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Impact of Characterizations of $L_r(X)$, $U_r(X)$ and $B_r(X)$ In Nano Regular B-Closed Sets In Nano Topological Spaces.

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ABSTRACT

The purpose of this paper is to study the impact of characterizations of Lower approximation space - $L_R(X)$, Upper approximation space - $U_R(X)$ and the Boundary region - $B_R(X)$ in Nano regular b-closed sets (Nrb-closed sets). And also to discuss an application based on the Nano regular b-closed sets (Nrb-closed sets).

Keywords: Lower approximation space, Upper approximation space and the Boundary region, Nano regular b-closed sets (Nrb-closed sets).

Mathematical Subject Classification 2010:54B05,54A05.

I. INTRODUCTION

In 1970,Levine [4] introduced the concept of generalized closed sets in topological space. Andrijevic [1] introduced a new class of generalized open sets namely, b-open sets in 1996.A.Narmadha and N.Nagaveni [5] introduced regular b-closed sets in 2012. In 2013, A.Narmadha, N.Nagaveni and T.Noiri [6] introduced regular b-open sets. The notion of nano topology was introduced by Lellis Thivagar[2] in 2013 and also he established certain weak forms of nano open sets such as nano α -open sets, nano semi-open sets and nano pre open sets. In 1991 Pawlak Z introduced new set theory called rough set theory [7]. And also he discovered some application in rough set theory in 2002 [8]. Later Lellis thivagar introduced a new topology rough topology in terms of lower, upper approximations and boundary. And then the concepts of nano topological basis was applied for finding the deciding factors in data analysis [3]. In 2022 P.Srividhya and T.Indira[9] introduced Nano regular b-open sets and Nanoregular b-closed sets.In this paper we studied the Characterizations of Lower approximation space - $L_R(X)$, Upper approximation space - $U_R(X)$ and the Boundary region - $B_R(X)$ in Nano regular b-closed sets (Nrb-closed sets). to discuss an application based on the Nano regular b-closed sets (Nrb-closed sets). And also an application based on the Nano regular b-closed sets (Nrb-closed sets) was discussed.

II. PRELIMINARIES

Definition: 2.1 [2] Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as indiscernibility relation. Then U is divided into disjoint equivalence classes. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U,R) is said to be the approximation space. Let$X\subseteq U$. Then

- The lower approximation of X with respect to R is the set of all objects which can be for certainly classified as X with respect to R and is denoted by $L_R(X)$.

$$L_R(X) = \bigcup_{x\in U} \{R(x):R(x)\subseteq X\}$$

- The upper approximation of X with respect to R is the set of all objects which can be possibly classified as X with respect to R and is denoted by $U_R(X)$.

$U_R(X) = \bigcup_{x \in U} \{R(x):R(x) \cap X \neq \phi\}$

- The boundary region of X with respect to R is the set of all objects which can be classified neither as X nor as not –X with respect to R and is denoted by $B_R(X)$.

$B_R(X) = U_R(X) - L_R(X)$.

Definition: 2.2 [2] Let U be the universe, R be an equivalence relation on U and $\tau_R(X) = \{U, \phi, L_R(X), U_R(X), B_R(X)\}$ where $X \subseteq U$. Then $\tau_R(X)$ satisfies the following axioms:

- U and $\phi \in \tau_R(X)$.
- The union of the elements of any sub collection of $\tau_R(X)$ is in $\tau_R(X)$.
- The intersection of the elements of any finite sub collection of $\tau_R(X)$ is in $\tau_R(X)$. Then $\tau_R(X)$ is called the nano topology on U with respect to X, $(U, \tau_R(X))$ is called the nano topological space.

Elements of the nano topology are known as nano open sets. The complement of elements of the nano open sets are called as nano closed sets.

Definition: 2.3 [2] If $(U, \tau_R(X))$ is a nano topological space with respect to X where $X \subseteq U$ and if $A \subseteq U$, then

- The nano interior of a set A is defined as the union of all nano open sets contained in A and it is denoted by Nint(A).
- The nano closure of a set A is defined as the intersection of all nano closed sets containing A and it is denoted by Ncl(A).

Definition: 2.4 [9]A subset A of a nano topological spaces $(U, \tau_R(X))$ is said to be "nano regular b-closed" (briefly nano rb-closed) if $Nrcl(A) \subset G$ whenever $A \subset G$ and G is nano b-open in U.

Definition: 2.5[9]The union of all nano regular b-open sets contained in A is nano regular b-interior of A and it is denoted by Nrbint(A).

The intersection of all nano regular b-closed sets containing A is called the nano regular b-closure of A and it is denoted by Nrbcl(A).

Remark:2.6 [2]

In general, Nano open sets = $\{U, \phi, L_R(X), U_R(X), B_R(X)\}$ and Nano closed sets $= \{U, \phi, [L_R(X)]^C, [U_R(X)]^C, [B_R(X)]^C\} = \{U, \phi, U_R(X^C), L_R(X^C), L_R(X) \cup L_R(X^C)\}$.

Proposition: 2.7 [2]

1) Ncl(U) = Nint(U) = U.
2) $Ncl(\phi) = Nint(\phi) = \phi$.
3) $Ncl(L_R(X)) = [B_R(X)]^C = L_R(X) \cup L_R(X^C)$, $Nint(L_R(X)) = L_R(X)$.
4) $Ncl(U_R(X)) = U$, $Nint(U_R(X)) = U_R(X)$.
5) $Ncl(B_R(X)) = [L_R(X)]^C = U_R(X^C)$, $Nint(B_R(X)) = B_R(X)$.

III. IMPACT OF CHARACTERIZATIONS OF $L_R(X)$, $U_R(X)$ AND $B_R(X)$

IN NANO REGULAR b-CLOSED SETS.

In this section we are going to see the impact of characterizations of $L_R(X)$, $U_R(X)$ and $B_R(X)$ in Nrb-closed sets.Before getting the Nrb-closed sets, we have to find nanob-open sets and nano regular closed sets.

From the set U the following subsets may be thepossibilities of being nano b-open sets and nano regular closed sets.

(i) U, ϕ.

(ii) $L_R(X)$, $[L_R(X)]^C$, Subset of $L_R(X)$, Superset of $L_R(X)$ and any non-empty subset A of U which has the non-empty intersection with $L_R(X)$.

(iii) $U_R(X)$, $[U_R(X)]^C$, Subset of $U_R(X)$, Superset of $U_R(X)$ and any non-empty subset A of U which has the non-empty intersection with $U_R(X)$.

(iv) $B_R(X)$, $[B_R(X)]^C$, Subset of $B_R(X)$, Superset of $B_R(X)$ and any non-empty subset A of U which has the non-empty intersection with $B_R(X)$.

Theorem: 3.1 If $L_R(X) = U_R(X) = X$ then U, ϕ, $[B_R(X)]^C$are the only Nrb-closed sets.

Proof: Let $L_R(X) = U_R(X) = X$. Then $\tau_R(X) = \{U, \phi, L_R(X)\}$ are the nano open sets.

Nano closed sets $= \{U, \phi, [L_R(X)]^C\} = \{U, \phi, U_R(X^C)\}$.

To find Nrb-closed set, we have to find the Nano b-open sets and nano regular closed sets.Since every nano open set is nano b-open, we get $\{U, \phi, L_R(X)\}$ are nano b-open sets. Along with them,

(i) Let $A = L_R(X)$, then $Nint(A) = L_R(X)$, $Ncl(A) = U$.

$\therefore Ncl(Nint(A)) \cup Nint(Ncl(A)) = Ncl(L_R(X)) \cup Nint(U) = U \cup U = U$.

$\Rightarrow A \subseteq Ncl(Nint(A)) \cup Nint(Ncl(A)) \therefore A = L_R(X)$ is nano b-open

