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Formulation of a Performance Measure forHeterogenic Vehicle RoutingProblems (HVRP) with Intermittent Customers and an Enforced Split Deliveries

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ABSTRACT:
Recent trends in businesses coupled with numerous customers entering the supply chain with different issues spreading across customers' conditions of service, vehicles with conditions, and problems associated with routes are of great concern to logistic providers. This paper aims to formulate a model for Heterogenic Vehicle Routing Problems (HVRP) with Intermittent Customers and Enforced Split Delivery resulting from Customers' Vehicular Preference, Road Time, Vehicle Weight, and Vehicle Height Restrictions. The formulated HVRP objective function emphasizes robustness and demonstrates an amplified functional architecture that will minimize the total travel and running cost, maximize the clientele's priorities, and cater to the marginal difference in vehicle carriage capacity extensively discussed. Solving problems of this nature could be tasking hence,requires optimizing along different directions considering the service choices and the customers' intermittencies in routing problems. Reasons for these are the uncertainties that accord real-life business situations that make life dynamic, opening the vista that brought about intermittencies in VRP. To achievethis, the paper formulates dynamics that fuse the Late Request Customers (LRC) into the Earliest Request Customers (ERC). The paper also considers the effects of Forced Splitting against classical splitting orchestrated by road restrictions. It analyses the fused LRC and encapsulates road restrictions as they affect thedelivery of goods and services to customers.

Keywords: Heterogeneous Fleet, Intermittent Customers, Enforced Split Deliveries, Road Restriction, Reoptimization, Reactivation, Early Request Customers (ERC), Late Request Customers (LRC).

I. INTRODUCTION

One of the most interesting problems thatmany researchers have put effortsintoformulating and developing a new methodology to solve is the Vehicle Routing Problem (VRP).The VRP has been acclaimed as a combinatorial integer programming problem searching for the optimal set of routes for a set of vehicles that depart from the depot to deliverto one or more customers at specified locations. The typical performance measure it aims at achieving includesminimizing running costs, fulfilling customers' requests, maximizing profits, and maximizing the utilities involved in the production and distribution process.

The classical model of the VRP that was first introduced in 1959 according to [1] has been extensively studied and spread to several types such as Capacitated Vehicle Routing Problem (CVRP), Vehicle Routing Problem with Time Windows (VRPTW), Vehicle Routing Problem with Deliveries and Pickups (VRPDP), Split Delivery Vehicle Routing Problem (SDVRP), and lots more. The VRP variants such as time windows, dynamic situations, pickup and delivery, split deliveries, etc., appear as added constraints that can be included in the model to represent the border on the real business. The variants of the VRP and different techniques developed over time for solving this class of problems have been studied theoretically and applied practically.

As manufacturing companies and distribution outlets realized the significant factors contributed by transportation expenses to the overall cost of goods and the application of computer-generated solutions hasincreased significantly the quest to study VRP.As opined by [2] and [3], the cost of transportation is significant for both developed and developing countries where logistics sectors like road and freight transport and distribution services are rated to be underperforming. Although VRP was mostly applied in product distribution, some were also applicable in service deliveries such as school children's bus services as in [4] and [5], worker's shuttle buses, and other public transport as shown in [6], and marine border security patrol.

The major contributions of this paperinclude the following:

- generate relations for different classes ofcustomers' intermittencies in VRP;
- develop a model and mathematical formulation strengthened with valid inequalities that fuse heterogeneous vehicles into VRP against the classical homogeneous vehicles used;
- formulate a dynamic that incorporates the customers' intermittencies and the heterogeneous vehicles into the classical VRP;
- outline a computational layout required to solve the problem to optimality.

Consequent to this, the paper is organized as follows: section 2 stresses the requisites of the objective function concept of intermittencies in VRP with the formulation of relations for the Degree of Dynamisms. Section 3 focuses on the ideas surrounding Heterogenic VRP with the simulation of the model equations. Insection 4, the fundamentals of HVRPwill be highlighted alongside split delivery and road restrictions that will lead to the objective function formulation in section 5. The conclusion comes up in section 6.

II. REQUISITES FOR THE OBJECTIVE FUNCTION FORMULATION

The formulation of the HVRP objective function requires the vista of key ingredients which are Customers' Intermittencies and Heterogeneity of vehicles to be used which are the pivots of the formulation. These will be the core of the discussion in this section.

2.1 Concept of Intermittencies in VRP

The classical VRP is characterized by the quantity the customers require and their locations are known beforethe vehicles leave the depot. Intermittent customers are the customers whose requests are placed after the vehicle has departed the depot. Intermittent customers' quantities and locations come after the dispatched manager has sent the vehicles out for the day's tour.

Intermittency has become inevitable if a business grows especially with the advent of technological advancement. The invention and improvement in information technology have greatly contributed to the workability of this concept. It has become less difficult due to the use of network facilities, the Global System for Mobile Communication, and the Global Positioning System. Otherwise, its attainment would have been a mirage and not feasible.

According to [7] and [8], all the customers who have indicated their interest before setting out of the vehicle at the depot will be referred to as the Early Request Customers (ERC) with

$$\mathrm{ERC} = \sum_{i=1}^{N} \mathrm{ERC}_i \qquad (1)$$

where N stands for the number of customers in the set. Let the anticipatory customers that enter into the link after the vehicle has departed the depot be regardedas Late Request Customers (LRC) with

$$\mathrm{LRC} = \sum_{i=1}^{L} \mathrm{LRC}_{N_S}^{i} \qquad (2)$$

where the number of anticipatory customers is L, the customers already serviced before LRCare servedare presented by N_S, and $\mathrm{LRC}_{N_S}^{i}$ is the set of LRC.

Within the specified time frame, each vehicleis expected to reach the customers at a service point. The entire tour will begin and will ultimately end at the depot. According to [8], at least one ERC must be served before any of the LRC is serviced,and all the ERCswaiting in line must be served on the day. As the tour continues for the day, a set of LRCs may stochastically request to be served. The LRCquantity, time, and location are not known until the dispatched manager makes their requests known. The location, time, and quantity that the LRCrequires follow a known probability scheme.

When a vehicle sets out to service customers, the dispatch manager decides which of the occurred requests' subsetswill be assigned to a particular vehicle, and as much as the vehicleis still within the service period, a record of the entire tour is kept.

When a vehicle is designated to a particular LRC, the vehicle is expected to service the remaining ERC on that route within the time frame. The dispatcher aims at maximizing the number of LRCs to which the vehicle is assigned subject to the Degree of Dynamism(DD). Depending on the number of ERC and their corresponding entering time, the author in [9] suggested a potent means of determining the degrees of dynamism. To adhere to the priorities and road restrictions, there is a need to adaptively restructure the extant service pattern to take care of the LRC since the problem is intermittently dynamic.

