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Estimating the value of H_0 by using Type1A Supernovae and Bayesian statistics to a precision of 2.1%

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ABSTRACT
This paper presents a new estimation of the Hubble Constant H_0 to a precision of 2.1% using machine learning and intricate datasets . Most precise measurements are inferred from data of the Cepheid dis- tances to Type 1A Supernovae (SNe Ia) host galaxies and distances of SNe Ia in galaxies with observable recession due to the expansion of the universe as per the Hubble Law. Up-to-date H_0 measurements tend to have systematic uncertainties, thus the need for intricate datasets. To have a closer estimate, recent extensive data from Reiss et al [13, 14] and Dhawan et al has been used as we analyse SNe Ia as standard candles in Cepheid galaxies in the NIR(near infrared region), where luminosity variations in the supernovae and extinction by dust are both reduced. We use Bayesian statistics for analysing the data and inference as it works efficiently and provides scope in machine learning, future pre- dictions and works with data with limitations.
Coupled with the best complementary sources of Cepheid calibration,
2.1% precision is reached and
$H_0 = (71.77 \pm 2.2_{err} kms^1 Mpc^1)$
is calculated.
The results differ slightly from the gravitational-wave measurement of H_0 [2], and for the statistical uncertainty, colour corrections, dust extinction and a myriad of other systematic uncertainties can be the predominant reason.

I. THEORY

In the middle of the 2nd century BCE, Greek astronomer Hipparchus pioneered the use of a method known as parallax to calculate the dis- tance to the moon.
The idea of parallax is that objects appear to shift when viewed from different angles. Since parallax shift is more with objects closer to us than farther, [1] , it’s easier to measure the distance stars near us.

This begins the cosmic ladder, and then we transcend to Cepheids and standard candles. Using parallax we estimate distances to nearby stars, then using that we find distance estimates to Cepheid standard can- dles. From the period luminosity law, we’re able to calculate absolute magnitude of these Cepheids. We use this data to get an estimate of M_0, the theorised average for SNe Ia luminosity because we know that SNE Ia [7, 8, 9] have the same absolute magnitude,calibrating the cos- mic ladder. Using the calibrated M_0 we use the inverse square law and the relation m_k-M_k=μ_k to get estimates of the distance to far galaxies [13, 14, 15, 16, 17, 24]. We then use the Hubble flow [10, 11, 12]dataset to get an expression for H_0 in terms of θ equivalent to terms with redshift calculated from spectroscopy and the apparent magnitude observed. Then, estimate M_0 and θ simultaneously in one likelihood function

1.1 The Cepheid Set

Using the distance to (1) these stars calculated by parallax, further stars can be measured by use of standard candles and Cepheids. The ab- solute magnitude M of Cepheids is derived from the period-luminosity relation, and the apparent magnitude m is essentially observed. The data for the same, from Dhawan et al; is used to calibrate M_0.We de- scribe the Cepheid calibrator set for k Cepheids observed,we then use the following relation,

$$\hat{m}_k - \hat{M}_k = \hat{\mu}_k \qquad (2)$$

Here, due to Law of Large Numbers, for

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each SNe Ia, we calculate $\hat{\mu}_k$ and $\hat{m}_k$ and construct normal distribution functions with deviations due to measurement errors being σ_{μ_k} and σ_{m_k} as such:

$$\hat{m}_k \sim N(m_k, \sigma^2_{m_k}) \quad (3)$$

$$\hat{\mu}_k \sim N(\mu_k, \sigma^2_{\mu_k}) \quad (4)$$

Since

$$Var(\hat{M}_k) = Var(\hat{m}_k) + Var(\hat{\mu}_k) - Covar(\hat{\mu}_k, \hat{m}_k) \quad (5)$$

$$\Rightarrow \sigma_{err}{}^2 = \sigma_{\mu_k}{}^2 + \sigma_{m_k}{}^2 \quad (6)$$

Since Covar($\hat{\mu}_k$,$\hat{m}_k$)=0, because they are independent random variables, where σ_{err} is the standard deviation for normal distribution of $\hat{M}_k$ values around mean M_k [6, 5], i.e.

$$\hat{M}_k \sim N(M_k, \sigma^2_{err}) \quad (7)$$

and

$$\hat{m}_k - \hat{\mu}_k \sim N(M_0, (\sigma^2_{err} + \sigma^2_{int})) \quad (8)$$

$$\Rightarrow \hat{m}_k - \hat{\mu}_k \sim N(M_0, (\sigma_{\mu_k}{}^2 + \sigma_{m_k}{}^2 + \sigma^2_{int})) \quad (9)$$

where M_0 is the theorised average for Sne Ia luminosity.
The likelihood for parameters M_0 and σ_{int} is derived simultaneously with the hubble flow set, done next.