(ii) Let $A \subset L_R(X)$ and $A \neq \phi$, then $Nint(A) = \phi$, $Ncl(A) = U$.
($\because L_R(X) = U_R(X) \Rightarrow L_R(X^C) = U_R(X^C) \Rightarrow L_R(X) \not\subseteq U_R(X^C)$).
$\therefore Ncl(Nint(A)) \cup Nint(Ncl(A)) = Ncl(\phi) \cup Nint(U) = \phi \cup U = U$.
$\Rightarrow A \subseteq Ncl(Nint(A)) \cup Nint(Ncl(A))$
$\therefore A \subset L_R(X)$ is nano b-open$\Rightarrow$ Any subset of $L_R(X)$ is nano b-open.
(iii) Let $A \neq U$, $A \supset L_R(X)$, then $Nint(A) = L_R(X)$, $Ncl(A) = U$.
$\therefore Ncl(Nint(A)) \cup Nint(Ncl(A)) = Ncl(L_R(X)) \cup Nint(U) = U \cup U = U$.
$\Rightarrow A \subseteq Ncl(Nint(A)) \cup Nint(Ncl(A))$.
$\therefore A \supset L_R(X)$ is nano b-open$\Rightarrow$ Any superset of $L_R(X)$ is nano b-open.
(iv) Let A be the subset of U which has the non-empty intersection with $L_R(X)$.
(i.e) $L_R(X) \cap A \neq \phi$. Then $Nint(A) = \phi$, $Ncl(A) = U$.
$\therefore Ncl(Nint(A)) \cup Nint(Ncl(A)) = Ncl(\phi) \cup Nint(U) = \phi \cup U = U$.
$\Rightarrow A \subseteq U \Rightarrow A \subseteq Ncl(Nint(A)) \cup Nint(Ncl(A))$. $\therefore A \cap L_R(X) \neq \phi$ is nano b-open
(v) Let $A = [L_R(X)]^C$, then $Nint(A) = \phi$, $Ncl(A) = [L_R(X)]^C = U_R(X^C)$.
$\therefore Ncl(Nint(A)) \cup Nint(Ncl(A)) = Ncl(\phi) \cup Nint([L_R(X)]^C) = \phi \cup \phi = \phi$.
$\Rightarrow A \not\subseteq Ncl(Nint(A)) \cup Nint(Ncl(A))$.$\therefore A = [L_R(X)]^C$ is not a nano b-open
Here $L_R(X) = U_R(X)$, therefore from (i), (ii), (iii), (iv) and (v) we get,
(vi) $A = U_R(X)$ is nano b-open
(vii) $A \subset U_R(X)$ is nano b-open $\Rightarrow$ Any subset of $U_R(X)$ is nano b-open.
(viii) $A \supset U_R(X)$ is nano b-open$\Rightarrow$ Any superset of $U_R(X)$ is nano b-open.
(ix) Let A be the subset of U which has the non-empty intersection with $U_R(X)$.
(i.e) $U_R(X) \cap A \neq \phi$. $\therefore A \cap U_R(X) \neq \phi$ is nano b-open
(x) $A = [U_R(X)]^C$ is not a nano b-open
(xi) Let $A = B_R(X) = \phi$, then $Nint(A) = \phi$, $Ncl(A) = \phi$.
$\therefore Ncl(Nint(A)) \cup Nint(Ncl(A)) = Ncl(\phi) \cup Nint(\phi) = \phi \cup \phi = \phi$.
$\Rightarrow A \not\subseteq Ncl(Nint(A)) \cup Nint(Ncl(A))$.$\therefore A = B_R(X)$ is not a nano b-open
(xii) Let $A \subset B_R(X)$, Since $B_R(X) = \phi$, there is no subset for ϕ. So this case cannot be defined.
(xiii) Let $A \neq U$, $A \supset B_R(X)$, here $B_R(X) = \phi$ therefore the supersets of $B_R(X)$ are the remaining subsets of U except ϕ. For these sets the Ncl and Nint will be the corresponding Ncl and Nint of thatsupersets.$\therefore A \supset B_R(X)$ is nano b-open whether its superset is nano b-open.
$\Rightarrow$ Any superset of $B_R(X)$ is nano b-open based on its super set.
(xiv) Let A be the subset of U which has the non-empty intersection with $B_R(X)$.
(i.e) $B_R(X) \cap A \neq \phi$. Here $B_R(X) = \phi$,
$\therefore B_R(X) \cap A = \phi$ which is a contradiction to our assumption that A be the subset of U which has the non-empty intersection with $B_R(X)$.So this case cannot be defined.
(xv) Let $A = [B_R(X)]^C = [\phi]^C = U$, then $Nint(A) = U$, $Ncl(A) = U$.
$\therefore Ncl(Nint(A)) \cup Nint(Ncl(A)) = Ncl(U) \cup Nint(U) = U \cup U = U$.
$\Rightarrow A \subseteq Ncl(Nint(A)) \cup Nint(Ncl(A))$.$\therefore A = [B_R(X)]^C$ is nano b-open
From the above cases, we get $A = L_R(X)$, $A \subset L_R(X)$, $A \supset L_R(X)$, $A \cap L_R(X) \neq \phi$, $A = U_R(X)$, $A \subset U_R(X)$, $A \supset U_R(X)$, $A \cap U_R(X) \neq \phi$ and $A = [B_R(X)]^C$ are the nanob-open sets.
$\therefore$ {U, ϕ, $L_R(X)$, $A \subset L_R(X)$, $A \supset L_R(X)$, any set $A \cap L_R(X) \neq \phi$, $U_R(X)$, $A \subset U_R(X)$, $A \supset U_R(X)$, any set $A \cap U_R(X) \neq \phi$, $[B_R(X)]^C$}are the only nanob-open sets.

To find Nano regular closed sets, We know that if $A = Ncl(Nint(A))$ then A is Nano regular closed.
(i) Let $A = L_R(X)$, then $Ncl(Nint(L_R(X)) = Ncl(L_R(X)) = U \neq A$.
$\therefore A = L_R(X)$ is not a nano regular closed.
(ii) Let $A \subset L_R(X)$ and $A \neq \phi$, then $Ncl(Nint(A)) = Ncl(\phi) = \phi \neq A$.
$\therefore A \subset L_R(X)$ is not a nano regular closed.
$\Rightarrow$ Any subset of $L_R(X)$ is not a nano regular closed.
(iii) Let $A \supset L_R(X)$ and $A \neq U$, then $Ncl(Nint(A)) = Ncl(L_R(X)) = U \neq A$.
$\therefore A \supset L_R(X)$ is not a nano regular closed.

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⇒ Any superset of $L_R(X)$ is not a nano regular closed.
(iv) Let A be the subset of U which has the non-empty intersection with $L_R(X)$.
(i.e) $L_R(X) \cap A \neq \phi$. Then $Ncl(Nint(A)) = Ncl(\phi) = \phi \neq A$.
∴ $A \cap L_R(X) \neq \phi$ is not a nano regular closed.
(v) Let $A = [L_R(X)]^C$, then $Ncl(Nint([L_R(X)]^C) = Ncl(\phi) = \phi \neq A$.
∴$A = [U_R(X)]^C$ is not a nano regular closed.
Here $L_R(X) = U_R(X)$, therefore from (i), (ii), (iii), (iv) and (v) we get,
(vi) $A = U_R(X)$ is not a nano regular closed.
(vii) $A \subset U_R(X)$ is not a nano regular closed.
⇒ Any subset of $U_R(X)$ is not a nano regular closed.
(viii) $A \supset U_R(X)$ is not a nano regular closed.
⇒ Any superset of $U_R(X)$ is not a nano regular closed.
(ix) Let A be the subset of U which has the non-empty intersection with $U_R(X)$.
(i.e) $U_R(X) \cap A \neq \phi$. ∴$A \cap L_R(X) \neq \phi$ is not a nano regular closed.
(x) $A = [U_R(X)]^C$ is not a nano regular closed.
(xi) Let $A = B_R(X)$, Here $B_R(X) = \phi$, then $Ncl(Nint(B_R(X)) = Ncl(\phi) = \phi \neq A$.
∴ $A = B_R(X)$ is not a nano regular closed.
(xii) Let $A \subset B_R(X)$, Since $B_R(X) = \phi$, there is no subset for ϕ.
So this case cannot be defined.
(xiii) Let $A \neq U$, $A \supset B_R(X)$, here $B_R(X) = \phi$ therefore the supersets of $B_R(X)$ are the remaining subsets of U except ϕ.
For these sets the Ncl and Nint will be the corresponding Ncl and Nint of that supersets.
∴$A \supset B_R(X)$ is nano regular closed whether its superset is nano regular closed.
⇒ Any superset of $B_R(X)$ is nano regular closed based on its super set.
(xiv) Let A be the subset of U which has the non-empty intersection with $B_R(X)$.
(i.e) $B_R(X) \cap A \neq \phi$. Here $B_R(X) = \phi$,
∴ $B_R(X) \cap A = \phi$ which is a contradiction to our assumption that A be the subset of U which has the non-empty intersection with $B_R(X)$. So this case cannot be defined.
∴ $A \cap B_R(X) \neq \phi$ is not a nano regular closed.
(xv) Let $A = [B_R(X)]^C$, then $Ncl(Nint([B_R(X)]^C) = Ncl(U) = U = A$.
Since $B_R(X) = \phi$ then $[B_R(X)]^C = [\phi]^C = U$
∴$A = [B_R(X)]^C$ is a nano regular closed.
Therefore U, ϕ, $[B_R(X)]^C$ are the only nano regular closed sets.
By the definition of Nrb-closed set, A subset A ofU is said to be Nrb-closed if $Nrcl(A) \subseteq G$ whenever $A \subseteq G$ and G is nano b-open in U.
For $A = [B_R(X)]^C$, $Nrcl(A) = [B_R(X)]^C = U$.
Here $Nrcl(A) = U$ for every non-emptysubset A, since U and ϕ are the only nano regular closed sets.
∴ $Nrcl(A) \not\subset G$ for every non-empty subset A of U except Uand $[B_R(X)]^C$.
Hence $\{U, \phi, [B_R(X)]^C\}$ are the only Nrb-closed sets.

Theorem: 3.2 If $L_R(X) = \phi$, $U_R(X) = U$, then U, ϕ are the only Nrb-closed sets.
Proof: If $L_R(X) = \phi$, $U_R(X) = U$, then the nano topology becomes $\{U, \phi\}$.
∴The proof is obvious.

Theorem: 3.3 If $L_R(X) = U_R(X) = U$, then U, ϕ are the only Nrb-closed sets.
Proof: If $L_R(X) = U_R(X) = U$, then the nano topology $\tau_R(X) = \{U, \phi\}$.
∴The proof is obvious.

