		B <sub>2</sub>	0.06		0.0032									
		B <sub>3</sub>	0.11			0.0059								
		B <sub>4</sub>	0.03				0.0016							
		B <sub>5</sub>	0.02					0.0011						
		B <sub>6</sub>	0.09						0.0048					
		B <sub>7</sub>	0.11							0.0059				
		B <sub>8</sub>	0.12								0.0064			
		B <sub>9</sub>	0.03									0.0016		
		B <sub>10</sub>	0.23											0.0122
N <sub>7</sub>	0.0345	B <sub>1</sub>	0.17	0.0059										
		B <sub>2</sub>	0.04		0.0014									
		B <sub>3</sub>	0.10			0.0035								
		B <sub>4</sub>	0.04				0.0014							
		B <sub>5</sub>	0.03					0.0010						
		B <sub>6</sub>	0.10						0.0035					
		B <sub>7</sub>	0.11							0.0038				
		B <sub>8</sub>	0.12								0.0041			
		B <sub>9</sub>	0.03									0.0010		
		B <sub>10</sub>	0.26											0.0090
N <sub>8</sub>	0.0276	B <sub>1</sub>	0.18	0.0050										
		B <sub>2</sub>	0.04		0.0011									
		B <sub>3</sub>	0.13			0.0036								
		B <sub>4</sub>	0.04				0.0011							
		B <sub>5</sub>	0.04					0.0011						
		B <sub>6</sub>	0.12						0.0033					
		B <sub>7</sub>	0.09							0.0025				
		B <sub>8</sub>	0.11								0.0030			
		B <sub>9</sub>	0.03									0.0008		
		B <sub>10</sub>	0.22											0.0061

Table 15: Joint Probabilities Outcomes at Third (3rd) Iteration Continued

States of Nature (N <sub>i</sub> )	Prior Probability P(N <sub>i</sub> )	Outcomes (B <sub>j</sub> )	Conditional Probability P(B <sub>j</sub> /N <sub>i</sub> )	Joint Probability P(B <sub>j</sub> ∩N <sub>i</sub> ) = P(N <sub>i</sub> ) P(B <sub>j</sub> /N <sub>i</sub> )																	
				B <sub>1</sub>	B <sub>2</sub>	B <sub>3</sub>	B <sub>4</sub>	B <sub>5</sub>	B <sub>6</sub>	B <sub>7</sub>	B <sub>8</sub>	B <sub>9</sub>	B <sub>10</sub>								
N <sub>9</sub>	0.0281	B <sub>1</sub>	0.16	0.0045																	
		B <sub>2</sub>	0.08		0.0022																
		B <sub>3</sub>	0.09			0.0025															
		B <sub>4</sub>	0.07				0.002														
		B <sub>5</sub>	0.07					0.002													
		B <sub>6</sub>	0.10						0.0028												
		B <sub>7</sub>	0.09							0.0025											
		B <sub>8</sub>	0.10								0.0028										
		B <sub>9</sub>	0.03										0.0008								
		B <sub>10</sub>	0.21																		0.0059
N <sub>10</sub>	0.7881	B <sub>1</sub>	0.38	0.2995																	
		B <sub>2</sub>	0.02		0.0158																
		B <sub>3</sub>	0.06			0.0473															
		B <sub>4</sub>	0.01				0.0079														
		B <sub>5</sub>	0.02					0.0158													
		B <sub>6</sub>	0.09						0.0709												
		B <sub>7</sub>	0.10								0.0788										
		B <sub>8</sub>	0.10									0.0788									
		B <sub>9</sub>	0.03										0.0236								
		B <sub>10</sub>	0.19																		0.1497
<b>Marginal Probability =</b>				<b>0.3363</b>	<b>0.0288</b>	<b>0.0709</b>	<b>0.0188</b>	<b>0.0267</b>	<b>0.0912</b>	<b>0.1000</b>	<b>0.1019</b>	<b>0.0291</b>	<b>0.1961</b>								

**Discussion of Results in Table 15:**

(i). The joint probabilities outcomes were calculated by multiplying prior probability of each states of nature by the conditional probability outcomes and adding of the result of each of them to obtain the marginal probability values as shown on Table 15.