1.2 The Hubble Flow Set

From a cosmological model, we can infer H_0 with the reverse distance ladder from cosmic microwave backgrounds and high redshift observa- tions. We measure redshift constant Z_i for i observed SNe 1A by spec- troscopy,and then assume theorised average absolute magnitude to be M_0, which we caliberate using the Cepheid set and observe apparent magnitudes $\hat{m}_i$, we calculate the distance d_i by the following relation (the Hubble law):

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$$d_i = \frac{cZ_i}{H_0} \tag{10}$$

We also let:

$$\theta = 5\log_{10}(\frac{H_0}{100}) \tag{11}$$

We know for the distance modulus μ_i

$$\mu_i = 25 - \theta + 5\log_{10}(\frac{cZ_i}{100}) \tag{12}$$

$$\Rightarrow \hat{m}_i - M_0 - err_{int} - err_{m_i} = 25 - \theta + 5\log_{10}(\frac{cZ_i}{100}) \tag{13}$$

$$\Rightarrow \hat{m}_i - 25 - 5\log_{10}(\frac{cZ_i}{100}) = M_0 + err_{int} + err_{m_i} - \theta \tag{14}$$

Assuming

$$\hat{m}_i - 25 - 5\log_{10}(\frac{cZ_i}{100})$$

to be $\hat{F}_i$, we obtain the normal function:

$$\hat{F}_i \sim N(M_0 - \theta, ({\sigma_{m_i}}^2 + \sigma_{int}^2)) \tag{15}$$

Now we create the total likelihood function as:

To find the maximum likelihood estimates, we differentiate the log of likelihoods with respect to the parameters Mo and theta we get:

Now we create the total likelihood function as:

$$L(M_0, \theta, \sigma_{int}) = (\prod_{k=1}^{K} N(M_0, ({\sigma_{\mu_k}}^2 + {\sigma_{m_k}}^2 + \sigma_{int}^2))) \times (\prod_{i=1}^{N} N(M_0 - \theta, ({\sigma_{m_i}}^2 + \sigma_{int}^2))) \tag{16}$$

To find the maximum likelihood estimates, we differentiate the log of likelihoods with respect to the parameters Mo and theta we get:

$$M_{0_{MLE}} = \frac{\sum_{k=1}^{K} \hat{M}_k ({\sigma_{\mu_k}}^2 + {\sigma_{m_k}}^2 + \sigma_{int}^2)^{-1}}{\sum_{k=1}^{K}(({\sigma_{\mu_k}}^2 + {\sigma_{m_k}}^2 + \sigma_{int}^2))^{-1}} \tag{17}$$

and

$$\theta_{MLE} = \frac{\sum_{k=1}^{K} \hat{M}_k ({\sigma_{\mu_k}}^2 + {\sigma_{m_k}}^2 + \sigma_{int}^2)^{-1}}{\sum_{k=1}^{K}(({\sigma_{\mu_k}}^2 + {\sigma_{m_k}}^2 + \sigma_{int}^2))^{-1}} - \frac{\sum_{i=1}^{N} \hat{F}_i ({\sigma_{\mu_i}}^2 + \sigma_{int}^2)^{-1}}{\sum_{i=1}^{N} ({\sigma_{\mu_i}}^2 + \sigma_{int}^2)^{-1}} \tag{18}$$

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1.3The Metropolis Hastings Algorithm

We calculate the variance in θ and M_0 with fisher information and poste- rior density then using importance sampling and MCMC [4],[18, 19, 20], we implement the metropolis-hastings algorithm [21, 22, 23] to deduce which proposed values of theta and M_0 (after considering the variations) we should accept or reject. We do this by calculating r, the ratio of pos- terior probabilities of newly generated parameter versus the posterior probability of previous value of the parameter. If $r > 1$, we accept the value, other wise we draw a uniform random number u in the interval of 0 to 1 and if $r > u$ we accept the value, if not then we reject the value.