Theorem: 3.4 If $L_R(X) = \phi$, $U_R(X) \neq \phi$, then U, ϕ are the only Nrb-closed sets.
Proof: If $L_R(X) = \phi$, $U_R(X) \neq \phi$ then $\tau_R(X) = \{U, \phi, U_R(X)\}$ = Nano open sets.
Nano closed sets = $\{U, \phi, [U_R(X)]^C\} = \{U, \phi, L_R(X^C)\}$.
Since every nano open set is nano b-open, we get $\{U, \phi, U_R(X)\}$are nano b-open sets.
Along with them,
(i) Let $A = L_R(X) = \phi$, then $Nint(A) = \phi$, $Ncl(A) = \phi$.
∴$Ncl(Nint(A)) \cup Nint(Ncl(A)) = Ncl(\phi) \cup Nint(\phi) = \phi \cup \phi = \phi$.
⇒$A \not\subseteq Ncl(Nint(A)) \cup Nint(Ncl(A))$.∴$A = L_R(X)$ is not a nano b-open

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(ii) Let $A \subset L_R(X)$, Since $L_R(X) = \phi$, there is no subset for ϕ. So this case cannot be defined.

(iii) Let $A \neq U$, $A \supset L_R(X)$, here $L_R(X) = \phi$ therefore the supersets of $L_R(X)$ are the remaining subsets of U except ϕ. For these sets the Ncl and Nint will be the corresponding Ncl and Nint of that supersets.$\therefore A \supset L_R(X)$ is nano b-open whether its superset is nano b-open.

$\Rightarrow$ Any superset of $L_R(X)$ is nano b-open based on its super set.

(iv) Let A be the subset of U which has the non-empty intersection with $L_R(X)$.

(i.e) $L_R(X) \cap A \neq \phi$. Here $L_R(X) = \phi$, $\therefore L_R(X) \cap A = \phi$ which is a contradiction to our assumption that A be the subset of U which has the non-empty intersection with $L_R(X)$. So this case cannot be defined.

(v) Let $A = [L_R(X)]^C = [\phi]^C = U$, then $Nint(A) = U$, $Ncl(A) = U$.

$\therefore Ncl(Nint(A)) \cup Nint(Ncl(A)) = Ncl(U) \cup Nint(U) = U \cup U = U$.

$\Rightarrow A \subseteq Ncl(Nint(A)) \cup Nint(Ncl(A)).\therefore A = [L_R(X)]^C$ is nano b-open

(vi) Let $A = U_R(X)$, then $Nint(A) = U_R(X)$, $Ncl(A) = U$.

$\therefore Ncl(Nint(A)) \cup Nint(Ncl(A)) = Ncl(U_R(X)) \cup Nint(U) = U \cup U = U$.

$\Rightarrow A \subseteq Ncl(Nint(A)) \cup Nint(Ncl(A)).\therefore A = U_R(X)$ is nano b-open

(vii) Let $A \subset U_R(X)$ and $A \neq \phi$, then $Nint(A) = \phi$, $Ncl(A) = U$.

$\therefore Ncl(Nint(A)) \cup NintNcl(A)) = Ncl(\phi) \cup Nint(U) = \phi \cup U = U$.

$\Rightarrow A \subseteq Ncl(Nint(A) \cup Nint(Ncl(A))$.

$\therefore A \subset U_R(X)$ is nano b-open $\Rightarrow$ Any subset of $U_R(X)$ is nano b-open.

(viii) Let $A \neq U$, $A \supset U_R(X)$, then $Nint(A) = U_R(X)$, $Ncl(A) = U$.

$\therefore Ncl(Nint(A)) \cup Nint(Ncl(A)) = Ncl(U_R(X)) \cup Nint(U) = U \cup U = U$.

$\Rightarrow A \subseteq Ncl(Nint(A)) \cup Nint(Ncl(A))$.

$\therefore A \supset U_R(X)$ is nano b-open $\Rightarrow$ Any superset of $U_R(X)$ is nano b-open.

(ix) Let A be the subset of U which has the non-empty intersection with $U_R(X)$. (i.e) $U_R(X) \cap A \neq \phi$. Then $Nint(A) = \phi$, $Ncl(A) = U$.

$\therefore Ncl(Nint(A)) \cup Nint(Ncl(A)) = Ncl(\phi) \cup Nint(U) = \phi \cup U = U$.

$\Rightarrow A \subseteq Ncl(Nint(A)) \cup Nint(Ncl(A)).\therefore A \cap U_R(X) \neq \phi$ is nano b-open

(x) Let $A = [U_R(X)]^C$, then $Nint(A) = \phi$, $Ncl(A) = [U_R(X)]^C = L_R(X^C)$.

$\therefore Ncl(Nint(A)) \cup Nint(Ncl(A)) = Ncl(\phi) \cup Nint([U_R(X)]^C) = \phi \cup \phi = \phi$.

$\Rightarrow A \not\subseteq Ncl(Nint(A)) \cup Nint(Ncl(A)).\therefore A = [U_R(X)]^C$ is not a nano b-open

Here $U_R(X) = B_R(X)$, therefore from (vi), (vii), (viii), (ix) and (x) we get,

(xi) $A = B_R(X)$ is nano b-open.

(xii) $A \subset B_R(X)$ is nano b-open $\Rightarrow$ Any subset of $B_R(X)$ is nano b-open.

(xiii) $A \supset B_R(X)$ is nano b-open $\Rightarrow$ Any superset of $B_R(X)$ is nano b-open.

(xiv) Let A be the subset of U which has the non-empty intersection with $B_R(X)$.

$\therefore A \cap B_R(X) \neq \phi$ is nano b-open

(xv) $A = [B_R(X)]^C$ is not a nano b-open

From the above cases, we get $A = [L_R(X)]^C$, $A = U_R(X)$, $A \subset U_R(X)$, $A \supset U_R(X)$, $A \cap U_R(X) \neq \phi$, $A = B_R(X)$, $A \subset B_R(X)$, $A \supset B_R(X)$ and $A \cap B_R(X) \neq \phi$ are the nanob-open sets.

$\therefore$ $\{U, \phi$, $A = [L_R(X)]^C$, $A = U_R(X)$, $A \subset U_R(X)$, $A \supset U_R(X)$, $A \cap U_R(X) \neq \phi$, $A = B_R(X)$, $A \subset B_R(X)$, $A \supset B_R(X)$ and $A \cap B_R(X) \neq \phi\}$ are the only nano b-open sets.

(i) Let $A = L_R(X)$, then $Ncl(Nint(L_R(X)) = Ncl(\phi) = \phi \neq A$.

$\therefore A = L_R(X)$ is not a nano regular closed.

(ii) Let $A \subset L_R(X)$, Since $L_R(X) = \phi$, there is no subset for ϕ. So this case cannot be defined.

(iii) Let $A \neq U$, $A \supset L_R(X)$, here $L_R(X) = \phi$ therefore the supersets of $L_R(X)$ are the remaining subsets of U except ϕ. For these sets the Ncl and Nint will be the corresponding Ncl and Nint of that supersets.

$\therefore A \supset L_R(X)$ is nano regular closed whether its superset is nano regular closed.

$\Rightarrow$ Any superset of $L_R(X)$ is nano regular closed based on its super set.

(iv) Let A be the subset of U which has the non-empty intersection with $L_R(X)$.

(i.e) $L_R(X) \cap A \neq \phi$. Here $L_R(X) = \phi$,

$\therefore L_R(X) \cap A = \phi$ which is a contradiction to our assumption that A be the subset of
which has the non-empty intersection with $B_R(X)$. So this case cannot be defined.

$\therefore A \cap L_R(X) \neq \phi$ is not a nano regular closed.

(v) Let $A = [L_R(X)]^C$, then $Ncl(Nint([L_R(X)]^C) = Ncl(U) = U = A$.

Since $L_R(X) = \phi$ then $[L_R(X)]^C = [\phi]^C = U$. $\therefore A = [L_R(X)]^C$ is a nano regular closed.

(vi) Let $A = U_R(X)$, then $Ncl(Nint(A)) = Ncl(U_R(X)) = U \neq A$.

$\therefore A = U_R(X)$ is not a nano regular closed.

(vii) Let $A \subset U_R(X)$ and $A \neq \phi$, then $Ncl(Nint(A)) = Ncl(\phi) = \phi \neq A$. $\therefore A \subset U_R(X)$ is not a nano regular closed $\Rightarrow$ Any subset of $U_R(X)$ is not a nano regular closed.

(viii) Let $A \neq U$, $A \supset U_R(X)$, then$Ncl(Nint(A)) = Ncl(U_R(X)) = U \neq A$. $\therefore A \supset U_R(X)$ is not a nano regular closed. $\Rightarrow$ Any superset of $U_R(X)$ is not a nano regular closed.

(ix) Let A be the subset of U which has the non-empty intersection with $U_R(X)$.

(i.e) $U_R(X) \cap A \neq \phi$. Then$Ncl(Nint(A)) = Ncl(\phi) = \phi \neq A$.

$\therefore A \cap U_R(X) \neq \phi$ is not a nano regular closed.

(x) Let $A = [U_R(X)]^C$, then $Ncl(Nint(A)) = Ncl(\phi) = \phi \neq A$.

$\therefore A = [U_R(X)]^C$ is not a nano regular closed.

