

# Dictionary learning and VMD based seismic denoising algorithm

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**ABSTRACT:** Seismic data usually contain a large amount of noise, in order to effectively remove noise and improve the signal-to-noise ratio of seismic signals, a compressive-aware denoising seismic algorithm with joint dictionary learning and VMD is proposed. Firstly, the VMD algorithm is used to decompose the seismic signal into several intrinsic modal components (IMF) with different center frequencies, and then the noise-dominated modal components are discarded and the high-noise components are removed directly. Then, the signal dominant modal components are discarded and the high noise components are removed directly, and the signal is finally combined with ODL to learn the dominant modal components and the transitional modal components, and then the signal is sparsely denoised. The experimental results show that VMD has good adaptiveness for non-stationary signal decomposition and can be well applied to seismic signals. The combination of the two methods can achieve better denoising effect. The signal-to-noise ratio of the simulated signals and the actual seismic data processed by the proposed method is significantly better than that of the traditional single denoising method.

**KEYWORDS:** Variational Modal Decomposition, Seismic Signal Denoising, Signal to Noise Ratio, Dictionary Learning, ODL

## I. INTRODUCTION

As oil and gas exploration proceeds, the acquisition environment becomes more and more complex, and a large amount of noise and missing seismic traces will occur during the acquisition process, which affects the subsequent seismic signal analysis, so the missing information needs to be reconstructed. The compressive sensing theory shows that when the signal is sparse, the complete signal can be reconstructed from samples that are

far below the sampling number of the Nyquist sampling theorem by a nonlinear reconstruction algorithm. The variational modal decomposition proposed by Konstantin et al [3] is an adaptive, nonrecursive decomposition method that decomposes the signal into a finite number of modal components and determines the center frequency and bandwidth of each modal component to achieve effective separation of each component of the signal.... The dictionary learning based seismic data denoising method is one of the research contents of this paper. The dictionary learning-based seismic data denoising method is better than the mathematical transform-based seismic data denoising method, but the classical K-SVD algorithm is less efficient in denoising computation. In this paper, the online dictionary learning algorithm is used to overcome the large-scale data problem, and the VMD algorithm is combined to improve the seismic data denoising algorithm.

Therefore, the two methods are combined to propose a new seismic signal denoising method. Firstly, the seismic signal is firstly decomposed by VMD algorithm combined with autocorrelation analysis and peak judgment to get the dominant mode, noise dominant mode and transition mode, then the noise dominant mode is discarded and the high noise component is directly eliminated, finally the dominant mode and transition mode are learned by combining with ODL dictionary learning algorithm, finally the sparse denoising is performed respectively, and finally the combined signal is reconstructed. Finally, the seismic signal denoising is completed.

## II. DICTIONARY LEARNING ALGORITHMS AND VMD THEORY

### Dictionary learning algorithm

The seismic data containing noise can be expressed as

$$X = W + n$$

where  $X$  is the measured noisy seismic data,  $W$  is the noiseless seismic data to be estimated and  $n$  is the noise.  $W$  is reconstructed based on the combination of the dictionary  $D$  and the sparse coefficient  $\alpha$ . Therefore, the noise-bearing seismic data are abstracted as

$$x = D\alpha + \epsilon$$

According to sparse representation theory, the sparser the seismic data, the better the denoising effect, and the denoising problem can be transformed into a minimization problem. And it has been shown that the convex parametrization performs better than the  $l_0$  parametrization in online dictionary learning. Lee et al. relax the  $l_0$  parametrization to the  $l_1$  parametrization, and the dictionary learning problem is generalized as

$$\min_{D, \alpha} \frac{1}{2} \|X - D\alpha\|_2^2 + \lambda \|\alpha\|_1$$

where  $\lambda$  is the regularized equilibrium parameter.

The objective function of the ODL algorithm is

$$\min_{D \in \Phi, \alpha \in R^n} \frac{1}{n} \sum_{i=1}^n \left( \frac{1}{2} \|x_i - D\alpha_i\|_2^2 + \lambda \|\alpha_i\|_1 \right)$$

The dictionary is obtained by training the seismic data  $X$  containing noise. Preprocessing is performed first, and learning full-size data dictionary atoms is not computationally easy to handle because it requires a large number of training samples and atoms of each dimension are learned. The trained dictionary will also be too large to be used in practice because the overcomplete dictionary contains more atoms than even its dimensions. Patch-based methods can overcome this limitation by dividing the data into small pieces and denoising  $X$  into  $n$  data samples separately. Figure 1 shows the online dictionary learning algorithm. Assume that the samples in the training set are independently and identically distributed and obey the  $p(x)$  distribution.