The most effective and efficient way to achieve this is to re-optimize and re-activate the ERCresolution then, the LRC is infiltrated into the existing ERC solution process. For proper planning, possible future business expansion, and cost-effectiveness, it is expedient to consider the anticipatory customers for some reasons. To solve a dynamical pickup and delivery problem [10] proposes a double-horizon heuristic that focuses on short-term and long-term goals by minimizing the entire distance traveled and by maximizing the slack time respectively to accommodate servicing of the LRC. However, [11], [12], and [13] investigated the waiting techniquesadopted by the vehicles and improved the solution profile by forcing the vehicles to wait at certain places merely to buy time.

The Intermittent VRP with stochastic requests according to [14],varies in their levels of uncertainty. These variations are specifiedin the number of LRCsthat might place orders when the ERC is to be serviced. The authors in [7] and [15] tagged the rate of uncertainty as Degree of Dynamism and Dynamical Degree (DD) respectively and depicted the DD as:

$$\mathrm{DD} = \frac{\mathrm{LRC}}{\mathrm{OC}} \qquad (3)$$

where the Overall Customers (OC) is the total number of ERC and LRC. The DD will be considered based on its peculiarity:

(i) In the first kind, as presented by [8], the entire LRCwill come after all the ERCmust have been serviced. The relations (1) and (2) thus give rise to:

$$\mathrm{OC} = \mathrm{ERC} + \mathrm{LRC} = \sum_{i=1}^{N} \mathrm{ERC_i} + \sum_{i=1}^{L} \mathrm{LRC}_{N_S}^{i} \quad (4)$$

This first category is usually straightforward to address compared to othercategories because the initially planned ERC tour is not warped in any manner. With or without the LRC in this case, all the ERC are kept unaltered, treated, and serviced first. From (3) and (4), the DD in this category is given by:

$$\mathrm{DD}_1 = \frac{\mathrm{LRC}}{\mathrm{OC}} = \frac{\mathrm{LRC}}{\mathrm{ERC} + \mathrm{LRC}} = \frac{\sum_{i=1}^{L} \mathrm{LRC}_{N_S}^{i}}{\sum_{i=1}^{N} \mathrm{ERC}_i + \sum_{i=1}^{L} \mathrm{LRC}_{N_S}^{i}} \quad (5)$$

(ii) The second kind of DDis when the LRC comes after some of the ERC, i.e.ERC_1. The remaining ERC to be serviced is ERC_2 given by

$$\mathrm{ERC}_2 = ERC - ERC_1 \qquad (6)$$

is serviced after all possible LRCmust have been serviced thus:

$$OC = ERC_1 + LRC + ERC_2 \qquad (7)$$

From the second case of the DD,the resulting $OC$is given by:

$$OC = \sum_{i=1}^{E_1} ERC_i + \sum_{i=1}^{L} LRC_{N_S}^{i} + \sum_{i=E_1+1}^{N-E_1} ERC_i \quad (8)$$

where $E_1 < N$, represents part of the ERC that has been serviced after which the LRC request will be met and L represents the maximum possible number of $LRCs$ that can be serviced such that the initial ERCtour plan will not be affected.

Then, from (3) and (8), the DD in the second case is given by:

$$DD_2 = \frac{LRC}{OC} = \frac{LRC}{ERC_1 + LRC + ERC_2} \qquad (9)$$

$$DD_2 = \frac{\sum_{i=1}^{L} LRC_{N_S}^{i}}{\sum_{i=1}^{E_1} ERC_i + \sum_{i=1}^{L} LRC_{N_S}^{i} + \sum_{i=G_1+1}^{N-E_1} ERC_i} \qquad (10)$$

(iii) In the third case, the LRC comes intermittently within the ERC with a proviso that the LRC can't come before the first ERC. This case allows for the $LRC$to comein at different intervals that could be regular or irregular. In totality, the LRC so involved are presented as:

$$LRC = \sum_{i=1}^{L_1} LRC_{E_1}^{i} + \sum_{i=1}^{L_2} LRC_{E_2}^{i} + \cdots + \sum_{i=1}^{L_{n-1}} LRC_{E_{n-1}}^{i} + \sum_{i=1}^{L_n} LRC_{E_n}^{i} \qquad (11)$$

where $L_1, L_2, \ldots, L_n \epsilon L$ represent various anticipatory customers that might intermittently come up, $E_1, E_2, \ldots, E_{n-1}, E_n$ represent the number of customers serviced when the anticipatory customer's request comes in, and $E_1 + E_2 + \cdots + E_n = N$.and the corresponding OC for the third case is given by:

$$OC = ERC_1 + LRC_1 + ERC_2 + LRC_2 + \cdots + LRC_y + ERC_M \qquad (12)$$

where $M \leq (N-1)$ is the possible number of ERC serviced in the course of the routing and $y \leq (L-1)$ is the number of intermittentLRCserviced. With

$$ERC = ERC_1 + ERC_2 + ERC_3 + \cdots + ERC_M = \sum_{i=1}^{E_1} ERC_i + \sum_{i=E_1+1}^{E_2=N-E_1} ERC_i + \sum_{i=E_2+1}^{E_3=N-E_2} ERC_i + \cdots + \sum_{i=E_2+1}^{E_n=N-E_{n-1}} ERC_i + \sum_{i=E_n+1}^{N-E_n} ERC_i \qquad (13)$$

and

$$LRC = LRC_1 + LRC_2 + \cdots + LRC_y$$

$$LRC = \sum_{i=1}^{L_1} LRC_{E_1}^{i} + \sum_{i=1}^{L_2} LRC_{E_2}^{i} + \sum_{i=1}^{L_3} LRC_{E_3}^{i} + \cdots + \sum_{i=1}^{L_3} LRC_{E_{n-1}}^{i} + \sum_{i=1}^{L_n} LRC_{E_n}^{i} \qquad (14)$$

The resulting $OC$from the third case is then given as:

$$OC = \sum_{i=1}^{E_1} ERC_i + \sum_{i=1}^{L_1} LRC_{E_1}^{i} + \sum_{i=E_1+1}^{E_2=N-E_1} ERC_i + \sum_{i=1}^{L_2} LRC_{E_2}^{i} + \sum_{i=E_2+1}^{E_3=N-E_2} ERC_i + \sum_{i=1}^{L_3} LRC_{E_3}^{i} + \cdots + \sum_{i=1}^{L_n} LRC_{E_n}^{i} + \sum_{i=E_n+1}^{N-E_n} ERC_i \qquad (15)$$