Here $U_R(X) = B_R(X)$, therefore from (vi), (vii), (viii), (ix) and (x) we get,

(xi) $A = B_R(X)$ is not a nano regular closed.

(xii) $A \subset B_R(X)$ is not a nano regular closed.

$\Rightarrow$ Any subset of $B_R(X)$ is not a nano regular closed.

(xiii) $A \supset B_R(X)$ is not a nano regular closed.

$\Rightarrow$ Any superset of $B_R(X)$ is not a nano regular closed.

(xiv) Let A be the subset of U which has the non-empty intersection with $B_R(X)$.

(i.e) $B_R(X) \cap A \neq \phi$. $\therefore A \cap B_R(X) \neq \phi$ is not a nano regular closed.

(xv) $A = [B_R(X)]^C$ is not a nano regular closed.

Therefore U, ϕ , $[L_R(X)]^C$ are the only nano regular closed sets.

Here $Nrcl(A) = U$ for every non-empty subset A except $[L_R(X)]^C$, since U and ϕ are the only nano regular closed sets.

For $A = [L_R(X)]^C$, $Nrcl(A) = [L_R(X)]^C$.But we know that $[L_R(X)]^C = U$.

$\therefore Nrcl(A) \not\subset G$ for every non-empty subset A of U except U and $[L_R(X)]^C$.

Hence $\{U, \phi\}$ are the only Nrb-closed sets.

Theorem: 3.5

If $L_R(X) \neq \phi$, $U_R(X) = U$ and $B_R(X) \neq \phi$, then U, ϕ , $[L_R(X)]^C$ and $L_R(X)$ are the only Nrb-closed sets.

Proof:

If $L_R(X) \neq \phi$, $U_R(X) = U$, then $\tau_R(X) = \{U, \phi$, $L_R(X), B_R(X)\}$ = Nano open sets, Nano closed sets $= \{U, \phi, [L_R(X)]^C, [B_R(X)]^C\} = \{U, U_R(X^C), L_R(X) \cup L_R(X^C)\}$.

Since every nano open set is nano b-open, so we have $\{U, \phi$, $L_R(X), B_R(X)\}$ as nano b-open sets. Along with them,

(i) Let $A = L_R(X)$, then $Nint(A) = L_R(X)$, $Ncl(A) = [B_R(X)]^C$.

$\therefore Ncl(Nint(A)) \cup Nint(Ncl(A)) = Ncl(L_R(X)) \cup Nint([B_R(X)]^C)$

$= [B_R(X)]^C \cup [B_R(X)]^C = [B_R(X)]^C$.

$\Rightarrow A \subseteq Ncl(Nint(A)) \cup Nint(Ncl(A))$.

$\therefore A = L_R(X)$ is nano b-open

(ii) Let $A \subset L_R(X)$ and $A \neq \phi$, then $Nint(A) = \phi$, $Ncl(A) = [B_R(X)]^C$.

$\therefore Ncl(Nint(A)) \cup Nint(Ncl(A)) = Ncl(\phi) \cup Nint([B_R(X)]^C) = \phi \cup L_R(X) = L_R(X)$.

$\Rightarrow A \subseteq Ncl(Nint(A)) \cup Nint(Ncl(A))$.

$\therefore A \subset L_R(X)$ is nano b-open

$\Rightarrow$ Any subset of $L_R(X)$ is nano b-open.

(iii) Let $A \neq U$, $A \supset L_R(X)$, then $Nint(A) = L_R(X)$, $Ncl(A) = U$.

$\therefore Ncl(Nint(A)) \cup Nint(Ncl(A)) = Ncl(L_R(X) \cup Nint(U) = [B_R(X)]^C \cup U = U$.

$\Rightarrow A \subseteq Ncl(Nint(A)) \cup Nint(Ncl(A))$.

$\therefore A \supset L_R(X)$ is nano b-open

$\Rightarrow$ Any superset of $L_R(X)$ is nano b-open.

(iv) Let A be the subset of U which has the non-empty intersection with $L_R(X)$. (i.e) $L_R(X) \cap A \neq \phi$. Then $Nint(A) = \phi$, $Ncl(A) = U$.

$\therefore Ncl(Nint(A)) \cup Nint(Ncl(A)) = Ncl(\phi) \cup Nint(U) = \phi \cup U = U$.

$\Rightarrow A \subseteq Ncl(Nint(A)) \cup Nint(Ncl(A))$.

$\therefore A \cap L_R(X) \neq \phi$ is nano b-open

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(v) Let $A = [L_R(X)]^C = B_R(X) = U_R(X^C)$, then $Nint(A) = B_R(X)$, $Ncl(A) = U_R(X^C)$.
$\therefore Ncl(Nint(A)) \cup Nint(Ncl(A)) = Ncl\ (B_R(X)) \cup Nint(U_R(X^C))$
$= U_R(X^C) \cup U_R(X^C) = U_R(X^C)$.
$\Rightarrow A \subseteq Ncl(Nint(A)) \cup Nint(Ncl(A))$.
$\therefore A = [L_R(X)]^C$ is nano b-open
(vi) Let $A = U_R(X) = U$, then $Nint(A) = U$, $Ncl(A) = U$.
$\therefore Ncl(Nint(A)) \cup Nint(Ncl(A)) = Ncl\ (U) \cup Nint(U) = U \cup U = U$.
$\Rightarrow A \subseteq Ncl(Nint(A)) \cup Nint(Ncl(A))$.
$\therefore A = U_R(X)$ is nano b-open
(vii) Let $A \subset U_R(X)$. Here $U_R(X) = U$, therefore the subsets of $U_R(X)$ are the possible subsets of U. For these sets the Ncl and Nint will be the corresponding Ncl and Nint of that subsets.
$\therefore A \subset U_R(X)$ is nano b-open whether its superset is nano b-open.
$\Rightarrow$ Any subset of $U_R(X)$ is nano b-open based on its super set.
(viii) Let $A \neq \phi$, $A \supset U_R(X)$, Since $U_R(X) = U$, there is no superset for U. Sothis case cannot be defined.
(ix) Let A be the subset of U which has the non-empty intersection with $U_R(X)$.
(i.e) $U_R(X) \cap A \neq \phi$. Here $L_R(X) = U$, therefore all the possible subsets of U have the non-empty intersection with $U_R(X)$.
For these sets the Ncl and Nint will be the corresponding Ncl and Nint of that subsets.
$\therefore U_R(X) \cap A = \phi$ is nano b-open whether its subset is nano b-open.
(x) Let $A = [U_R(X)]^C = [U]^C = \phi$, then $Nint(A) = \phi$, $Ncl(A) = \phi$.
$\therefore Ncl(Nint(A)) \cup Nint(Ncl(A)) = Ncl\ (\phi) \cup Nint(\phi) = \phi \cup \phi = \phi$.
$\Rightarrow A \subseteq Ncl(Nint(A)) \cup Nint(Ncl(A))$.
$\therefore A = [U_R(X)]^C$ is nano b-open
(xi) Let $A = B_R(X) = U_R(X^C)$, then $Nint(A) = B_R(X)$, $Ncl(A) = U_R(X^C)$.
$\therefore Ncl(Nint(A)) \cup Nint(Ncl(A)) = Ncl\ (B_R(X)) \cup Nint(U_R(X^C))$
$= U_R(X^C) \cup U_R(X^C) = U_R(X^C)$.
$\Rightarrow A \subseteq Ncl(Nint(A)) \cup Nint(Ncl(A))$.
$\therefore A = B_R(X)$ is nano b-open
(xii) Let $A \subset B_R(X)$ and $A \neq \phi$, then $Nint(A) = \phi$, $Ncl(A) = U_R(X^C) = [L_R(X)]^C$.
$\therefore Ncl(Nint(A)) \cup Nint(Ncl(A)) = Ncl(\phi) \cup Nint([L_R(X)]^C) = \phi \cup B_R(X) = B_R(X)$.
$\Rightarrow A \subseteq Ncl(Nint(A)) \cup Nint(Ncl(A))$.
$\therefore A \subset B_R(X)$ is nano b-open
$\Rightarrow$ Any subset of $B_R(X)$ is nano b-open.
(xiii) Let $A \neq U$, $A \supset B_R(X)$, then $Nint(A) = B_R(X)$, $Ncl(A) = U$.
$\therefore Ncl(Nint(A)) \cup Nint(Ncl(A)) = Ncl(B_R(X) \cup Nint(U) = [L_R(X)]^C \cup U = U$.
$\Rightarrow A \subseteq Ncl(Nint(A)) \cup Nint(Ncl(A))$.
$\therefore A \supset B_R(X)$ is nano b-open
$\Rightarrow$ Any superset of $B_R(X)$ is nano b-open.
(xiv) Let A be the subset of U which has the non-empty intersection with $B_R(X)$. (i.e) $B_R(X) \cap A \neq \phi$. Then $Nint(A) = \phi$, $Ncl(A) = U$.
$\therefore Ncl(Nint(A)) \cup Nint(Ncl(A)) = Ncl(\phi) \cup Nint(U) = \phi \cup U = U$.
$\Rightarrow A \subseteq Ncl(Nint(A)) \cup Nint(Ncl(A))$.
$\therefore A \cap B_R(X) \neq \phi$ is nano b-open