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**Algorithm 1** The ODL Algorithm

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**Input:**  $x \in R^m \sim p(x)$  (random variable),  $\lambda \in R$  (regularization parameter),  $D_0 \in R^{m \times k}$  (initial dictionary),  $T$  (number of iterations)

**Output:**  $D_T$  (learned dictionary)

- 1:  $A_0 \leftarrow 0, B_0 \leftarrow 0$  (Initialization)
  - 2: **for**  $t = 1$  to  $T$  **do**
  - 3: Draw  $x_t$  from  $p(x)$
  - 4: Sparse coding; compute using LARS  

$$\alpha_t \triangleq \arg \min_{\alpha \in R^k} \frac{1}{2} \|x_t - D_{t-1} \alpha\|_2^2 + \lambda \|\alpha\|_1$$
  - 5:  $A_t \leftarrow A_{t-1} + \frac{1}{2} \alpha_t \alpha_t^T, B_t \leftarrow B_{t-1} + x_t \alpha_t^T$
  - 6: Compute  $D_t$ , using Algorithm 2, so that  

$$D_t \triangleq \arg \min_{D \in C} \frac{1}{t} \sum_{i=1}^t \frac{1}{2} \|x_i - D\alpha_i\|_2^2 + \lambda \|\alpha_i\|_1$$

$$= \arg \min_{D \in C} \frac{1}{t} \left( \text{Tr}(D^T D A_t) - \text{Tr}(D^T B_t) \right)$$
  - 7: **end for**
  - 8: **return**  $D_T$
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**VMD theory**

The VMD (Variational Mode Decomposition) algorithm [6] is a process of decomposing a signal into several finite-bandwidth sub-signals. It is an adaptive, non-recursive, quasi-orthogonal decomposition method, in which each sub-signal, ie, the IMF component, surrounds its respective center frequency. The essence of VMD is to construct a variational problem and find the optimal solution. It decomposes the input signal  $f(t)$  into  $k$  modal components  $u_k(t)$  such that the sum of the estimated bandwidths of the  $k$  modes is minimized. The constraint is that the sum of the modal components equals the input signal.

where  $u_k(t)$  is the AM and FM signal, and the expression is

$$u_k(t) = A_k(t) \cos(\phi_k(t))$$

The augmented Lagrangian expression is

$$L(\{u_k\}, \{\lambda\}, \lambda) = \alpha \sum_k \|\partial_t \left[ (\delta(t) + \int \pi u_k(t) e^{-jwkt} dt \right. \right. \\ \left. \left. + \|f(t) - \sum_k u_k(t)\|^2 + \langle \lambda(t), f(t) - \sum_k u_k(t) \rangle \right\|_2$$

**III. DICTIONARY LEARNING AND VMD BASED SEISMIC DENOISING ALGORITHM**

**Correlation coefficient**

Through VMD decomposition, the seismic signal can be transformed into a finite number of modal components. The modal component with small order corresponds to the low-frequency component of the signal, which can be considered as an effective component. The modal component with a large order corresponds to the high-frequency component of the signal, and is greatly affected by noise. The Pearson correlation coefficient method is used to solve the linear relationship between each component and the original data in turn. The calculation formula is:

$$\text{Corr}(x, y) = \frac{\text{Cov}(x, y)}{\sqrt{\text{Var}(x)}\sqrt{\text{Var}(y)}} \quad (13)$$

The smaller the value of  $|\text{Corr}(x, y)|$ , the smaller the correlation between the two variables.

#### Dictionary learning and VMD based seismic denoising algorithm

The technical process of this paper is implemented in the following steps, and the flow chart is shown in Fig. 1.

(1) Firstly, the VMD algorithm combined with autocorrelation analysis and peak judgment is used to decompose each mode, and the seismic signal is decomposed into several eigenmode components with different center frequencies by the VMD algorithm.

(2) Then the noise-dominated modes are discarded, and the high-noise components are directly removed.

(3) Combined with the ODL dictionary learning algorithm to learn the dominant mode and the transition mode respectively, and finally to perform sparse denoising respectively, and finally to reconstruct and merge the signals to finally complete the seismic signal denoising.