(ii) The marginal probabilities values are: 0.3363 for economic efficiency; 0.0288 for federal economic redistribution; 0.0709 for regional economic redistribution; 0.0188 for stateeconomic redistribution; 0.0267 for local economic redistribution; 0.0912 for social well-being; 0.1000 for youth empowerment; 0.1019 for environmental quality improvement; 0.0291 for gender equality and 0.1961 for security.

(iii) Comparing this third iteration with the results obtained from second iteration, B<sub>1</sub> (economic efficiency) increased from 0.2987 to 0.3363; B<sub>5</sub> (local economic redistribution) increased from

0.026 to 0.0267; B<sub>7</sub> (youth empowerment) increased from 0.0955 to 0.1000.

**4.4.3 Determination of Posterior Probability Outcomes on Third Iteration**

The Posterior Probability  $P(N_i/B_i) = P(N_i \cap B_i) / P(B_i)$  where P(B<sub>i</sub>) is the values of the marginal probabilities which is the total sum of each values of the joint probabilities outcomes  $P(N_i \cap B_i)$ .

The Posterior Probability Outcomes at second iteration on Table 12 is computed by dividing each states of nature (N<sub>i</sub>) for i = 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 by each values of marginal probability outcomes P(B<sub>i</sub>) for i = 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 under each group values.

The Posterior Probability is obtained by dividing each Joint Probability Outcomes with the total of each marginal Probability Outcomes as shown in Table 16.

Table 16: Posterior Probability Outcomes at Third Iteration

Outcomes B <sub>i</sub>	Probability P(B <sub>ii</sub> )	States of Nature (N <sub>i</sub> )	Joint Probability P(B <sub>i</sub> ∩N <sub>i</sub> ) = P(N <sub>i</sub> ) P(B <sub>i</sub> /N <sub>i</sub> )	Posterior Probability P(N <sub>i</sub> /B <sub>i</sub> ) = P(N <sub>i</sub> ∩B <sub>i</sub> ) / P(B <sub>i</sub> )
B <sub>1</sub>	0.3363	N <sub>1</sub>	0.0001	0.0001/0.3363 = 0.0003
		N <sub>2</sub>	0.0041	0.0041/0.3363 = 0.0122

		N <sub>3</sub>	0.0004	0.0004/0.3363 = 0.0012
		N <sub>4</sub>	0.0003	0.0003/ 0.3363 = 0.0009
		N <sub>5</sub>	0.0059	0.0059/0.3363= 0.0175
		N <sub>6</sub>	0.0106	0.0106/0.3363 = 0.0315
		N <sub>7</sub>	0.0059	0.0059/0.3363 = 0.0175
		N <sub>8</sub>	0.0050	0.0050/0.3363 = 0.0149
		N <sub>9</sub>	0.0043	0.0043/0.3363 = 0.0134
		N <sub>10</sub>	0.2995	0.2995/0.3363 = 0.8906
B <sub>2</sub>	0.0288	N <sub>1</sub>	0.0002	0.0002/0.0288 = 0.0069
		N <sub>2</sub>	0.0022	0.0022/0.0288 = 0.0764
		N <sub>3</sub>	0.0004	0.0004/0.0288 = 0.0139
		N <sub>4</sub>	0.0002	0.0002/0.0288 = 0.0069
		N <sub>5</sub>	0.0021	0.0021/0.0288 = 0.0729
		N <sub>6</sub>	0.0032	0.0032/0.0288 = 0.1111
		N <sub>7</sub>	0.0014	0.0014/0.0288 = 0.0486
		N <sub>8</sub>	0.0011	0.0011/0.0288 = 0.0417
		N <sub>9</sub>	0.0022	0.0022/0.0288 = 0.0764
				N <sub>10</sub>