(xv) Let $A = [B_R(X)]^C = L_R(X)$, then $Nint(A) = L_R(X)$, $Ncl(A) = [B_R(X)]^C$.
$\therefore Ncl(Nint(A)) \cup Nint(Ncl(A)) = Ncl\ (L_R(X)) \cup Nint([B_R(X)]^C)$
$= [B_R(X)]^C \cup [B_R(X)]^C = [B_R(X)]^C$.
$\Rightarrow A \subseteq Ncl(Nint(A)) \cup Nint(Ncl(A))$.
$\therefore A = [B_R(X)]^C$ nano b-open
From the above cases, we get $A = L_R(X)$, $A \subset L_R(X)$, $A \supset L_R(X)$, $A \cap L_R(X) \neq \phi$, $A = [L_R(X)]^C$, $A = U_R(X)$, $A = [U_R(X)]^C$, $A = B_R(X)$, $A \subset B_R(X)$, $A \supset B_R(X)$, $A \cap B_R(X) \neq \phi$ and $A = [B_R(X)]^C$ are the nano b-open sets.
$\therefore$ {U, ϕ, $A = L_R(X)$, $A \subset L_R(X)$, $A \supset L_R(X)$, $A \cap L_R(X) \neq \phi$, $A = [L_R(X)]^C$, $A = U_R(X)$, $A = B_R(X)$, $A \subset B_R(X)$, $A \supset B_R(X)$, $A \cap B_R(X) \neq \phi$, $A = [B_R(X)]^C$} are the only nano b-open sets.
(i) Let $A = L_R(X)$, then $Ncl(Nint(L_R(X)) = Ncl(L_R(X)) = [B_R(X)]^C = L_R(X) = A$
$(\because [B_R(X)]^C = L_R(X) \cup L_R(X^C)$.
$\therefore A = L_R(X)$ is nano regular closed.
(ii) Let $A \subset L_R(X)$ and $A \neq \phi$, then $Ncl(Nint(A)) = Ncl(\phi) = \phi \neq A$.
$\therefore A \subset L_R(X)$ is not a nano regular closed.
$\Rightarrow$ Any subset of $L_R(X)$ is not a nano regular closed.

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(iii) Let $A \supset L_R(X)$ and $A \neq U$, then $Ncl(Nint(A)) = Ncl(L_R(X)) = [B_R(X)]^C = L_R(X) \cup L_R(X^C) \neq A$.
$\therefore A \supset L_R(X)$ is not a nano regular closed.
$\Rightarrow$ Any superset of $L_R(X)$ is not a nano regular closed.
(iv) Let A be the subset of U which has the non-empty intersection with $L_R(X)$.
(i.e) $L_R(X) \cap A \neq \phi$. Then $Ncl(Nint(A)) = Ncl(\phi) = \phi \neq A$.
$\therefore$ $A \cap L_R(X) \neq \phi$ is not a nano regular closed.
(v) Let $A = [L_R(X)]^C$, then $Ncl(Nint([L_R(X)]^C) = Ncl(B_R(X)) = U_R(X^C) = A$.
Since $[L_R(X)]^C = U_R(X^C)$.
$\therefore A = [L_R(X)]^C$ is nano regular closed.
(vi) Let $A = U_R(X) = U$, then $Ncl(Nint(A)) = Ncl(U) = U$.
$\therefore A = U_R(X)$ is nano regular closed.
(vii) Let $A \subset U_R(X)$. Here $U_R(X) = U$, therefore the subsets of $U_R(X)$ are the possible subsets of U.
For these sets the Ncl and Nint will be the corresponding Ncl and Nint of that subsets.
$\therefore A \subset U_R(X)$ is nano regular closed whether its superset is nano regular closed.
$\Rightarrow$ Any subset of $U_R(X)$ is nano regular closed based on its super set.
(viii) Let $A \neq \phi$, $A \supset U_R(X)$, Since $U_R(X) = U$, there is no superset for U. So this case cannot be defined.
(ix) Let A be the subset of U which has the non-empty intersection with $U_R(X)$.
(i.e) $U_R(X) \cap A \neq \phi$. Here $L_R(X) = U$, therefore all the possible subsets of U have the non-empty intersection with $U_R(X)$.
For these sets the Ncl and Nint will be the corresponding Ncl and Nint of that subsets.
$\therefore$ $U_R(X) \cap A = \phi$ is nano regular closed whether its subset is nano regular closed.
(x) Let $A = [U_R(X)]^C = [U]^C = \phi$, thenNcl(Nint(A)) = $Ncl(\phi) = \phi = A$.
$\therefore A = [U_R(X)]^C$ is nano regular closed.
(xi) Let $A = B_R(X)$, then $Ncl(Nint(B_R(X)) = Ncl(B_R(X)) = [L_R(X)]^C = A$.
Since $[L_R(X)]^C = B_R(X)$.
$\therefore$ $A = B_R(X)$ is nano regular closed.
(xii) Let $A \subset B_R(X)$, then $Ncl(Nint(A)) = Ncl(\phi) = \phi \neq A$.
$\therefore A \subset B_R(X)$ is not a nano regular closed.
$\Rightarrow$ Any subset of $B_R(X)$ is not a nano regular closed.
(xiii) Let $A \supset B_R(X)$, then $Ncl(Nint(A)) = Ncl(B_R(X) = [L_R(X)]^C \neq A$.
$\therefore A \supset B_R(X)$ is not a nano regular closed.
$\Rightarrow$ Any superset of $B_R(X)$ is not a nano regular closed.
(xiv) Let A be the subset of U which has the non-empty intersection with $B_R(X)$.
(i.e) $B_R(X) \cap A \neq \phi$. Then $Ncl(Nint(A)) = Ncl(\phi) = \phi \neq A$.
$\therefore$ $A \cap B_R(X) \neq \phi$ is not a nano regular closed.
(xv) Let $A = [B_R(X)]^C$, $Ncl(Nint(A)) = Ncl(L_R(X)) = [B_R(X)]^C = A$.
Since $[B_R(X)]^C = L_R(X)$.
$\therefore$ $A = [B_R(X)]^C$ is nano regular closed.
Therefore $\{U, \phi, L_R(X), U_R(X), B_R(X), [L_R(X)]^C, [U_R(X)]^C, [B_R(X)]^C\}$ are the only nano regular closed sets.
(i) Let $A = L_R(X)$, $Nrcl(A) = L_R(X) \subseteq G$ whenever $A \subseteq G$ and G is nano b-open in U.
$\therefore$ $A = L_R(X)$ is Nrb-closed.
(ii) Let $A = U_R(X)$, $Nrcl(A) = U_R(X) \subseteq G$ whenever $A \subseteq G$ and G is nano b-open in U.
$\therefore$ $A = U_R(X)$ is Nrb-closed.
(iii) Let $A = B_R(X)$, $Nrcl(A) = B_R(X) \subseteq G$ whenever $A \subseteq G$ and G is nano b-open in U.
$\therefore$ $A = B_R(X)$ is Nrb-closed.
(iv) Let $A = [L_R(X)]^C$, $Nrcl(A) = [L_R(X)]^C \subseteq G$ whenever $A \subseteq G$ and G is nano b-open.
(v) Let $A = [U_R(X)]^C$, $Nrcl(A) = [U_R(X)]^C \subseteq G$ whenever $A \subseteq G$ and G is nano b-open.
(vi) Let $A = [B_R(X)]^C$, $Nrcl(A) = [B_R(X)]^C \subseteq G$ whenever $A \subseteq G$ and G is nano b-open.
$\therefore A = [L_R(X)]^C$ is Nrb-closed.
Here $Nrcl(A) \not\subset G$ for every non-empty subset A of U except the above cases.
$\therefore \{U, \phi, L_R(X), [L_R(X)]^C, U_R(X), [U_R(X)]^C, B_R(X), [B_R(X)]^C\}$ are the only Nrb-closed sets.
But we know that $U_R(X) = U$, $[U_R(X)]^C = \phi$, $L_R(X) = [B_R(X)]^C$ and $[L_R(X)]^C = B_R(X)$.
$\therefore$ $\{U, \phi, L_R(X), [L_R(X)]^C\}$ are the only Nrb-closed sets.

Theorem: 3.6

If $L_R(X) \neq \phi$, $U_R(X) \neq \phi$ and $B_R(X) \neq \phi$ then $U, \phi, [L_R(X)]^C, [U_R(X)]^C, [B_R(X)]^C$ are the only Nrb-closed sets.

Proof:

If $L_R(X) \neq \phi$, $U_R(X) \neq \phi$ and $B_R(X) \neq \phi$, then $\tau_R(X) = \{U, \phi, L_R(X), B_R(X), U_R(X)\} =$

Nano open sets, Nano closed sets $=\{U, \phi, [L_R(X)]^C, [B_R(X)]^C\} = \{U, \phi, U_R(X^C), L_R(X^C), L_R(X) \cup L_R(X^C)\}$.