Hence, the DD for the third case from (3), (11), and (15) is given by:

$$DD_3 = \frac{\sum_{i=1}^{L1} LRC_{E1}^{i} + \sum_{i=1}^{L2} LRC_{E2}^{i} + \sum_{i=1}^{L3} LRC_{E3}^{i} + \ldots + \sum_{i=1}^{L3} LRC_{En-1}^{i} + \sum_{i=1}^{Ln} LRC_{En}^{i}}{\sum_{i=1}^{E1} ERC_i + \sum_{i=1}^{L1} LRC_{E1}^{i} + \sum_{i=E1+1}^{E2=N-E1} ERC_i + \sum_{i=1}^{L2} LRC_{E2}^{i} + \sum_{i=E2+1}^{E3=N-E2} ERC_i + \sum_{i=1}^{L3} LRC_{E3}^{i} + \ldots + \sum_{i=1}^{Ln} LRC_{En}^{i} + \sum_{i=En+1}^{N-En} ERC_i} \qquad (16)$$

It is worthy of note that, in any of the three cases, the LRC cannot come before the first ERC is attended to.The DD is a yardstick to classify HVRP into stochastic requests. From [16] and [17], a moderate DD realized in practices including the distribution of oils, transportation of patients, and grocery deliveries. Its range of utilization with high-level DD entails emergency vehicles or courier services in [18] and [19]. A high-level DD is mostly found in practical applications which include: responsive demand transportation, same-day delivery, and shared mobility as opined by authors of [20], [21], and [22] respectively. For a more detailed classification of DSVRP applications similar to IVRP see [8] and [23].

2.2 Concept of Heterogeneity to Vehicle Routing Problems

The VRP variation that considers fleets of composite vehicles assigned to visit a set of customers whose geographical locations are known is called Heterogeneous VRP (HVRP). Heterogeneous vehicles involve fleets with varying capacities, fuel costs, and fixed costs.Keen consideration has shown that our day-to-day activities dealwith more heterogeneous VRP than the classical VRP where a homogeneous vehicle is used [24]. This category of routing problems that are quite frequent in logistic operations, takes into consideration the situations in our real-life settings where it is practically not possible to claim to have only one type of vehicle for distribution knowing fully well that there are several reasons why different types of vehicles are needed.

The idea of HVRP was long borne in [25]. Its conceptualization is salient in pragmatic terms since most customers' demands might not be met by one single type of vehicle hence, are served by several vehicles as in [26] and [27]. There are several factors thedistribution managers will want to employ a fleet of vehicles with varying capacities, sizes, fuel costs, and fixed costs. Such rationale for HVRP include:

a. traffic flow regulated for specific times, weight, height restriction, etc. The peculiarities of road conditions make it necessary for companies to utilizedifferent types of vehicles to service their customers' requests in good time;
b. while heavy-duty vehicles are considered cost-effective on long distances, they are sometimes not allowed to ply some routes in metropolises owing to environmental degradation, road maintenance, and traffic congestion, or they may not be used to visit some particular customers because of certain restrictions on the road infrastructures.

A typical HVRP embeds the dependency of traveling time at a specific time of the day. The authors in [28] opined that the HVRP is targeted at minimizing the total costs of touringthusreducing the elapsed time over the entire planning time frame.

The goal of the HVRP is to decide the most appropriate composition of the fleet of vehicles and fashion out a routing plan correspondently to minimize the total cost of transportation.

III. BASIC REQUISITES OF THE HVRP

To enable us to spell out the fundamentalsof HVRP, there is the necessity to view VRP over a given time horizon, T. Let $C = \{c_i \mid i = 0, 1, 2, \ldots, N\}$ represents N set of customers such that c_0 is the depot. Let $V = \{V_k \mid k = 1, 2, \ldots, M\}$ denotes M set of a heterogenic fleet of vehicles positioned at the depot where

$$\left.\begin{aligned} V_1 &= \{v_1^a \mid a = 1, 2, 3, \ldots, A\}, \\ V_2 &= \{v_2^b \mid b = 1, 2, 3, \ldots, B\} \\ &\vdots \\ V_M &= \{v_M^z \mid z = 1, 2, 3, \ldots, Z\} \end{aligned}\right\} \qquad (17)$$

be the subsets of vehicles in each heterogenic type of vehicle in set V where $A, B, \ldots, Z$ denotes the number of vehicles in each heterogenic set. By [7], every deuce of locations, (i, j), of two sequent nodes affirmed as customers, with $i, j \leq N$. Associated with $i \neq j$, is the travel duration, t_{ij}, from one customer c_i to the next customer, $c_j = c_{i+1}$,and the distance traveled by the vehicle, $d(i, j) = d_{ij}$,is symmetrical, i.e.$d_{ij} = d_{ji}$.

The following fundamental requirements are bounding on every customer, c_i:

- there should be a pre-determined quantity, $q(c_i)$, of the commodities or services that the customer requires to be delivered. The dispatch manager hence decides which of the M heterogeneous vehicles will be most appropriate to do the delivery. Note, not in all cases can a vehicle do the entire delivery. However, where one vehicle solely cannot do the entire supply requested by a customer then, the supply has to be split. The splitting will be done either among different types of vehicles, $V_1, V_2, V_3 \dots V_M$, or among the same types of vehicles, $v_1^1, v_1^2, v_1^3, \dots, v_1^A$.

- there should be a set-out time, t_{ij}, requisite by the vehicle, V, to traverse from either the depot, c_0, to the customer, c_i, or one customer, c_i, to service the next customer, c_j, to discharge the quantity, $q(c_i)$. It ultimately leaves the customer, c_j, for either the next customer, c_{j+1}, or returns to the base station if every customer along that path to which the vehicle is assigned has been attended to for the day or summarily the vehicle carriage quantity, $Q(V_k)$ for the day has exhausted.

Another way around, there could be situations where more than one vehicle of the same type, $v_1^1, v_1^2, v_1^3 \dots v_1^A$, or more than a vehicle of different types, $V_1, V_2, V_3 \dots V_M$, have to serve a customer. Such vehicles have to move directly to the service point from the depot. The duration, t_{ij}, required by the vehicles either moving from the base station, c_0, or from a particular customer, c_i, to the next customer, c_j, to discharge the quantity, $q(c_j)$, and ultimately leaves for another customer, c_{j+1}, or invariably return to the base station has to be specified.

The priority, δ, of the customer, $\delta(c_i)$, to be satisfied by the vehicle,V must be stated clearly. Every customer is serviced only from one depot with a heterogeneous and finite fleet of vehicles. These vehicles depart the depot and in the fullness of time go back to the depot after the last customer has been attended to. The set of vehicles, V, with different quantities each vehicle can carry is represented by $Q(V_k)$.

The following characteristics are peculiar to the vehicle:

- there is a fixed working period for the vehicle, $T(V_k)$, with the earliest or starting time, $T^s(V_k)$, and the finishing time, $T^f(V_k)$ ie $T^f(V_k) - T^s(V_k) = T(V_k)$
- there is a fixed cost, $FC(V_k)$, for every vehicle which is the wages of the driver and the loaders deployed to each vehicle per trip.
- the quantity that the vehicle can carry, $Q(V_k)$ must be known right at the depot.