Table 16 Posterior Probability Outcomes at Third Iteration Continued

Outcomes B <sub>i</sub>	Probability P(B <sub>ii</sub> )	States of Nature (N <sub>i</sub> )	Joint Probability P(B <sub>i</sub> ∩N <sub>i</sub> ) = P(N <sub>i</sub> ) P(B <sub>i</sub> /N <sub>i</sub> )	Posterior Probability P(N <sub>i</sub> /B <sub>i</sub> ) = P(N <sub>i</sub> ∩B <sub>i</sub> ) / P(B <sub>i</sub> )
B <sub>3</sub>	0.0709	N <sub>1</sub>	0.0002	0.0002/0.0709 = 0.0028
		N <sub>2</sub>	0.0029	0.0029/0.0709 = 0.0409
		N <sub>3</sub>	0.0005	0.0005/0.0709 = 0.0071
		N <sub>4</sub>	0.0004	0.0004/0.0709 = 0.0056
		N <sub>5</sub>	0.0041	0.0041/0.0709 = 0.0578
		N <sub>6</sub>	0.0059	0.0059/0.0709 = 0.0832
		N <sub>7</sub>	0.0035	0.0035/0.0709 = 0.0494
		N <sub>8</sub>	0.0036	0.0036/0.0709 = 0.0508
		N <sub>9</sub>	0.0025	0.0025/0.0709 = 0.0353
				N <sub>10</sub>
B <sub>4</sub>	0.0188	N <sub>1</sub>	0.0001	0.0001/0.0188= 0.0053
		N <sub>2</sub>	0.0029	0.0029/0.0188= 0.1543
		N <sub>3</sub>	0.0003	0.0003/0.0188= 0.0160
		N <sub>4</sub>	0.0003	0.0003/0.0188= 0.0160
		N <sub>5</sub>	0.0010	0.0010/0.0188= 0.0532
		N <sub>6</sub>	0.0016	0.0016/0.0188= 0.0811
		N <sub>7</sub>	0.0014	0.0014/0.0188= 0.0213
		N <sub>8</sub>	0.0011	0.0011/0.0188= 0.0585
		N <sub>9</sub>	0.0020	0.0020/0.0188= 0.1064
				N <sub>10</sub>
B <sub>5</sub>	0.0267	N <sub>1</sub>	0.0001	0.0001/0.0267= 0.0037
		N <sub>2</sub>	0.0029	0.0029/0.0267= 0.1086
		N <sub>3</sub>	0.0003	0.0003/0.0267= 0.0112
		N <sub>4</sub>	0.0003	0.0003/0.0267= 0.0112
		N <sub>5</sub>	0.0021	0.0021/0.0267= 0.0787
		N <sub>6</sub>	0.0011	0.0011/0.0267= 0.0412
		N <sub>7</sub>	0.0010	0.0010/0.0267= 0.0375
		N <sub>8</sub>	0.0011	0.0011/0.0267= 0.0412
				N <sub>9</sub>



B <sub>6</sub>	0.0912	N <sub>10</sub>	0.0158	0.0158/0.0267 = 0.5918
		N <sub>1</sub>	0.0002	0.0002/0.0912 = 0.0022
		N <sub>2</sub>	0.0017	0.0017/0.0912 = 0.0186
		N <sub>3</sub>	0.0004	0.0004/0.0912 = 0.0044
		N <sub>4</sub>	0.0005	0.0005/0.0912 = 0.0055
		N <sub>5</sub>	0.0031	0.0031/0.0912 = 0.0340
		N <sub>6</sub>	0.0048	0.0048/0.0912 = 0.0526
		N <sub>7</sub>	0.0035	0.0035/0.0912 = 0.0384
		N <sub>8</sub>	0.0033	0.0033/0.0912 = 0.0362
		N <sub>9</sub>	0.0028	0.0028/0.0912 = 0.0307
B <sub>7</sub>	0.1	N <sub>10</sub>	0.0709	0.0709/0.0912 = 0.7774
		N <sub>1</sub>	0.0002	0.0002/0.10 = 0.0020
		N <sub>2</sub>	0.0017	0.0017/0.10 = 0.017
		N <sub>3</sub>	0.0004	0.0004/0.10 = 0.004
		N <sub>4</sub>	0.0004	0.0004/0.10 = 0.004
		N <sub>5</sub>	0.0038	0.0038/0.10 = 0.038
		N <sub>6</sub>	0.0059	0.0059/0.10 = 0.059
		N <sub>7</sub>	0.0038	0.0038/0.10 = 0.038
		N <sub>8</sub>	0.0025	0.0025/0.10 = 0.025
		N <sub>9</sub>	0.0025	0.0025/0.10 = 0.025
		N <sub>10</sub>	0.0788	0.0788/0.10 = 0.788