Since every nano open set is nano b-open, we get $\{U, \phi, L_R(X), B_R(X), U_R(X)\}$ are nano b-open sets. Along with them,

(i) Let $A = L_R(X) = [B_R(X)]^C$, then $Nint(A) = L_R(X)$, $Ncl(A) = [B_R(X)]^C$.
$\therefore Ncl(Nint(A)) \cup Nint(Ncl(A)) = Ncl(L_R(X)) \cup Nint([B_R(X)]^C)$
$= [B_R(X)]^C \cup [B_R(X)]^C = [B_R(X)]^C$.
$\Rightarrow A \subseteq Ncl(Nint(A)) \cup Nint(Ncl(A))$.
$\therefore A = L_R(X)$ is nano b-open

(ii) Let $A \subset L_R(X)$ and $A \neq \phi$, then $Nint(A) = \phi$, $Ncl(A) = [B_R(X)]^C$
$\therefore Ncl(Nint(A)) \cup Nint(Ncl(A)) = Ncl(\phi) \cup Nint([B_R(X)]^C) = \phi \cup L_R(X) = L_R(X)$
$\Rightarrow A \subseteq Ncl(Nint(A)) \cup Nint(Ncl(A))$.
$\therefore A \subset L_R(X)$ is nano b-open
$\Rightarrow$ Any subset of $L_R(X)$ is nano b-open.

(iii) Let $A \neq U$, $A \supset L_R(X)$, then $Nint(A) = L_R(X)$, $Ncl(A) = U$.
$\therefore Ncl(Nint(A)) \cup Nint(Ncl(A)) = Ncl(L_R(X)) \cup Nint(U) \Rightarrow U \cup U = U$.
$\Rightarrow A \subseteq Ncl(Nint(A)) \cup Nint(Ncl(A))$.
$\therefore A \supset L_R(X)$ is nano b-open
$\Rightarrow$ Any superset of $L_R(X)$ is nano b-open.

(iv) Let A be the subset of U which has the non-empty intersection with $L_R(X)$. (i.e) $L_R(X) \cap A \neq \phi$. Then $Nint(A) = \phi$, $Ncl(A) = U$.
$\therefore Ncl(Nint(A)) \cup Nint(Ncl(A)) = Ncl(\phi) \cup Nint(U) = \phi \cup U = U$.
$\Rightarrow A \subseteq Ncl(Nint(A)) \cup Nint(Ncl(A))$.
$\therefore A \cap L_R(X) \neq \phi$ is nano b-open

(v) Let $A = [L_R(X)]^C = B_R(X) = U_R(X^C)$, then $Nint(A) = B_R(X)$, $Ncl(A) = U_R(X^C)$.
$\therefore Ncl(Nint(A)) \cup Nint(Ncl(A)) = Ncl(B_R(X)) \cup Nint(U_R(X^C))$
$= U_R(X^C) \cup U_R(X^C) = U_R(X^C)$.
$\Rightarrow A \subseteq Ncl(Nint(A)) \cup Nint(Ncl(A))$.
$\therefore A = [L_R(X)]^C$ is nano b-open

(vi) Let $A = U_R(X)$, then $Nint(A) = U_R(X)$, $Ncl(A) = U$.
$\therefore Ncl(Nint(A)) \cup Nint(Ncl(A)) = Ncl(U_R(X)) \cup Nint(U) = U \cup U = U$.
$\Rightarrow A \subseteq Ncl(Nint(A)) \cup Nint(Ncl(A))$.
$\therefore A = U_R(X)$ is nano b-open

(vii) Let $A \subset U_R(X)$ and $A \neq \phi$, then $Nint(A) = \phi$, $Ncl(A) = U$.
$\therefore Ncl(Nint(A)) \cup Nint(Ncl(A)) = Ncl(\phi) \cup Nint(U) = \phi \cup U = U$.
$\Rightarrow A \subseteq Ncl(Nint(A)) \cup Nint(Ncl(A))$.
$\therefore A \subset U_R(X)$ is nano b-open
$\Rightarrow$ Any subset of $U_R(X)$ is nano b-open.

(viii) Let $A \neq U$, $A \supset U_R(X)$, then $Nint(A) = U_R(X)$, $Ncl(A) = U$.
$\therefore Ncl(Nint(A)) \cup Nint(Ncl(A)) = Ncl(U_R(X)) \cup Nint(U) \Rightarrow U \cup U = U$.
$\Rightarrow A \subseteq Ncl(Nint(A)) \cup Nint(Ncl(A))$.
$\therefore A \supset U_R(X)$ is nano b-open
$\Rightarrow$ Any superset of $U_R(X)$ is nano b-open.

(ix) Let A be the subset of U which has the non-empty intersection with $U_R(X)$.
(i.e) $U_R(X) \cap A \neq \phi$. Then $Nint(A) = \phi$, $Ncl(A) = [L_R(X)]^C$.
$\therefore Ncl(Nint(A)) \cup Nint(Ncl(A)) = Ncl(\phi) \cup Nint([L_R(X)]^C)$
$= \phi \cup B_R(X) = B_R(X)$.
$\Rightarrow A \nsubseteq Ncl(Nint(A)) \cup Nint(Ncl(A))$.
$\therefore A \cap L_R(X) \neq \phi$ is not a nano b-open.

(x) Let $A = [U_R(X)]^C$, $Nint(A) = \phi$, $Ncl(A) = [U_R(X)]^C$.
$\therefore Ncl(Nint(A)) \cup Nint(Ncl(A)) = Ncl(\phi) \cup Nint(\phi) = \phi \cup \phi = \phi$.
$\Rightarrow A \nsubseteq Ncl(Nint(A)) \cup Nint(Ncl(A))$.
$\therefore A = [U_R(X)]^C$ is not a nano b-open.

(xi) Let $A = B_R(X) = [L_R(X)]^C$, then $Nint(A) = B_R(X)$, $Ncl(A) = [L_R(X)]^C$.
$\therefore Ncl(Nint(A)) \cup Nint(Ncl(A)) = Ncl(B_R(X)) \cup Nint([L_R(X)]^C)$
$= [L_R(X)]^C \cup B_R(X) = [L_R(X)]^C$.
$\Rightarrow A \subseteq Ncl(Nint(A)) \cup Nint(Ncl(A))$.
$\therefore A = B_R(X)$ is nano b-open

(xii) Let $A \subset B_R(X)$ and $A \neq \phi$, then $Nint(A) = \phi$, $Ncl(A) = [L_R(X)]^C$.
$\therefore Ncl(Nint(A)) \cup Nint(Ncl(A)) = Ncl(\phi) \cup Nint([L_R(X)]^C) = \phi \cup B_R(X) = B_R(X)$.
$\Rightarrow A \subseteq Ncl(Nint(A)) \cup Nint(Ncl(A))$.
$\therefore A \subset B_R(X)$ is nano b-open.
$\Rightarrow$ Any subset of $B_R(X)$ is nano b-open

(xiii) Let $A \neq U$, $A \supset B_R(X)$, then $Nint(A) = B_R(X)$, $Ncl(A) = L_R(X^C)$.