II. DATA AND ANALYSIS

2.1 Data

Table 1- The Cepheid calibrator set:

supernova name	host galaxy name	$\hat{m}$	σ_m	$\hat{\mu}$	σ_μ
SN2001el	NGC1448	12.837	0.022	31.311	0.045
SN2002fk	NGC1309	13.749	0.010	32.523	0.055
SN2003du	UGC9391	14.325	0.056	32.919	0.063
SN2005cf	NGC5917	13.791	0.025	32.263	0.102
SN2007af	NGC5584	13.446	0.003	31.786	0.046
SN2011by	NGC3972	13.218	0.040	31.587	0.070
SN2011fe	M101	10.464	0.009	29.135	0.045
SN2012cg	NGC4424	12.285	0.017	31.080	0.292
SN2015F	NGC2442	13.081	0.024	31.511	0.053

Table 2- The Hubble Flow set:

supernova name	host galaxy name	redshift	$\hat{m}$	σ_m
SN2004eo	NGC6928	0.015259	15.496	0.010
SN2005M	NGC2930	0.025441	16.475	0.017
SN2005el	NGC1819	0.015044	15.439	0.007
SN2005eq	MCG010906	0.028336	16.793	0.059
SN2005kc	NGC7311	0.014468	15.390	0.008
SN2005ki	NGC3332	0.019887	16.111	0.014
SN2006ax	NGC3663	0.017908	15.719	0.010
SN2006et	NGC232	0.022288	16.061	0.019
SN2006hx	Abell168	0.044533	17.779	0.077
SN2006le	UGC3218	0.018374	15.935	0.010
SN2006lf	UGC3108	0.012037	14.945	0.220
SN2007S	UGC5378	0.015244	15.346	0.018
SN2007as	PGC026840	0.018486	15.864	0.016
SN2007bd	UGC4455	0.031624	17.105	0.028
SN2007ca	MCG023461	0.014471	15.568	0.006
SN2008bc	PGC90108	0.015623	15.542	0.009
SN2008hv	NGC2765	0.013816	15.232	0.013
SN2009ad	UGC3236	0.028587	16.880	0.031
SN2009bv	MCG062939	0.038302	17.552	0.028
SN2010ag	UGC10679	0.033461	17.202	0.019
SN2010ai	Coma	0.022102	16.597	0.027
PTF10bjs	MCG092183	0.030573	17.033	0.029
SN2010ju	UGC3341	0.015020	15.600	0.018
SN2010kg	NGC1633	0.017021	15.822	0.019
PTF10mwb	SDSSJ171750.05+405252.5	0.031004	16.995	0.026
PTF10ufj	2MASXJ02253767+2445579	0.076676	19.298	0.079
SN2011ao	IC2973	0.012164	14.885	0.028

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2.2 Analysis

In our Bayesian analysis we take flat improper priors: uniform on $H_0 > 0$, and scale-free on $\sigma_{int} > 0$, with $p(\sigma_{int}) = 1/\sigma_{int}$.The code, is available at https://github.com/Azulversa612/Estimating-Hubble-Constant

After using Bayesian stats and the MCMC around it,we find a sample mean H_0 = 71.77±2.1 km s^1 Mpc^1.

III. FURTHER SCOPE AND REFERENCES

We estimate the Hubble constant to a precision of 2.1 with H_0= 71.77 H-band "peak" is better in terms of dust uncertainties but it's longer in time and not as concise as in J, thus we resort to observations in the J-band peak [3].

The statistical uncertainty can be improved by more objects in both sets: more Cepheid calibrated SNe Ia and more Hubble-flow SNe Ia.

We anticipate further improvements in estimation of H_0 with a larger calibrator set of SNe Ia with Cepheid distances, more Hubble flow SNe Ia with NIR light curves, and choosing better estimators or methods of optimization.

The most fascinating aspect is how well the standard candle ap- proach with use of Bayesian stats works.Trying to work out physics explaining both inter and intra galactic measures of H_0, and increasing number of observations in each set and coming up with new/different parameters or methods (like interpreting gravitational waves) would help unravel cosmological mysteries and yield to results taking us closer in the ladder for the unified field equation.

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