Table 16: Posterior Probability Outcomes at Third Iteration Continued

Outcomes B <sub>i</sub>	Probability P(B <sub>ii</sub> )	States of Nature N <sub>i</sub>	Joint Probability P(B <sub>i</sub> ∩N <sub>i</sub> )=P(N <sub>i</sub> ) P(B <sub>i</sub> /N <sub>i</sub> )	Posterior Probability P(N <sub>i</sub> /B <sub>i</sub> ) = P(N <sub>i</sub> ∩B <sub>i</sub> ) / P(B <sub>i</sub> )
B <sub>8</sub>	0.1019	N <sub>1</sub>	0.0002	0.0002/0.1019 = 0.0020
		N <sub>2</sub>	0.0017	0.0017/0.1019 = 0.0167
		N <sub>3</sub>	0.0004	0.0004/0.1019 = 0.0039
		N <sub>4</sub>	0.0005	0.0005/0.1019 = 0.0049
		N <sub>5</sub>	0.0038	0.0038/0.1019 = 0.0373
		N <sub>6</sub>	0.0064	0.0064/0.1019 = 0.0628
		N <sub>7</sub>	0.0041	0.0041/0.1019 = 0.0402
		N <sub>8</sub>	0.0030	0.0030/0.1019 = 0.0294
		N <sub>9</sub>	0.0028	0.0028/0.1019 = 0.0275
		N <sub>10</sub>	0.0788	0.0788/0.1019 = 0.7733
B <sub>9</sub>	0.0291	N <sub>1</sub>	0.00003	0.00003/0.0291 = 0.0103
		N <sub>2</sub>	0.0005	0.0005/0.0291 = 0.0172
		N <sub>3</sub>	0.00008	0.00008/0.0291 = 0.0027
		N <sub>4</sub>	0.0001	0.0001/0.0291 = 0.0034
		N <sub>5</sub>	0.0010	0.0010/0.0291 = 0.0344
		N <sub>6</sub>	0.0016	0.0016/0.0291 = 0.0550
		N <sub>7</sub>	0.0010	0.0010/0.0291 = 0.0344
		N <sub>8</sub>	0.0008	0.0008/0.0291 = 0.0275
		N <sub>9</sub>	0.0008	0.0008/0.0291 = 0.0275
		N <sub>10</sub>	0.0236	0.0236/0.0291 = 0.8110
B <sub>10</sub>	0.1961	N <sub>1</sub>	0.0003	0.0003/0.1961 = 0.0015
		N <sub>2</sub>	0.0034	0.0034/0.1961 = 0.0173
		N <sub>3</sub>	0.0008	0.0008 / 0.1961 = 0.0041
		N <sub>4</sub>	0.0011	0.0011/0.1961 = 0.0056
		N <sub>5</sub>	0.0076	0.0076/0.1961 = 0.0388
		N <sub>6</sub>	0.0122	0.0122/0.1961 = 0.0622

	N <sub>7</sub>	0.0090	0.0090/0.1961 = 0.0459
	N <sub>8</sub>	0.0061	0.0061/0.1961 = 0.0311
	N <sub>9</sub>	0.0059	0.0059/0.1961 = 0.0301
	N <sub>10</sub>	0.1497	0.1497/0.1961 = 0.7634

**Discussion of Result in Table 16:**

(i). The Posterior Probability was computed by dividing each states of nature total joint probabilities (referred to as marginal probabilities) by probability values of each outcomes for each of the objectives (B<sub>i</sub>) for B<sub>1</sub>, B<sub>2</sub>, B<sub>3</sub>, B<sub>4</sub>, B<sub>5</sub>, B<sub>6</sub>, B<sub>7</sub>, B<sub>8</sub>, B<sub>9</sub> and B<sub>10</sub> as stated before for all N<sub>1</sub>, N<sub>2</sub>, N<sub>3</sub>, N<sub>4</sub>, N<sub>5</sub>, N<sub>6</sub>, N<sub>7</sub>, N<sub>8</sub>, N<sub>9</sub> and N<sub>10</sub> for each set of B<sub>i</sub>.