Leaning on the layout requirements for both the customers and the fundamental characteristics of the vehicles, the under-listed assumptions are curled out thus:

- the variable cost, VC_{ij}, is the servicing cost from one customer c_i to the next customer $c_j = c_{i+1}$;
- the time, t_{ij}, is the traveling time from one customer c_i to the next customer $c_j = c_{i+1}$. If all the road/traffic restrictions remain the same, the traveling time is assumed to be symmetrical, $t_{ij} = t_{ji}$,
- $R_i = \{r_i(1), \dots, r_i(S)\}$ stands for the set of routes for the vehicles, V, the number of customers serviced along a route is represented by S, and $r_i(S)$represents the ithcustomer visited. It is also assumed that every route terminates at the depot hence, $r_i(S+1) = 0$;
- the interval at which the vehicle parked to discharge the goods and the warehouse/store of each customer is assumed to be equal hence, unloading time per unit item is kept constant.

3.1 Split Delivery Classifications

Irrespective of whether the vehicle used is homogenous or heterogeneous, in practical settings [29] pointed out that split deliveries occur when the demand of a customer cannot be met by just a vehicle. Findings have shown that there are cases whereby the demand of customers outweighs the carrying capacity of the vehicles. However, by the design and formulation of VRP, a vehicle is not allowed to visit a customer more than oncea day hence, necessitates split delivery.

Besides the primary factor for splitting, other reasons for split deliveries are discussed in what follows thus:

- the quantity, q, that is demanded by a customer, $q(c_i)$, is more than the carriage capacity, Q, of the vehicle, $Q(v_k^a)$. Given by the relation:

$$q[c_i(v_k^a)] > Q(v_k^a) \quad (18)$$

This would lead to the delivery being done by more vehicles of the same types (homogeneous):

$$SD_1(c_i) = Q(v_k^a) + Q(v_{k+1}^a) - q[c_i(v_k^a)] \geq q(c_i) \quad (19)$$

On the other hand, the delivery could be done by different vehicles (heterogeneous):

$$SD_2(c_i) = Q(v_k^a) + Q(v_k^{a+1}) - q[c_i(v_k^a)] \geq q(c_i) \quad (20)$$

where $Q(v_{k+1}^a)$ and$Q(v_k^{a+1})$is the carrying capacity of vehicles v_{k+1}^aandv_k^{a+1} respectively, $q[c_i(v_k^a)]$ is the quantity to be supplied or delivered to customer c_i, $SD_1(c_i)$ and $SD_2(c_i)$ are split deliveries while $[Q(v_{k+1}^a)$ or$Q(v_k^{a+1})]$ appropriately, is the splitting quantity to be delivered by the same or different type of vehicle respectively to the same customer c_i.

- the vehicle, v_k^a, has serviced some customers say, $c_1, c_2, \dots, c_{N-n}$, along the route, $r_i(S_i)$, with the

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quantities, $q(c_1), q(c_2), \dots, q(c_{N-n})$, where the number of customers that have been serviced on the router$_i(S_i)$ is$n < N$. The quantities delivered by the vehicle v_k^a to the customers $c_1, c_2, \dots, c_{N-n}$, is given by:

$$q[c_1(v_k^a)] + q[c_2(v_k^a)] + \dots + q[c_{N-n}(v_k^a)] = \sum_{i=1}^{N-n} q[c_i(v_k^a)]. \quad (21)$$

Then, the quantities to be delivered by the vehicle v_k^a to the next customer c_{N-n+1}, is given by: $q[c_{N-n+1}(v_k^a)]$ and the total quantity to be delivered by vehiclev_k^a is given by:

$$\sum_{i=1}^{N-n} q[c_i(v_k^a)] + q[c_{N-n+1}(v_k^a)] = \sum_{i=1}^{N-n+1} q[c_i(v_k^a)]. \quad (22)$$

With (22), if the quantity $q[c_{N-n+1}(v_k^a)]$, which would have been delivered to the next customers, c_{N-n+1}, along the same route is not sufficient i.e.

$$\sum_{i=1}^{N-n+1} q[c_i(v_k^a)] > Q(v_k^a) \quad (23)$$

then, it occasioned a split delivery to be undertaken by another vehicle of the same type, v_{k+1}^aas:

$$\{\sum_{i=1}^{N-n} q[c_i(v_k^a)]\} + q[c_{N-n+1}(v_{k+1}^a)] \quad (24)$$

or another vehicle of adifferent type,v_k^{a+1} as:

$$\{\sum_{i=1}^{N-n} q[c_i(v_k^a)]\} + q[c_{N-n+1}(v_k^{a+1})]. \quad (25)$$

From (24) and (25), the split quantity, $q[c_{N-n+1}(v_{k+1}^a)]$ or $q[c_{N-n+1}(v_k^{a+1})]$ required by c_{N-n+1} is the remaining quantity to have been delivered by v_k^a which is:

$$Q(v_k^a) - \sum_{i=1}^{N-n} q[c_i(v_k^a)] + q[c_{N-n+1}] = q[c_{N-n+1}(v_{k+1}^a)] \text{or } q[c_{N-n+1}(v_k^{a+1})] \quad (26)$$

Since the quantity that will be delivered by $Q(v_{k+1}^a)$ or$Q(v_k^{a+1})$ cannot be determined in advancethen:

$$SD_3(c_i) = Q(v_k^a) + Q(v_{k+1}^a) + \dots + Q(v_M^a) - \sum_{i=1}^{N-n} q[c_i(v_k^a)] = \sum_{k=1}^{M} Q[(v_k^a)] - \sum_{i=1}^{N-n} q[c_i(v_k^a)] \geq q[c_{N-n+1}] \quad (27)$$

Or with

$$SD_4(c_i) = Q(v_k^a) + Q(v_k^{a+1}) + \dots + Q(v_k^A) - \sum_{i=1}^{N-n} q[c_i(v_k^a)] = \sum_{a=1}^{A} Q[(v_k^a)] - \sum_{i=1}^{N-n} q[c_i(v_k^a)] \geq q[c_{N-n+1}] \quad (28)$$

A more complex case is when (27) and (28) cannot meet the quantity required by c_{N-n+1} leaving us with the option of having:

$$Q(v_k^a) + Q(v_{k+1}^a) + \dots + Q(v_M^a) + Q(v_k^{a+1}) + \dots - \sum_{i=1}^{N-n} q[c_i(v_k^a)] \geq q[c_{N-n+1}]$$

$$SD_5(c_i) = \{\sum_{k=1}^{M} Q[(v_k^a)]\} + Q(v_k^{a+1}) + \dots - \sum_{i=1}^{N-n} q[c_i(v_k^a)] \geq q[c_{N-n+1}] \quad (29)$$

Conversely, it has

$$Q(v_k^a) + Q(v_k^{a+1}) + \dots + Q(v_k^A) + Q(v_{k+1}^a) + \dots - \sum_{i=1}^{N-n} q[c_i(v_k^a)] \geq q[c_{N-n+1}]$$

$$SD_6(c_i) = \{\sum_{a=1}^{A} Q[(v_k^a)]\} + Q(v_{k+1}^a) + \dots - \sum_{i=1}^{N-n} q[c_i(v_k^a)] \geq q[c_{N-n+1}] \quad (30)$$

From the above, a customer to whom the split delivery condition applies can only be linked to one of (19), (20), (27), (28), (29), or (30). Where $SD_c(c_i)$ represents the customers split delivery. It must be noted thatnot all customersare bound to be faced withsplit delivery situations. Since a customer can only fulfillone set of time and quantity priority conditions, the interplay between the time and the quantity priorities will be discussed via the tree diagram in [30].