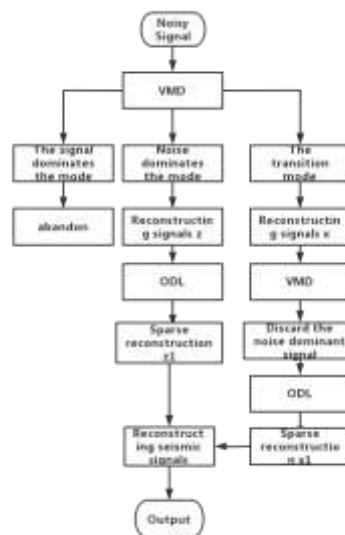


Fig.2 The technical process

## IV. EXPERIMENTATION

### Experimental simulation

To evaluate the effectiveness of this denoising method, the denoising effect is compared with other traditional single denoising effects. First, a single synthetic seismic record is generated, and Gaussian white noise with different decibels is added to this signal, and then hard-threshold wavelet denoising, soft-threshold wavelet denoising, VMD denoising, and the proposed online dictionary learning algorithm combined with VMD denoising are used respectively, and the signal-to-noise ratio is calculated to compare the denoising effect. Figure 3 shows the VMD decomposition IMFs of one seismic record, Figure

4 shows the noisy signal with SNR = 5 dB, and Figure 5 shows the noisy signal after the online dictionary learning algorithm combined with VMD denoising, Table 1 shows the wavelet hard threshold denoising results, wavelet soft threshold denoising results, the denoising results of VMD, the signal-to-noise ratio has been improved to some extent, but it can be seen that the wavelet hard threshold denoising and soft threshold denoising The waveform amplitude is also attenuated, and some effective signals are lost. Figure 5 shows the denoising results of the improved method proposed by the author. From Figure 6,7, we can see that the noise is largely removed and the amplitude loss of wavelet hard and soft threshold denoising is

overcome. In order to evaluate the denoising effect of the improved denoising method under different signal-to-noise ratios, the following experiments were conducted again. The signal-to-noise ratios of four different white noises with different energy levels and different signal-to-noise ratios were calculated by using the wavelet hard threshold method, the wavelet soft threshold method, the

VMD decomposition and reconstruction method, and the improved denoising method, respectively. The signal-to-noise ratio before and after denoising is shown in Table 1. The comparison of the denoising effect of various methods under different noise energies confirms the effectiveness of the improved denoising method proposed in this paper.