(ii) For example, the objective B<sub>1</sub> (Economic efficiency) and the values of posterior probabilities under it were: N<sub>1</sub> (states of nature) = 0.0003, N<sub>2</sub> = 0.0122, N<sub>3</sub> = 0.0012, N<sub>4</sub> = 0.0009, N<sub>5</sub> = 0.0175, N<sub>6</sub> = 0.0315, N<sub>7</sub> = 0.0175, N<sub>8</sub> = 0.0149, N<sub>9</sub> = 0.0134 and N<sub>10</sub> = 0.8906. It follows the same pattern for B<sub>2</sub> (Federal economic redistribution), B<sub>3</sub> (Regional economic redistribution), B<sub>4</sub> (Stateeconomic redistribution), B<sub>5</sub> (Local economic redistribution), B<sub>6</sub> (Social well-being), B<sub>7</sub> (Youth empowerment), B<sub>8</sub> (Enviromental quality empromvement), B<sub>9</sub> (Gender equality) and B<sub>10</sub> (Security). These are shown on Table 16.

(iii) Comparing the results from the second iterations, there are reductions in the posterior probability outcomes while on the purpose of reservoir/gullies, the value increased from 0.7881 to 0.8906.

**4.4.4 Determination of Forecast Outcomes for Objectives/Benefits at Third (3rd) Iteration (Posterior Expected Opportunity Loss)**

This is determined by calculating the forecast outcomes for the objectives/benefits which is the sum of the multiplication of each respective value of the posterior probability results with the conditional opportunity loss of each of the states of nature to get the Expected Opportunity Loss (EOL). The sum totals of each set of values are referred to as the Posterior Expected Opportunity Loss for each of the benefits. The Conditional Opportunity Loss is obtained for each states of nature by subtracting each net benefit (B<sub>i</sub>) from the highest benefits of each group. For example, B<sub>1</sub> (economic efficiency); the COL for N<sub>1</sub> = 8.73 – 3.65 = 5.08; the COL for N<sub>2</sub> = 8.73 – 4.84 = 3.89; the COL for N<sub>3</sub> = 8.73 – 6.36 = 2.37; the COL for N<sub>4</sub> = 8.73 – 3.6 = 5.13; the COL for N<sub>5</sub> = 8.73 – 3.44 = 5.29; the COL for N<sub>6</sub> = 8.73 – 4.37 = 4.36 etc. the COL for N<sub>7</sub> = 8.73 – 4.05 = 4.68; the COL for N<sub>8</sub> = 8.73 – 4.22 = 4.51; the COL for N<sub>9</sub> = 8.73 – 1.12 = 7.61; etc. The Forecast Outcomes for the Benefits (Posterior Expected Opportunity Loss) are obtained as the sum of the multiple of each Posterior Probabilities with the Conditional Opportunity Loss (COL) as shown in Table 17

Table 17: Forecast Outcomes for Objectives/Benefits at Third (3rd) Iteration (Posterior Expected Opportunity Loss)