From the priority interplay, if a customer, c_i, is not serviced, according to [30],both the Time Priorityand the Quantity Priority of the customer, $PT_a(c_i) = 0$ and $PQ_b(c_i) = 0$ else, $PT_a(c_i) = 1$ and $PQ_b(c_i) = q(c_i)$. With this, the sum of all the time and quantity priorities for all the customers,$c_1, c_2, \dots, c_N$, is given by:

$$\sum_{i=1}^{N} \delta(c_i) = \sum_{i=1}^{N} (\sum_{a=1}^{4} \sum_{b=1}^{4} PT_a(c_i) \times PQ_b(c_i)) \quad (31)$$

Consequently, if the quantity the customer required, $q(c_i)$ exceeds the carriage capacity of the vehicle or fails to satisfy all the road restriction conditions then, $PQ_b(c_i)$ will require a split delivery, $SD_c(c_i)$. Whenever there is splitting, the sum of the time windows and quantities priorities delivered to the customers is given by:

$$\sum_{i=1}^{N} \delta(c_i) = \sum_{i=1}^{N} (\sum_{a=1}^{4} \sum_{c=1}^{6} PT_a(c_i) \times SD_c(c_i)) \quad (32)$$

3.2 Road Restrictions on Vehicles

Transportation plays a significant rolein our day-to-day activities. The growth of transport-related energy consumption, congestion, and its adverse effects on the environment have attracted global concerns. With an increasing vehicular flow on the highway, it has led to traffic jams, pollution, road degradation, and lots more. The Traffic planners and managements tend to put an embargoes or restriction on highways based on one reason or the other in different ways. Therefore, continuous concerted efforts on VRP will not be directed towards reducing

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the transportation cost only but, contribute to our environmental protection.

Some roads are more susceptible to damage than others based on poor drainage, weather conditions, and other variables. The relevant authorities place an embargo on such roads due to weather conditions and frost testing. Road bans are effective tools for preventing road damage, reducing maintenance costs, and ensuring roadways remain safe for all motorists.

Road restrictions can be grouped under the following:

- Road Time Restriction, $TR(V_k)$: The $TR(V_k)$ disallows the movement of vehicles at a particular time in some places and restricts the movements of some types of vehicles on some paths to checkmate the traffic flow in the areas.
- Vehicular Weight Restriction, $WR(V_k)$: The $WR(V_k)$ is placed on some roads to regulate the weight of vehicles that traverse such roads purposely to keep the road infrastructure from further damage to the roador the total breakdown of the road.
- Vehicular Height Restriction, $HR(V_k)$: Not all roads allowfor any height of the vehicles. Height restrictions are setin motion to regulate either heavy-duty vehicles or vehicles with too high consignment from plying a particular routeto avoid degradation, total damage, or mishapto the road.

Customer's Preference for a vehicle, $CP(V_k)$:When a road ban occurs, signs indicating the allowed axle percentages are publicized, and the ban is scrutinized and enforced to ensure compliance. Such restrictions are not for life instead, the authority in charge fixes them. Once the road has been repaired and classified as structurally sound then, load restrictions can be rescinded. Not until lately, split delivery has been a function of a vehicle not being able to supply all the quantity that a particular customer requires. However, it has become apparent that there could be forced split delivery which results from situations where: there is the Customer's Preferred Vehicle to do the delivery; there is Road Time Restriction; Vehicle Weight Restriction limiting the carriage of the vehicle and there is Vehicle Height Restriction limiting the height of consignment that the vehicle can carry irrespective of whether the load is light. In a case where a vehicle fails to satisfy all four road restriction conditions, ultimately, the delivery will have to be split among vehicles. Such resulting splitting is referred to as Forced Split Delivery. While the next section will dwell on the inclusion of the priorities into the formulated HVRP objective function, it is expedient to look at the connectivity between the road restriction conditions and the priorities as it ducktails to Forced Split Delivery in the flowchart in Figure 1.

IV. FORMULATION OF THE HVRP OBJECTIVE FUNCTION WITH INTERMITTENT CUSTOMERS

Time Windows and Quantities priorities that came up in practical situations in HVRP with intermittent customers is a multi-objective function. Here, J_1 is to calculate the distance of the customer or the carriage cost to each of the customers, J_2 is to determine the vehicle's fixed cost andJ_3 is aimed at solving for the priorities of the customers.

If the vehicle v_k^a has to visit customer $c_j = c_{i+1}$ just after visiting customer c_i then, $\xi_{ijk} = 1$ otherwise, $\xi_{ijk} = 0$. So, the distance traveled or carriage cost and the fixed cost respectively are given as:

$$MinJ_1 = \alpha \sum_{j=1}^{N+L} \sum_{k=1}^{M} \Big(\sum_{i=1}^{E_1} d_{ij}(c_i) + \sum_{i=1}^{L_1} d_{ij}(c_{E_1}^{i}) + i{=}E1{+}1E2{=}N{-}E1\, dij(ci) + i{=}1L2\, dij(cE2i) + \ldots + i{=}1Lndij(cEni) + i{=}En{+}1N{-}Endij(ci)\,\xi ijk \quad (33)$$

$$MinJ_2 = \beta \sum_{k=1}^{M} \sum_{i=0}^{N} \sum_{j=1}^{N} FC(v_k^a)\xi_{ijk} \quad (34)$$

If (18) holds, $FC(v_k^a)$ in (31) becomes

$$FC(v_k^a) = FC(\sum_{k=1}^{M} Q[(v_k^a)] - \sum_{i=1}^{N-n} q[c_i(v_k^a)]) \quad (35a)$$

Or

$$FC(\sum_{a=1}^{A} Q[(v_k^a)] - \sum_{i=1}^{N-n} q[c_i(v_k^a)]) \quad (35b)$$

If (35a) or (35b) is inputted in (34), it gives rise to

$$MinJ_2 = \beta \sum_{k=1}^{M} \sum_{i=0}^{N} \sum_{j=1}^{N} FC(\sum_{k=1}^{M} Q[(v_k^a)] - i{=}1N{-}nqcivka\xi ijk \quad (35c)$$

Or

$$MinJ_2 = \beta \sum_{k=1}^{M} \sum_{i=0}^{N} \sum_{j=1}^{N} FC(\sum_{a=1}^{A} Q[(v_k^a)] - i{=}1N{-}nqcivka)\xi ijk \quad (35d)$$