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$\therefore$Ncl(Nint(A)) $\cup$ Nint(Ncl(A)) = Ncl(L_R(X))$\cup$Nint(U) $\Rightarrow$ U $\cup$U = U.
$\Rightarrow$A $\subseteq$Ncl(Nint(A)) $\cup$ Nint(Ncl(A)).
$\therefore$A$\supset$$B_R$(X) is nano b-open
$\Rightarrow$ Any superset of B_R(X) is nano b-open.
(xiv) Let A be the subset of U which has the non-empty intersection with B_R(X). (i.e) B_R(X) $\cap$ A $\neq \phi$. Then Nint(A) = ϕ , Ncl(A) = $[L_R(X)]^C$.
$\therefore$Ncl(Nint(A)) $\cup$ Nint(Ncl(A)) = Ncl(ϕ) $\cup$ Nint($[L_R(X)]^C$)
= ϕ $\cup B_R$(X) = B_R(X).
$\Rightarrow$A $\not\subseteq$Ncl(Nint(A)) $\cup$ Nint(Ncl(A)).
$\therefore$ A $\cap$ L_R(X) $\neq \phi$ is not a nano b-open
(xv) Let A = $[B_R(X)]^C$, Nint(A) = L_R(X), Ncl(A) = $[B_R(X)]^C$.
$\therefore$Ncl(Nint(A)) $\cup$ Nint(Ncl(A)) = Ncl(L_R(X)) $\cup$ Nint($[B_R(X)]^C$)
= $[B_R(X)]^C \cup L_R(X)$ = $[B_R(X)]^C$.
$\Rightarrow$A $\subseteq$Ncl(Nint(A)) $\cup$ Nint(Ncl(A)).
$\therefore$ A = $[B_R(X)]^C$ is nano b-open.
From the above cases, we get A = L_R(X), A $\subset$ L_R(X), A$\supset$ L_R(X), A $\cap$ L_R(X) $\neq \phi$, A = $[L_R(X)]^C$, A = U_R(X), A $\subset$ U_R(X), A$\supset$ U_R(X),A = $[U_R(X)]^C$ A = B_R(X), A $\subset$ B_R(X), A$\supset$ B_R(X), and A =$[B_R(X)]^C$ are the nano b-open sets.
$\therefore$ {U, ϕ , A = L_R(X), A $\subset$ L_R(X), A$\supset$ L_R(X), A $\cap$ L_R(X) $\neq \phi$, A = $[L_R(X)]^C$, A = U_R(X), A $\subset$ U_R(X), A$\supset$ U_R(X),A = $[U_R(X)]^C$ A = B_R(X), A $\subset$ B_R(X), A$\supset$ B_R(X), A =$[B_R(X)]^C$} are the only nano b-open sets.
(i) Let A = L_R(X) ,then Ncl(Nint(A)) = Ncl (L_R(X))= $[B_R(X)]^C$$\neq$ A.
$\therefore$A =L_R(X) is not a nano regular closed.
(ii) Let A $\subset$$L_R$(X) and A$\neq\phi$, then Ncl(Nint(A)) = Ncl(ϕ) = ϕ $\neq$ A.
$\therefore$A $\subset$$L_R$(X) is not a nano regular closed.
$\Rightarrow$ Any subset of L_R(X) is not a nano regular closed.
(iii) Let A $\neq$ U, A $\supset$$L_R$(X), then Ncl(Nint(A)) = Ncl(L_R(X)) = U$\neq$ A.
$\therefore$A$\supset$$L_R$(X) is not a nano regular closed.
$\Rightarrow$ Any superset of L_R(X) is not a nano regular closed.
(iv) Let A be the subset of U which has the non-empty intersection with L_R(X).
(i.e) L_R(X) $\cap$ A $\neq \phi$.
Then Ncl(Nint(A)) = Ncl(ϕ) = ϕ $\neq$ A.
$\therefore$ A $\cap$ L_R(X) $\neq \phi$ is not a nano regular closed.
(v) Let A = $[L_R(X)]^C$ = B_R(X) = $U_R(X^C)$,
Then Ncl(Nint(A)) = Ncl (B_R(X)) = $U_R(X^C)$ = A.
$\therefore$A = $[L_R(X)]^C$ is nano regular closed.
(vi) Let A = U_R(X), then Ncl(Nint(A)) = Ncl (U_R(X)) = U$\neq$ A..
$\therefore$A = U_R(X) is not a nano regular closed.
(vii) Let A $\subset$$U_R$(X) and A$\neq\phi$, then Ncl(Nint(A)) = Ncl(ϕ) = ϕ $\neq$ A.
$\therefore$A $\subset$$U_R$(X) is not a nano regular closed.
$\Rightarrow$ Any subset of U_R(X) is not a nano regular closed.
(viii) Let A $\neq$U, A $\supset$$U_R$(X), then Ncl(Nint(A)) = Ncl(U_R(X)) = U$\neq$ A.
$\therefore$A$\supset$$U_R$(X) is not a nano regular closed.
$\Rightarrow$ Any superset of U_R(X) is not a nano regular closed.
(ix) Let A be the subset of U which has the non-empty intersection with U_R(X).
(i.e) U_R(X) $\cap$ A $\neq \phi$. Then Ncl(Nint(A))= Ncl(ϕ) = ϕ $\neq$ A.
$\therefore$ A $\cap$ L_R(X) $\neq \phi$ is not a nano regular closed.
(x) Let A = $[U_R(X)]^C$, thenNcl(Nint(A)) = Ncl(ϕ)= ϕ $\neq$ A.
$\therefore$ A = $[U_R(X)]^C$ is not a nano regular closed.
(xi) Let A = B_R(X), then Ncl(Nint(A)) = Ncl (B_R(X)) = $[L_R(X)]^C$$\neq$ A.
$\therefore$A = B_R(X) is not a nano regular closed.
(xii) Let A$\subset$$B_R$(X) and A$\neq\phi$, thenNcl(Nint(A)) = Ncl(ϕ) = ϕ $\neq$ A.
$\therefore$ A$\subset$$B_R$(X) is not a nano regular closed.
$\Rightarrow$ Any subset of B_R(X) is not a nano regular closed.
(xiii) Let A $\neq$ U, A $\supset$$B_R$(X), then Ncl(Nint(A)) = Ncl(L_R(X)) = U$\neq$ A.
$\therefore$A$\supset$$B_R$(X) is not a nano regular closed.
$\Rightarrow$ Any superset of B_R(X) is not a nano regular closed.
(xiv) Let A be the subset of U which has the non-empty intersection with B_R(X).
(i.e) B_R(X) $\cap$ A $\neq \phi$. Then Ncl(Nint(A)) = Ncl(ϕ) = ϕ $\neq$ A.
$\therefore$ A $\cap$ L_R(X) $\neq \phi$ is not a nano regular closed.
(xv) Let A = $[B_R(X)]^C$, thenNcl(Nint(A)) = Ncl(L_R(X)) = $[B_R(X)]^C$ = A.
$\therefore$ A = $[B_R(X)]^C$ is nano regular closed.
From the above cases, we get A = $[L_R(X)]^C$, A = $[B_R(X)]^C$ are the nano regular closed sets.

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∴ $\{U, \phi, [L_R(X)]^C, [B_R(X)]^C\}$ are the only nano regular closed sets.

(i) Let $A = [L_R(X)]^C$, $Nrcl(A) = [L_R(X)]^C \subseteq G$ whenever $A \subseteq G$ and G is nano b-open.

$\therefore A = [L_R(X)]^C$ is Nrb-closed.

(ii) Let $A = [B_R(X)]^C$, $Nrcl(A) = [B_R(X)]^C \subseteq G$ whenever $A \subseteq G$ and G is nano b-open.

$\therefore A = [B_R(X)]^C$ is Nrb-closed.

(iii) Let $A = [U_R(X)]^C$, $Nrcl(A) = [B_R(X)]^C \subseteq G$ whenever $A \subseteq G$ and G is nano b-open.

$\therefore A = [U_R(X)]^C$ is Nrb-closed.

For the remaining cases $Nrcl(A) \not\subset G$, since $\{U, \phi, [L_R(X)]^C, [B_R(X)]^C\}$ are the only nano regular closed sets.

∴ $\{U, \phi, [L_R(X)]^C, [U_R(X)]^C, [B_R(X)]^C\}$ are the only Nrb-closed sets.

IV. APPLICATIONS OF NANO REGULAR B-CLOSED SETS

Definition: 4.1 [3] Let (U, A) be an information system, where A is divided into a set C of condition attributes and a set D of decision attribute. A subset R of C is said to be a core, if Nrb-closed sets of R= Nrb-closed sets of C and Nrb-closed sets of C≠Nrb-closed sets of C – {r}for all r ∈ R. That is, a core is a minimal subset of attributes which is such that none of its elements can be removed without affecting the classification power of attributes.

TO FIND THE KEY FACTORS FOR A HEALTHY LIFE:

In this section we discussed an application based on Nano regular b-closed sets. We found the essential key factors for an expression is odd or even.

The general algorithm for identify the key factors is given below.

ALGORITHM:

Step 1: Let U be the finite universe, A be the finite set of attributes which is divided into two classes such that condition attributes and decision attributes where condition attributes are denoted by C and decision attributes are denoted by D. Then R is the equivalence relation on U corresponding to the condition attributes C and a subset X of U. The given tabular column was represented by the datas, in which coloumns are the attributes and rows are the objects. The entries of the table are known as the attribute values.

Step 2: Find the lower approximation $L_C(X)$, upper approximation $U_C(X)$ and the boundary region $B_C(X)$ of X with respect to R.

Step 3: Find the nano topology $\tau_C(X)$ on U and the nano regular b-closed sets corresponding to the conditional attribute set C.

Step 4: Remove an attribute x from C and find the lower and upper approximations and the boundary region of X with respect to the equivalence relation on C - {x}.

Step 5: Find the nano topology $\tau_{C-\{X\}}(X)$ on U and the nano regular b-closed sets for C - {x}.

Step 6: Repeat steps 4 and 5 for all attributes in C.

Step 7: Those attributes in C for which Nano regular b-closed sets of C≠Nano regular b-closed sets of C – {x} form the CORE.

EXAMPLE 4.2:

Consider an expression a + 2b – 4c + 6d.

In this case the expression $= \begin{cases} \text{Odd}, & \text{When a is odd} \\ \text{Even}, & \text{When a is even} \end{cases}$

∴ a is the decision variable for the expression is to be odd or even.

Now, for this same expression, we put some values either odd or even for a, b, c, d and e respectively. Then we get the answer.

By using our general algorithm we find the key variable for the expression is odd or even.

VARIOUS SETS	**a**	**b**	**c**	**d**	**e**	**a + 2b – 4c + 6d = ODD / EVEN**
A	odd	odd	even	odd	odd	**ODD**
B	even	odd	even	odd	odd	**EVEN**
C	odd	odd	even	odd	odd	**ODD**

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D	odd	odd	odd	even	odd	**ODD**
E	even	odd	even	odd	odd	**EVEN**
F	odd	odd	even	odd	odd	**ODD**
G	even	even	odd	even	odd	**EVEN**

Here A, B, C, D, E, F, G are the set of variables and A = {a,b,c,d,e} are the set of attributes. “Odd” or “Even” are the entries in the tables which is known as attribute values.

The attributes in A are the condition attributes and a + 2b – 4c + 6d = ODD / EVEN be the decision attribute. Condition attribute is denoted by C such as C = {a, b, c, d, e} and decision attribute is denoted by D ={O / E}. The family of classes, U/C corresponding to C is given by U/R(C) = {{A, C, F},{B, E},{D},{G}}.

Case 1 : (If the expressionis odd)
Let X = {A, C, D, F}, the set of variables where the expression is odd. Then $L_C(X)= U_C(X)$={A, C, D, F} and $B_C(X) = U_C(X) - L_C(X)=\emptyset$.Then$\tau_C(X)$) ={U,∅,{A, C, D, F}}. Then the Nano regular b-closed sets ={U,∅}.