States of Nature	B <sub>1</sub> (Economic Efficiency)			B <sub>2</sub> (Federal Economic Redistribution)			B <sub>3</sub> (Regional Economic Redistribution)			B <sub>4</sub> (State Economic Redistribution)			B <sub>5</sub> (Local Economic Redistribution)		
	Prob.	COL	EOL	Prob.	COL	EOL	Prob.	COL	EOL	Prob.	COL	EOL	Prob.	COL	EOL
N <sub>1</sub>	0.0003	5.08	0.0015	0.0069	0	0	0.0028	4.59	0.0129	0.0033	17.47	0.0926	0.0037	4.75	0.0176
N <sub>2</sub>	0.0122	3.89	0.0475	0.0764	5.83	0.4454	0.0409	4.79	0.1959	0.1543	19.94	3.0767	0.1086	15.95	1.7322
N <sub>3</sub>	0.0012	2.37	0.0028	0.0139	3.78	0.0525	0.0071	3.09	0.0219	0.0160	15.31	0.2450	0.0112	9.70	0.1086
N <sub>4</sub>	0.0009	5.13	0.0046	0.0069	3.70	0.0255	0.0056	5.35	0.0300	0.0160	17.38	0.2813	0.0112	18.28	0.2047
N <sub>5</sub>	0.0175	5.29	0.0926	0.0729	4.09	0.2982	0.0578	5.61	0.3243	0.0532	17.53	1.9326	0.0787	15.88	1.2498
N <sub>6</sub>	0.0315	4.36	0.1373	0.1111	7.92	0.8799	0.0832	4.57	0.3802	0.0811	14.38	1.1662	0.0412	13.00	0.5356
N <sub>7</sub>	0.0175	4.68	0.0819	0.0486	7.33	0.3562	0.0494	4.91	0.2426	0.0213	14.81	0.3155	0.0375	10.45	0.3919
N <sub>8</sub>	0.0149	4.51	0.0672	0.0417	6.99	0.2915	0.0508	4.76	0.2418	0.0585	13.57	0.7938	0.0412	11.13	0.4586
N <sub>9</sub>	0.0134	7.61	0.1020	0.0764	12.01	0.9176	0.0353	8.0	0.2824	0.1064	22.44	2.3876	0.0749	18.96	1.4201
N <sub>10</sub>	0.89475	0	0	0.5486	2.43	1.3331	0.6671	0	0	0.4202	0	0	0.5918	0	0
<b>Posterior EOL</b>			<b>0.5374</b>	<b>4.5999</b>			<b>1.7328</b>			<b>9.2913</b>			<b>6.1191</b>		
States of Nature	B <sub>6</sub> (Social Well-being)			B <sub>7</sub> (Youth Empowerment)			B <sub>8</sub> (Environmental Quality Improvement)			B <sub>9</sub> (Gender Equality)			B <sub>10</sub> (Security)		
	Prob.	COL	EOL	Prob.	COL	EOL	Prob.	COL	EOL	Prob.	COL	EOL	Prob.	COL	EOL
N <sub>1</sub>	0.0022	2.69	0.0059	0.002	9.01	0.0180	0.0020	2.87	0.0057	0.0103	4.07	0.0419	0.0015	0	0
N <sub>2</sub>	0.0186	16.54	0.3076	0.017	22.00	0.374	0.0167	13.77	0.2300	0.0172	11.25	0.1935	0.0173	77.26	1.3383
N <sub>3</sub>	0.0044	11.92	0.0524	0.004	15.58	0.0623	0.0039	6.51	0.0254	0.0027	10.00	0.027	0.0041	80.76	0.3311
N <sub>4</sub>	0.0055	18.73	0.1030	0.004	22.32	0.0901	0.0049	13.63	0.0668	0.0034	11.68	0.0397	0.0056	79.56	0.4455
N <sub>5</sub>	0.0340	20.57	0.6994	0.038	22.61	0.8592	0.0373	13.52	0.5043	0.0344	11.49	0.3953	0.0388	79.44	3.0823
N <sub>6</sub>	0.0526	13.44	0.7069	0.059	15.37	0.9068	0.0628	7.22	0.4534	0.0550	9.12	0.5016	0.0622	62.93	3.9142
N <sub>7</sub>	0.0384	11.80	0.4531	0.038	14.61	0.5552	0.0402	9.65	0.3879	0.0344	10.42	0.3584	0.0459	62.38	2.8632
N <sub>8</sub>	0.0362	10.77	0.3901	0.025	13.69	0.3423	0.0294	8.06	0.2370	0.0275	9.68	0.2662	0.0311	62.21	1.9347
N <sub>9</sub>	0.0307	19.22	0.5901	0.025	22.61	0.5653	0.0275	14.57	0.4007	0.0275	15.82	0.4351	0.0301	77.15	2.3222
N <sub>10</sub>	0.7774	0	0	0.788	0	0	0.7733	0	0	0.8110	0	0	0.7634	41.69	31.8261
<b>Posterior EOL</b>			<b>3.3085</b>	<b>3.7732</b>			<b>2.3112</b>			<b>2.6187</b>			<b>48.0576</b>		

**Discussion of Results on Table 17:**

(i) The total Posterior Expected Opportunity Loss (EOL) for the objectives are; ₦0.5374 billion for economic efficiency; ₦4.5999 billion for federal economic redistribution; ₦1.7328 billion for regional economic redistribution; ₦9.2913 billion for state economic redistribution; ₦6.1191 billion for local economic redistribution; ₦3.3085 billion for social well-being; ₦3.7732 billion for youth empowerment; ₦2.3112 billion for environmental

quality improvement; ₦2.6187 billion for gender equality; and ₦48.0576 billion for security.

**4.4.5 Determination of Expected Value of Sample Information (EVSI) Outcomes at Third (3rd) Iteration.**

The Expected Value of Sample Information (EVSI) is calculated by multiplying Posterior Expected Opportunity Loss (EOLs) values with the marginal probabilities as shown on Table 18.