Or

$$J_2 = \beta \sum_{k=1}^{M} \sum_{i=0}^{N} \sum_{j=1}^{N} FC(\{\sum_{k=1}^{M} Q[(v_k^a)]\} + Qvka{+}1{+}\ldots{-}i{=}1N{-}nqcivka)\xi ijk \quad (35e)$$

Or

$$MinJ_2 = \beta \sum_{k=1}^{M} \sum_{i=0}^{N} \sum_{j=1}^{N} FC(\{\sum_{a=1}^{A} Q[(v_k^a)]\} + Qvk{+}1a{+}\ldots{-}i{=}1N{-}nqcivka)\xi ijk \quad (35f)$$

$$MaxJ_3 = \gamma \sum_{i=0}^{N} \sum_{j=0}^{N} \sum_{k=1}^{M} \delta(c_i)\xi_{ijk} \quad (36)$$

$$MaxJ_4 = \sum_{i=1}^{N+L} \sum_{k=1}^{M} \Big(\sum_{i=1}^{E_1} c_i + \sum_{i=1}^{L_1} c_{E_1}^{i} + i{=}E1{+}1E2{=}N{-}E1\, ci + i{=}1L2\, cE2i + \ldots + i{=}1LncEni + i{=}En{+}1N{-}Enci\xi ijk \quad (37)$$

where α, β, and γ are specified constants for weighting corresponding terms to J_1,J_2, and J_3 as opined by [24], and A stands for any of the set in $\{A, B, \ldots, Z\}$.

The HVRP objective function with priorities which this paper aimed at formulating is the one found on combining all the objectives in (33), (34), and (32) in place of (36) and (37) as:

$$J = MinJ_1 + MinJ_2 + MaxJ_3 + MaxJ_4 \qquad (38)$$

$$J = \alpha \sum_{i=1}^{N+L} \sum_{k=1}^{M} \left(\sum_{i=1}^{E_1} d_{ij}(c_i) + \sum_{i=1}^{L_1} d_{ij}(c_{E_1}^{i}) + \sum_{i=E_1+1}^{E_2=N-E_1} d_{ij}(c_i) + \sum_{i=1}^{L_2} d_{ij}(c_{E_2}^{i}) + \cdots + \sum_{i=1}^{L_n} d_{ij}(c_{E_n}^{i}) + \sum_{i=E_n+1}^{N-E_n} d_{ij}(c_i) \right) \xi_{ijk} + \beta \sum_{i=0}^{N} \sum_{j=0}^{N} \sum_{k=1}^{M} \left(FC \left(\sum_{k=1}^{M} Q[(v_k^a)] - \sum_{i=1}^{N-n} q[c_i(v_k^a)] \right) \right) \xi_{ijk}$$

$$+\gamma \sum_{i=1}^{N} \left(\sum_{a=1}^{4} \sum_{c=1}^{6} PT_a(c_i) \times SD_c(c_i) \right) \xi_{ijk} + \sum_{i=1}^{N+L} \sum_{k=1}^{M} \left(\sum_{i=1}^{E_1} c_i + \sum_{i=1}^{L_1} c_{E_1}^{i} + \sum_{i=E_1+1}^{E_2=N-E_1} c_i + \right.$$

i=1L2cE2i+…+i=1$LncEni$+i=En+1N−$Enci\xi ijk$

(39)

However, when split delivery is involved, we combine the three sub-objectives: (33), only one of (35c), (35d), (35e) or (35f) as appropriate in place of (34), and (36) according to the peculiarity of the problem.

Subject to: $\sum_{i=0}^{N} \sum_{j=1}^{N} \sum_{k=1}^{M} \xi_{ijk} \le 1, \ j = 1, \dots, N$ (40)

$\sum_{i=0}^{N} \sum_{p=1}^{N} \sum_{k=1}^{M} \xi_{ipk} - \sum_{p=1}^{N} \sum_{j=2}^{N} \sum_{k=1}^{M} \xi_{pjk}$ (41)

$\sum_{i=0}^{N} \sum_{j=1}^{N} \sum_{k=1}^{M} q(c_i)\xi_{ijk} \le Q(v_k) \ k = 1, \dots,$ (42)

$\sum_{i=0}^{N} \sum_{j=1}^{N} \sum_{k=1}^{M} t_{ij} \xi_{ijk} \le T_k^f - T_k^s,$ (43)

$\sum_{i=0}^{N} \sum_{j=1}^{N} \sum_{k=1}^{M} \xi_{ijk} \le 1,$ (44)

$y_i - y_j + N \sum_{i=0}^{N} \sum_{j=1}^{N} \sum_{k=1}^{M} \xi_{ijk} \le (N-1)$ (45)

$\xi_{ijk} \in \{0,1\} \ \forall \ i, j, k$ and , $p = 1, \dots, N$ (46)

The following constraints apply:

The constraint in (40) stresses a customer can only be serviced by a vehicle at most once a day. Constraint (41) states that a vehicle that visits a customer must leave the customer's place for another customer or back to the depot. Constraint (42) expresses the quantity a vehicle can carry on each route. However, where $q(c_i) > Q(v_k^a)$, splitting will be necessary. The constraint (43) is the working duration of each vehicle on each route. Constraint (44) states that a vehicle can only be used at most once a day. Where y_i is an arbitrary constant, and by [15] the relation (45) is the sub-tour-elimination condition attached to the Travelling Salesman Problem (TSP) and VRP as opined by [16] and [17]. This is aimed at forcing each route to pass through the depot. Constraint (46) is the integrality conditions.

From the formulated HVRP objective above, should the HVRP aim to determine the priorities alone then, the series in (33), (34), and (37) are set as zero in (39). If the HVRP is aimed at determining the priorities and the costs then, the series (37) will be set as zero. When the target is to calculate the intermittencies, (33), (34), (35c), (35d), (35e), and (35f) are all set at zero in (39) but, if the aim is to compute the variable cost, fixed cost, the priorities, and the intermittencies then, (39) holds.

The formulated HVRP objective function and its operations are shown in the flow chart in Figure 1 below.