Step 1:

(i) Remove the attribute “ a ” from C, U/R (C –a) = {{A, B, C, E, F},{D},{G}}then the lower and upper approximations of X corresponding to C-{a} are given by $L_{C-\{a\}}(X)$ = {D},$U_{C-\{a\}}(X)$ = {A, B, C, D, E, F}and the boundary region is given by $B_{C-\{a\}}(X)$ ={A, B, C, E, F}. Then the nano topology $\tau_{C-\{a\}}(X)$) = {U,∅, {D}, {A, B, C, D, E, F},{A, B, C, E, F}}and the Nrb-closed sets of C – {a} = {U,∅, {D}, {A, B, C, D, E, F},{A, B, C, E, F}}≠ Nrb-closed sets of C.

(ii) Remove the attribute “ b ” from C, U/R (C –b) = {{A, C, F},{B, E},{D},{G}}then the lower and upper approximations of X corresponding to C-{b} are given by $L_{C-\{b\}}(X) = U_{C-\{b\}}(X)$ = {A, C, D, F}and the boundary region is given by $B_{C-\{b\}}(X)$ =∅.Then the nano topology $\tau_{C-\{b\}}(X)$) = {U,∅,{A, C, D, F}} and the Nrb-closed sets of C – {b} = {U,∅}=Nrb-closed sets of C.

(iii) Remove the attribute “ c ” from C, U/R (C –c) = {{A, C, F},{B, E},{D},{G}}then the lower and upper approximations of X corresponding to C-{c} are given by $L_{C-\{c\}}(X) = U_{C-\{c\}}(X)$ = {A, C, D, F}and the boundary region is given by $B_{C-\{c\}}(X)$ =∅. Then the nano topology $\tau_{C-\{c\}}(X)$) = {U,∅,{A, C, D, F}} and the Nrb-closed sets of C – {c} = {U,∅}= Nrb-closed sets of C.

(iv) Remove the attribute “ d ” from C, U/R (C –d) = {{A, C, F},{B, E},{D},{G}}then the lower and upper approximations of X corresponding to C-{d} are given by $L_{C-\{d\}}(X) = U_{C-\{d\}}(X)$ = {A, C, D, F}and the boundary region is given by $B_{C-\{d\}}(X)$ =∅ Then the nano topology $\tau_{C-\{d\}}(X)$) = {U,∅,{A, C, D, F}} and the Nrb-closed sets of C – {d} = {U,∅}= Nrb-closed sets of C.

(v) Remove the attribute “ e ” from C, U/R (C –e) = {{A, C, F},{B, E},{D},{G}}then the lower and upper approximations of X corresponding to C-{e} are given by $L_{C-\{e\}}(X) = U_{C-\{e\}}(X)$ = {A, C, D, F}and the boundary region is given by $B_{C-\{e\}}(X)$ =∅. Then the nano topology $\tau_{C-\{e\}}(X)$) = {U,∅,{A, C, D, F}} and the Nrb-closed sets of C – {e} = {U,∅}= Nrb-closed sets of C.

Step 2:
If R = C – {b, c, d, e} = {a},then $L_R(X)$= $U_R(X)$={A, C, D, F} and $B_R(X)$ = ∅.Then $\tau_R(X)$) ={U,∅,{A, C, D, F}}. Then the Nano regular b-closed sets of R = {U,∅} = Nano regular b-closed sets of C . From step 1, we get ,

(i) U/R (C – a) = {{A, B, C, E, F},{D},{G}}, $L_{C-\{a\}}(X)$ = {D}, $U_{C-\{a\}}(X)$ = {A, B, C, D, E, F} and $B_{C-\{a\}}(X)$ ={A, B, C, E, F}. Then $\tau_{C-\{a\}}(X)$) = {U,∅, {D}, {A, B, C, D, E, F},{A, B, C, E, F}} and the Nrb-closed sets of C – {a} = {U,∅, {D}, {A, B, C, D, E, F},{A, B, C, E, F}}≠ Nrb-closed sets of C.

From this we get, $\mathcal{B}_R(X) = \mathcal{B}_C(X)$ and $\mathcal{B}_{C - \{r\}} \neq \mathcal{B}_R(X)$ for every r in R.
∴ **CORE** = { a}

Case 2 : (If the expression is even)
Let X = {B, E, G}, the set of variables where the expression is even. Then $L_C(X)= U_C(X)$={B,E, G} and $B_C(X) = U_C(X) - L_C(X)=\emptyset$.Then$\tau_C(X)$) ={U,∅,{B, E, G}}. Then the Nano regular b-closed sets ={U,∅}.

Step 1:

(i) Remove the attribute " a " from C, U/R (C – a) = {{A, B, C, E, F},{D},{G}} then the lower and upper approximations of X corresponding to C -{a} are given by $L_{C-\{a\}}(X)$ = {G}, $U_{C-\{a\}}(X)$ = {A, B, C, E, F, G} and the boundary region is given by $B_{C-\{a\}}(X)$ ={A, B, C, E, F}. Then the nano topology $\tau_{C-\{a\}}(X)$) = {U,∅, {G}, {A, B, C, E, F, G},{A, B, C, E, F}} and the Nrb-closed sets of C – {a} = {U,∅, {G}, {A, B, C, E, F, G},{A, B, C, E, F}}≠ Nrb-closed sets of C.

(ii) Remove the attribute " b " from C, U/R (C – b) = {{A, C, F},{B, E},{D},{G}} then the lower and upper approximations of X corresponding to C -{b} are given by $L_{C-\{b\}}(X) = U_{C-\{b\}}(X)$ = {B, E, G} and the boundary region is given by $B_{C-\{b\}}(X)$ = ∅ .Then the nano topology $\tau_{C-\{b\}}(X)$) = {U,∅,{B, E, G}} and the Nrb-closed sets of C – {b} = {U,∅} = Nrb-closed sets of C.

(iii) Remove the attribute " c " from C, U/R (C – c) = {{A, C, F},{B, E},{D},{G}} then the lower and upper approximations of X corresponding to C -{c} are given by $L_{C-\{c\}}(X) = U_{C-\{c\}}(X)$ = { B, E, G } and the boundary region is given by $B_{C-\{c\}}(X)$ = ∅ .Then the nano topology $\tau_{C-\{c\}}(X)$) = {U,∅,{ B, E, G }} and the Nrb-closed sets of C – {c} = {U,∅} = Nrb-closed sets of C.

(iv) Remove the attribute " d " from C, U/R (C – d) = {{A, C, F},{B, E},{D},{G}} then the lower and upper approximations of X corresponding to C -{d} are given by $L_{C-\{d\}}(X) = U_{C-\{d\}}(X)$ = { B, E, G } and the boundary region is given by $B_{C-\{d\}}(X)$ = ∅.Then the nano topology $\tau_{C-\{d\}}(X)$) = {U,∅,{ B, E, G }} and the Nrb-closed sets of C – {d} = {U,∅} = Nrb-closed sets of C.

(v) Remove the attribute " e " from C, U/R (C – e) = {{A, C, F},{B, E},{D},{G}} then the lower and upper approximations of X corresponding to C -{e} are given by $L_{C-\{e\}}(X) = U_{C-\{e\}}(X)$ = { B, E, G } and the boundary region is given by $B_{C-\{e\}}(X)$ = ∅ .Then the nano topology $\tau_{C-\{e\}}(X)$) = {U,∅,{ B, E, G }} and the Nrb-closed sets of C – {e} = {U,∅} = Nrb-closed sets of C.

Step 2:
If R = C – {b, c, d, e} = {a},then $L_R(X)= U_R(X)$={B, E, G} and $B_R(X)$ = ∅Then $\tau_R(X)$) ={U,∅,{B, E, G}}. Then the Nano regular b-closed sets of R = {U,∅} = Nano regular b-closed sets of C . From step 1, we get ,

(i) U/R (C – a) = {{A, B, C, E, F},{D},{G}}, $L_{C-\{a\}}(X)$ = {G}, $U_{C-\{a\}}(X)$ = {A, B, C, E, F, G} and $B_{C-\{a\}}(X)$ ={A, B, C, E, F}. Then $\tau_{C-\{a\}}(X)$) = {U,∅, {G}, {A, B, C, E, F, G},{A, B, C, E, F}} and the Nrb-closed sets of C – {a} = {U,∅, {G}, {A, B, C, E, F, G},{A, B, C, E, F}}≠ Nrb-closed sets of C.

From this we get, $\mathcal{B}_R(X) = \mathcal{B}_C(X)$ and $\mathcal{B}_{C - \{r\}} \neq \mathcal{B}_R(X)$ for every r in R.
∴ **CORE** = { a}

From the core of the above two cases we conclude that the essential key factors for anexpression a + 2b – 4c + 6d = ODD / EVEN is " a ".

V. CONCLUSION

In this paper we studied the Impact of characterizations of Lower approximation space - $L_R(X)$, Upper approximation space - $U_R(X)$ and the Boundary region - $B_R(X)$ in Nano regular b-closed sets (Nrb-closed sets) in nano topological spaces.And an application based on Nano regular b-closed sets was discussed.

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