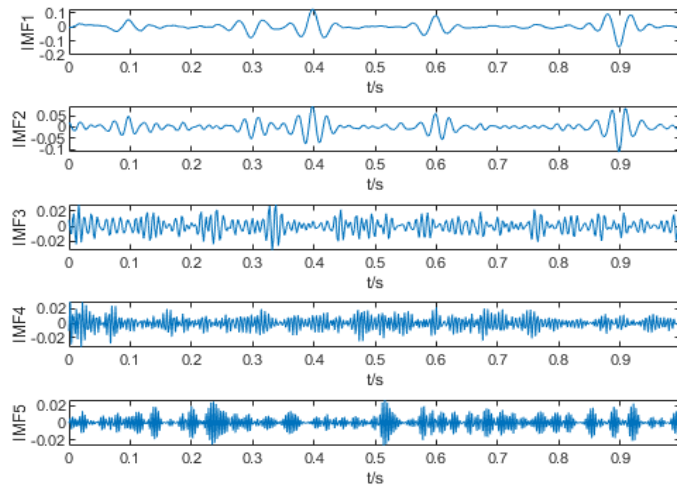


Fig.3 IMFs

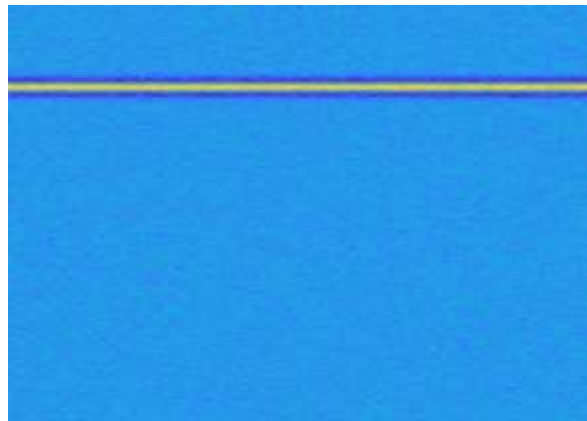


Fig.4 Noisy signal

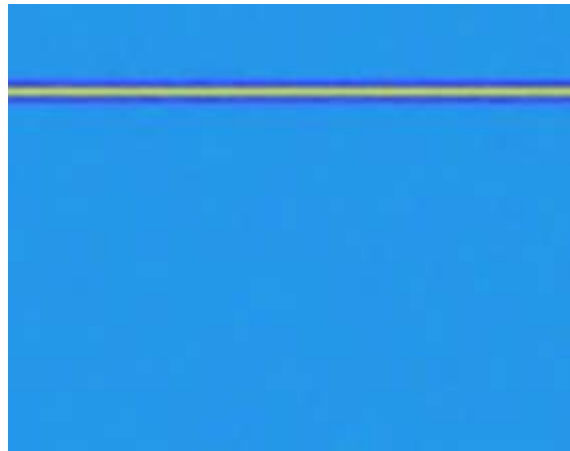


Fig.5 denosing signal

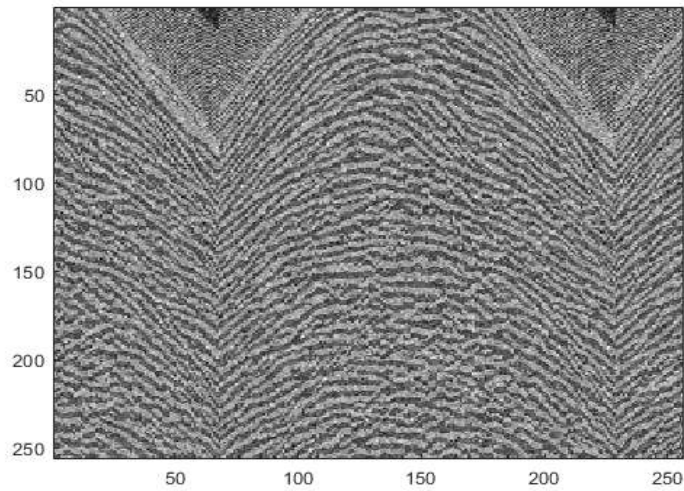


Fig.6 Noisy signal

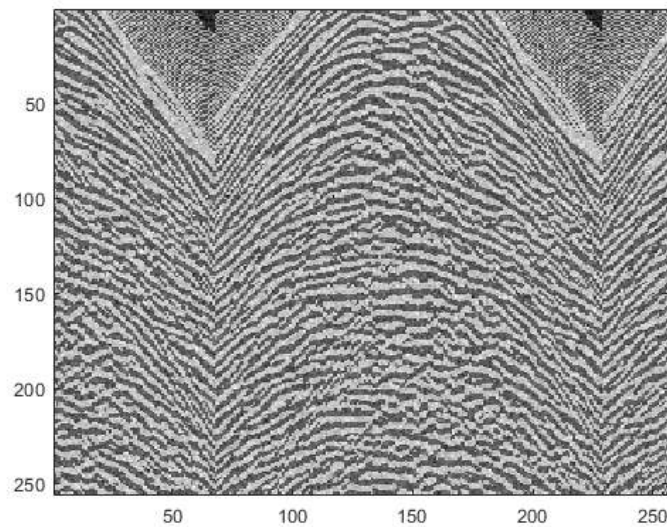


Fig7 Noisy signal



Tab.1 The denoising effect of adding different decibel Gaussian white noise

Noise+signal/dB	Wavelet hard threshold/dB	Wavelet soft threshold/dB	VMD/dB	ODL+VMD
5dB	7.6656	9.6044	8.4426	11.9781
7dB	9.6569	10.6763	10.2505	12.9184
9dB	11.7902	12.5597	13.4631	15.6709
11dB	14.8859	15.1193	15.4929	18.6141

## V. CONCLUSION

The online dictionary learning combined with VMD algorithm denoising algorithm overcomes the shortcomings of traditional soft threshold function and hard threshold function, and also adapts to large-scale seismic data. In this paper, we propose an algorithm that can combine correlation coefficient and peak calculation, VMD and dictionary learning algorithm to denoise the signal, so as to retain the effective signal as much as possible while denoising. At the same time, the VMD decomposition method is combined with the threshold denoising method to further improve the denoising effect by using the multi-scale adaptive decomposition property of VMD. It is proved that the improved method proposed in this paper can effectively denoise the seismic signal and improve the signal-to-noise ratio, and the denoising effect is better than the other three single denoising methods, and more details and effective information can be retained in the denoising process.

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