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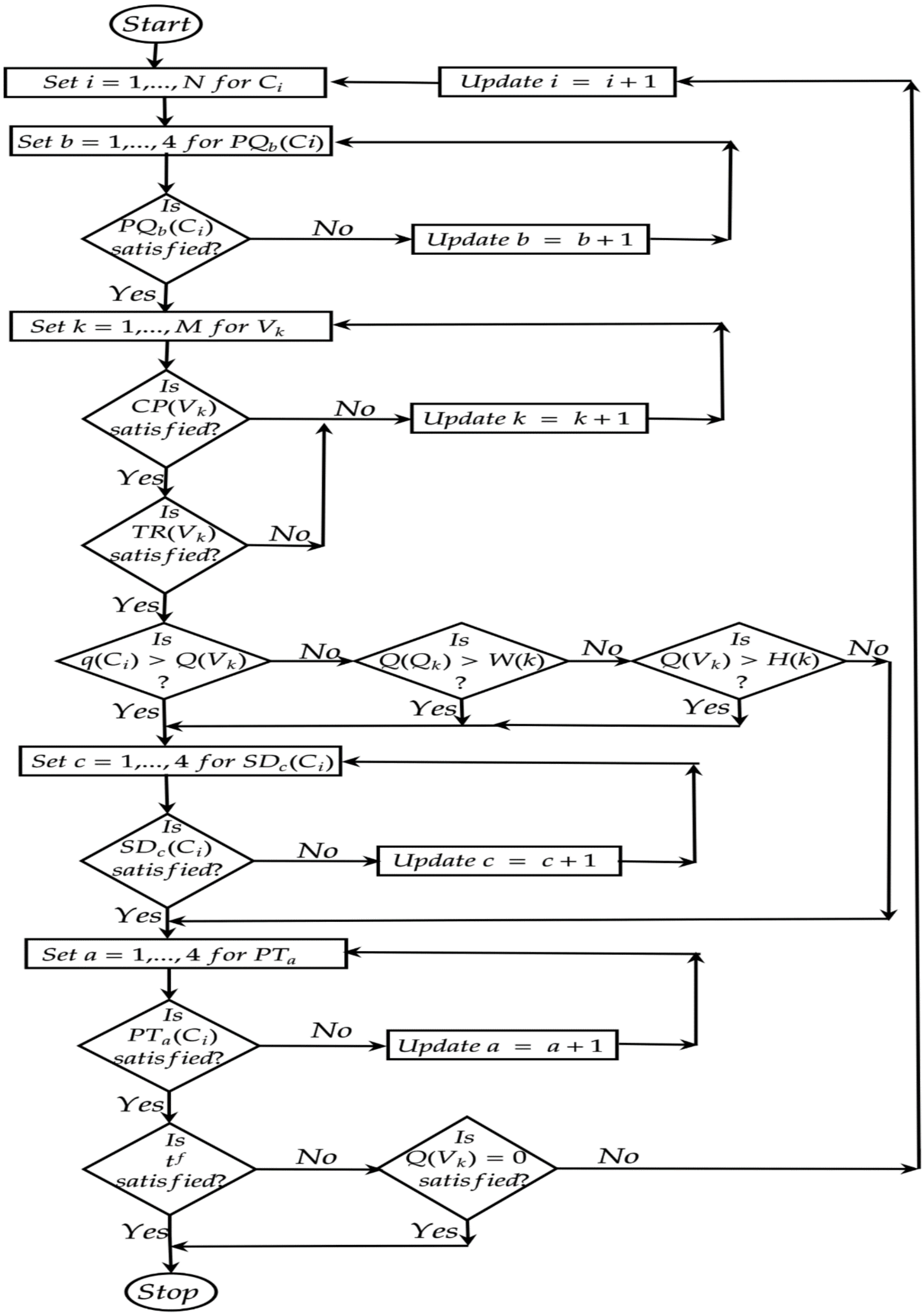

Figure 1:HVRP Flow chart

The chart shows the interconnectivity among the various splitting classifications, the road restrictions, and the classical VRP. All the parameters used have been declared in the previous sections of the work.

However, the central idea behind the HVRP is to assist the dispatch manager in planning the distribution/collection network ahead of time such that a customer gets the desired quantity and is delivered at the said time. It enables timely delivery, vehicle space, and capacity management of the vehicles. With these, it ensures that servicing of customers is based on the priorities such customers earlier set with a view to minimizing both the fixed and variable costs hence, maximizing the profit with the proviso that, should intermittent customers come in between the ERC, such customers' requests are also met without affecting the earlier planned routes, timing, and quantities.

V. CONCLUSION

Real-life situations that are characterized by changes daily have made HVRP with multiple priorities inevitable. As such, rather than a business or organization losing itscustomers to close competitors, more customers will be won hence, increasing the profit margin. The advent and improvement in information technology have greatly contributed to the solution to this class of problem. It has become less difficult due to the use of network facilities, a Global System for Mobile communication, and a Global Positioning System. Otherwise, its attainment would have been a mirage and not feasible.

The vehicle must carry along with it an anticipatory quantity and create room for additional anticipatory time to cover the supply and delivery. There should be information interconnectivity from the depot to customers via the vehicles in the chain.

While investigating HVRP with multiple priorities, randomly generated data will first be used against real-life data. Reasons for these are connected to: firstly, data randomly generated often enables an in-depth analysis. This is because the sets of data can be constructed such that other issues can be taken care of alongside. Secondly, most real-life HVRP with multiple priorities does not capture all the data needed for holistic analyses of the routing problem. The full information about the geographical locations of all the vehicles not known at the time the LRC request is received is one of the missing data items in our day-to-day business activities hence, necessitating randomly generated data.

This paper presents a way out to supply chain management and distribution problems currently besetting businesses all around the world.The formulation and development of aHVRP objective function with forced split deliveries orchestrated by road restrictions emphasize robustness, and efficiency and demonstrates an amplified architectural function thatminimizes the total traveling cost, minimizes the operational cost, maximizes the customers' preferences, and carter for the marginal difference that occur in vehicle carriagecapacity.

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REFERENCES

[1]. Dantzig, G. B. and Ramser, J. H., The Truck Dispatching Problem, Management Science,Vol. 6, No. 1, (1959), pp. 80-91.http://dx.doi.org/10.1287/mnsc.6.1.80
[2]. OECD, 2015. OECD Economic Surveys Indonesia March 2015 Overview, https://www.oecd.org/eco/surveys/Overview-Indonesia-2015.pdf.
[3]. World Economic Forum, The Global Competitiveness Report 2016-2017, <www.weforum.org/gcr
[4]. Li, L.Y.O. and Fu, Z., The school bus routing problem: a case study, Journal of theOperational Research Society, Vol. 53, No. 5, (2002). pp 552-558
[5]. Park, J. and Kim, B.I., The school bus routing problem: A review, European Journal ofOperational Research, Vol. 202, No. 2, (2010). pp. 311-319, https://EconPapers.repec.org/RePEc:eee:ejores:v:202:y:2010:i:2:p:311-319
[6]. Kechagiopoulos, P. N. and Beligiannis, G. N., Solving the Urban Transit Routing ProblemUsing a Particle Swarm Optimization Based Algorithm, Applied Soft Computing, Vol. 21, (2014), pp. 654-676.https://doi.org/10.1016/j.asoc.2014.04.005.
[7]. Adebayo, K. J., Aderibigbe, F.M. and Dele-Rotimi, A. O. On Vehicle Routing Problems (VRP) with a Focus on Multiple Priorities. American Journal of Computational Mathematics, 9 (5): (2019). 348-357, doi.org/10.4236/ajcm.2019.94025.
[8]. Adebayo, K. J. and Aderibigbe, F. M., On Dynamical Situations in Vehicle Routing Problems (DSVRP) with Multiple Priorities. American Journal of Traffic and Transportation Engineering, 6(1): (2021). 1-9. doi: 10.11648/j.ajtte.20210601.11.
[9]. Larsen, A., The dynamic vehicle routing problem. Ph.D. Thesis, Institute of Mathematical Modelling, Technical University of Denmark. (2001). https://www.researchgate.net/publication/260401175_The_Dynamic_Vehicle_Routing_Problem
[10]. Mitrovic-Minic, S., Krishnamurti, R., and Laporte, G., Double-horizon based heuristics for the dynamic pickup and delivery

problem with time windows. Transportation Research Part B: Methodological, 38(8): (2004a). 669–685. https://www.infona.pl/resource/bwmeta1.element.elsevier-b256505e-b4d8-3ec4-82a8-5fcbabfa8125