Table 18: Expected Value of Sample Information (EVSI) Outcomes at Third (3rd) Iteration

Outcomes B <sub>i</sub>	Marginal probability P( B <sub>i</sub> )	Expected Opportunity Loss (EOL)	Expected Value of Sample Information
B <sub>1</sub>	0.3363	0.5374	0.1807
B <sub>2</sub>	0.0288	4.5999	0.1325
B <sub>3</sub>	0.0709	1.7320	0.1228
B <sub>4</sub>	0.0188	9.2913	0.1747
B <sub>5</sub>	0.0267	6.1191	0.1634
B <sub>6</sub>	0.0912	3.3085	0.3017
B <sub>7</sub>	0.100	3.7732	0.3773
B <sub>8</sub>	0.1019	2.3112	0.2355
B <sub>9</sub>	0.0291	2.6187	0.0762
B <sub>10</sub>	0.1961	48.0576	9.4241
<b>TOTAL ( EVSI) =</b>			<b>₦11.1889 billion</b>

**Discussion of Results in Table 18:**

(i) The highest Expected opportunity loss of ₦48.0576 billion multiply by the marginal probability of 0.1961 results to ₦9.4241 billion of Expected Value of Sample Information under Reservoir /Gullies while the least is on Plantation/Forestry with the EVSI of ₦0.0762 billion.

(ii) The Expected Value of Sample Information (EVSI) is ₦11.1889 billion which indicates the money which can be paid for hiring the services of consultants for the River Basin operation yield for all the ten (10) purposes of Irrigation, Hydroelectric Power Generation, Water Supply, Navigation, Drainage/Dredging, Flood Control, Recreation/Tourism, Erosion Control, Plantation / Forestry, Reservoir/Gullies etc. respectively.

**V. CONCLUSION AND RECOMMENDATIONS**

The optimal utilization of river basin resources entails employment of all the purposes of Irrigation Agriculture, Hydro-electric power generation, Water supply, Navigation or Water transport, Drainage/Dredging, Flood control, Recreation/Tourism, Erosion control, Plantation/ Forestry and Reservoir /Gullies for the optimum benefits based on the objectives of Economic Efficiency, Federal Economic Redistribution,

Regional Economic Redistribution, State Economic Redistribution, Local Economic Redistribution, Social Well-being, Youth Empowerment, Environmental quality improvement, Gender Equality and Security.

- a) The Bayesian Decision Model analysis reveal that with a total of ₦12.50 billion released to Anambra-Imo River Basin for capital projects development from 2015 to 2020 for the multi-purpose/multi-objective projects will yield maximum Expected Monetary Value of ₦68.72 billion. This implies that with investment of ₦12.50 billion the river basin is expected to generate profit of ₦56.22 billion within the period. This is expected when there is perfect information or with data and the money appropriated for the purpose and objectives respectively.
- b) The expected profit with perfect information also increased from ₦45.56 billion without data at first iteration to ₦69.63 billion on third iteration.
- c) The Expected Value of Perfect Information reduced from ₦2.97 billion on first iteration to ₦0.9134 billion on third iteration while the expected value of Sample Information reduced from ₦28.69 billion on first iteration without data to ₦11.19 billion on third iteration. The expected value of Sample Information (EVSI)