[11]. Mitrovic-Minic, S. and Laporte, G., Waiting strategies for the dynamic pickup and delivery problem with time windows. Transportation Research Part B: Methodological, 38(7): (2004b), 635–655. https://ideas.repec.org/a/eee/transb/v38y2004i7p635-655.html

[12]. Branke, J., Middendorf, M., Noeth, G., and Dessouky, M., Waiting strategies for dynamic vehicle routing. Transportation Science, 39(3): (2005), 298–312. https://www.jstor.org/stable/25769252

[13]. Pureza, V. and Laporte, G., Waiting and Buffering Strategies for the Dynamic Pickup and Delivery Problem with Time Windows. INFOR, 46(3): (2008), 165–175. https://www.tandfonline.com/doi/abs/10.3138/infor.46.3.165

[14]. Bräysy, O. and Gendreau, M., Vehicle Routing Problem with Time Windows Part II: Metaheuristics. Transportation Science, Vol. 39, No. 1, (February 2005) pp. 119-139, DOI: 10.1287/trsc.1030.0057

[15]. Larsen, A., Madsen, O. B. G. and Solomon, M. M., Partially dynamic vehicle routingmodels and algorithms. Journal of the Operational Research Society, 53(6): (2002), 637–646. https://www.tandfonline.com/doi/abs/10.1057/palgrave.jors.2601352 16

[16]. Ritzinger, U., Puchinger, J. and Hartl, R. F., Dynamic Programming Based Metaheuristics for the dial-a-ride problem. Annals of Operations Research, (2014), 1–18. https://www.researchgate.net/publication/262415779_Dynamic_Programming_based_Metaheuristics_for_the_Dial-a-Ride_Problem

[17]. Ehmke, J. F. and Campbell, A. M., (2014). Customer acceptance mechanisms for home deliveries in metropolitan areas. European Journal of Operational Research, 233(1):193–207, https://www.researchgate.net/publication/270992093_Customer_acceptance_mechanisms_for_home_deliveries_in_metropolitan_areas

[18]. Maxwell, M. S., Restrepo, M., Henderson, S. G., and Topaloglu, H., (2010). Approximate dynamic programming for ambulance redeployment. INFORMS Journal on Computing, 22(2):266–281. https://people.orie.cornell.edu/shane/pubs/ADPforAmb.pdf

[19]. Thomas, B. W., Waiting strategies for anticipating service requests from known customer locations. Transportation Science, 41(3): (2007). 319–331. https://pubsonline.informs.org/doi/10.1287/trsc.1060.0183

[20]. Goodson, J. C., Thomas, B. W. and Ohlmann, J. W., Restocking-based Rollout Policies for the Vehicle Routing Problem with Stochastic Demand and Duration Limits. Transportation Science, 50(2): (2015),591–607. https://pubsonline.informs.org/doi/abs/10.1287/trsc.2015.0591

[21]. Voccia, S. A., Campbell, A. M., and Thomas, B. W., The same-day delivery problem for online purchases. Transportation Science. (2018). https://pubsonline.informs.org/doi/abs/10.1287/trsc.2016.0732

[22]. Brinkmann, J., Ulmer, M. W. and Mattfeld, D. C., Short-term strategies for stochastic inventory routing in bike sharing systems. Transportation Research Procedia, 10: (2015). 364–373. https://www.sciencedirect.com/science/article/pii/S2352146515002732

[23]. Ulmer, M. W., Approximate Dynamic Programming for Dynamic Vehicle Routing. Operations Research/Computer Science Interfaces Series. Springer. (2017), https://www.amazon.com/Approximate-Programming-Operations-Research-Interfaces-ebook/dp/B071DHNR19 http://www.iaeng.org/IJAM/issues_v52/issue_1/IJAM_52_1_14.pdf.

[24]. Fadda, P., Mancini, S., Serra, P., and Fancello., The Heterogeneous Fleet Vehicle Routing Problem with Draft Limits, Computer and Operations Research, (2022),https://doi.org/10.1016/j.cor.2022.106024

[25]. Golden, B., Assad, A., Levy, L., &Gheysens, F. G.,The fleet size and mix vehicle routing problemComputers &Oper. Res., 11, (1984). 49-66.DOI: 10.4236/ajor.2012.22018.

[26]. Hoff, A., Andersson, H., Christiansen, M., Hasle, G., and Lokketangen, A.An Integer Programming Model for Solving

Heterogeneous Vehicle Routing Problem with Hard Time Window considering Service Choice Computers & Operations Research, 37. (12), (2010),2041-2061. doi:10.1088/1757-899X/300/1/012023

[27]. Koc, C., T. Bektas, O. Jabali, G. Laporte., European Journal of Operational Research, 249, (2016),1-21.

[28]. Zainal Azis and Herman Mawengkang, Time-Dependent Heterogeneous Vehicle Routing Problem for Catering Service Delivery Problem, IOP Conf. Series: Journal of Physics: Conf. Series890 (2017) 012103 doi:10.1088/1742-6596/890/1/012103.

[29]. Olateju Samuel Olaniyi, Adebayo Kayode James, Ibrahim Abdullahi Adinoyi and Aderibigbe Felix Makanjuola, On the Application of a Modified Genetic Algorithm for Solving Vehicle Routing Problems with Time Windows and Split Delivery, IAENG International Journal of Applied Mathematics, 52(1), (2022),1-14.http://www.iaeng.org/IJAM/issues_v52/issue_1/IJAM_52_1_14.pdf.

[30]. Adebayo Kayode James, Aderibigbe Felix Makanjuola, Ibrahim Abdullahi Adinoyi, Olateju Samuel Olaniyi, On Formulation of the Vehicle Routing Problems Objective with Focus on Time Windows, Quantities and Split Delivery Priorities, International Journal of Applied Mathematics, 51(3), (2021),p680-687.http://www.iaeng.org/IJAM/issues_v51/issue_3/IJAM_51_3_26.pdf.