- of ₦11.19 billion is the maximum amount the river basin will pay for additional information for full utilization of the purpose and optimization of all the benefits.
- d) The Posterior Expected Opportunity Loss reduced drastically for the objectives. For example the Expected Opportunity Loss for benefit of security decreased from ₦114.91 billion on first iteration to ₦48.06 billion on third iteration.
- e) The River basin managers should use the Bayesian analysis to estimate expected monetary benefits for proper apportioning of available funds to various purposes and objectives in-order to realize optimal benefits from their investment in the light of the global climate change scenario and projections.
- f) There should be measures to encourage the use of green and clean energy while implementing the purpose/objectives in a multi-purpose/multi-objective Anambra-Imo River basin to reduce the impact of soil erosion, flood disaster, failure of reservoirs and dams, improve hydro-electric power generation, improve water supply, and check insecurity etc. that ravage the living environment.
- g) The implementation of these recommendations will be a fertile ground for the management of the river basin to generate revenue and financial benefits to the government, the community and social well-being of the inhabitants in the area.
- REFERENCES**
- [1]. Aaron, D. (2016). A Complete Guide To The Bayes Factor Test.
- [2]. Achenyo, I. (2015). The challenges for women entrepreneurs in Africa. The Pride of Africa's women Entrepreneur, Pp 1.
- [3]. Ahmadreza, S., Dastgerdi, M., Sargolini, S., Broussard, A., Allison, C. and Giuseppe, D. L. (2020). Climate Change and Sustaining Heritage Resources: A Framework for Boosting Cultural and Natural Heritage Conservation in Central Italy, **8**: 2-13.
- [4]. Babatolu, J.S. and Akinnubi, R.T. (2014). Influence of Climate Change in Niger River Basin J Development Authority Area on Niger Runoff, Nigeria. Journal of Earth Science Ohm Change, **5**: 230.
- [5]. Barrow, C. J. (1998). Dei^loggieng Planjjjng and! Management. WorldDevelopment,**26**(1): 1/11-186
- [6]. Barrow, C.J. (1985). River Basin Planning in Nigeria, Applied Geography, **5**(4):297-308.
- [7]. Bogardi, J.J. and Sutanto, A.A. (1992). Interactive Multiobjective Modeling of Discrete Decisions in Water Resources Planning, Water Resources Management: Modern Decision Techniques, edited by M. Benedi, K. Andah, R. Harboe, IAHR Proceedings, A. A. Balkema, Rotterdam, the Netherlands, Pp. 59-72.
- [8]. Bukar, A. C., Isah, F. A., Hassan, M. and Tijjani, C. (2010). A Conceptual Framework for Organization of River Basin Development and Management in Nigeria. European Journal of Engineering Research and Science. **4**(6)
- [9]. Calder, I.R. (1999). The Blue Resolution: Land Use and Integrated Water Resources Management, London UK: Earthscan.
- [10]. European Environmental Agency (EEA) (2007). Climate change and water adaptation issues. EEA Technical report No 2.
- [11]. Eme, L. C and Ohaji, E. (2019). Simulation Modeling in Prior-Posterior Decision Theory- A Case Study of Farmers' Decision Problem, Journal of Civil and Environmental Research, ISSN 2224- 5790 (Paper). Vol. 11 . No.2. [www.iiste.org](http://www.iiste.org)
- [12]. Eme, L. C. and Ohaji, E. (2019). Bayesian Decision Modeling in Watershed Management - Cross River Basin, Nigeria. Civil and Environmental Research. ISSN 2224- 5790 (Paper), ISSN; 2225-0514 (Online) Doi: 10;7176/CLR Vol. 11, No.2,
- [13]. Eme, L.C. (2010). Water Resources Engineering Development Scheme: Optimal Strategy for Multipurpose/Multi-objective Water Resources Engineering Development Scheme. African Journal of Engineering Research and Development, **3**(3): 27 – 33.
- [14]. Eme, L.C. (2015). Simulation Optimization for Model and Prototype using Non Parametric Method: A Case Study of Anambra/Imo River Basin Engineering Development Scheme. American Academic & Scholarly Research Journal, **7**(4): 63 – 61.
- [15]. Ezenweani, R. S. (2017). A Comprehensive Collation of River Basins in Nigeria: Benefits and River Basin

- Development Planning and Management. International Journal of Scientific & Engineering Research Volume A Issue 1.
- [16]. Federal Ministry of Water Resources (2003a). International Review of Water Resources Management and Capacity Building for Reform. International Analysis and Reforms Report-Final Report, Federal Ministry of Water Resources, Abuja.
- [17]. Harboe, R. (1992). Multicriteria Methods for Decision Making in Water Resources Systems, Water Resources Management: Modern Decision Techniques, edited by M. Benedini, K. Andah, R. Harboe, IAHR Proceedings, A. A. Balkema, Rotterdam, the Netherlands, Pp. 1-10.
- [18]. Molle, F., Philippus, W. and Phil, H. (2007). River Basin Development and Management. *Development and Change*, **32** (4): 40 – 52.
- [19]. Newson, M. (1997). Land, Water, and Development: Sustainable Management of River Basin Systems”. *Scottish Geographical Magazine*, **104**:56-66.
- [20]. United Nations, (1976). River Basin Development: Policies and Planning, Vol. I. Proc. of a UN Interregional Seminar on River Basin and Interbasin. UNDP/National Water Authority of Hungary, Washington, DC. Pp. 16-26.