

Mathematical Modeling of The Annual Average Daily Traffic on South Eastern Nigerian Highways using Environmental and Demographic Factors.

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ABSTRACT: In this research, the mathematical model of the annual average daily traffic (AADT) was developed using the environmental and demographic factors which were established based on a 20 - year sample collected from the Federal Ministry of Works and Housing and the Ministries of Budget and Planning. The environmental parameters were obtained using the degree of pavement flooding and pavement condition indices from two (2) selected highways. The demographic factors applied included the gross domestic products, GDP, and the population density, P. Linear and nonlinear regression equations were applied. The resultant of the linear regression equations obtained using the mathematical laboratory software (MATLAB) in the computation of the coefficients of the parameters includes; $T = 2.92 \times 10^4 + 689.57G + 2.82P, T =$

 $8.23 \times 10^5 + 0.02 \text{G} - 0.4 \text{P}$, while the non lonear multiple regression equations includes: $\ln T = 1.53 \times 10^4 \ln V - 2.54 \times 10^4 \ln D$, T =

 $1.1 \times 10^{6} D^{0.55}$

 $\frac{(10^{\circ}D^{\circ.33})}{V^{0.14}}$, for Umuahia – Ikot Ekpene expressway and Onitsha - Owerri Road respectively. The values of the coefficients of correlation, R, include (R^{G} = $0.3912, 0.5627, R^{P} = 0.7922, 0.4836, R^{V} = 0.9857,$ $0.6573, R^{D} = 0.9975, 0.4158)$ for Umuahia – Ikot Ekpene expressway and Onitsha - Owerri Road respectively. Hence, it can be deduced that the model factor that best fit into the equation for Umuahia - Ikot Ekpene expressway is the flood

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depth, while that of Onitsha - Owerri road is the volume of potholes. Thus flood depth and volume of potholes are the model parameters necessary for calibration of the AADT in Abia and Anambra states respectively.

KEYWORDS: volume of pot holes, regression model, mathematical laboratory, coefficient of correlation, pavement flooding.

I. **INTRODUCTION**

Over the past two decades, the focus of efforts in modeling and forecasting macroscopic traffic states has transitioned from univariate temporal correlation to multivariate temporal-spatial correlation and from linear to nonlinear forms. Those models may be loosely classified as statistical and non-statistical methods. This has caused most highway agencies to be assigning a considerable amount of their capital to traffic counting programs [1]. Temporal and spatial variations of traffic are studied using the traffic data volume collected. Important traffic parameters like the annual average daily traffic (AADT), design hourlyvolume (DHV), and average daily vehicle distance traveled (ADVDT) are also studied using the traffic volume data. Highway infrastructural design, planning, control, operation and management require these estimated traffic parameters [2]. Most of the universally applied counting programs by highway agencies include:



- 1 Continuous counting programs by permanent traffic counters (PTCs) which records traffic volume on hourly basis everyday throughout the year, and,
- 2 Sample counting programs by short duration counts (SDCs) which are periodically conducted (e.g., seasonally or sporadically) every year.
- 3 The feasibility of having continuous traffic counters on every road segments for the whole year is minimal. Thus, portable counters are used to perform short period traffic counts for the roads without PTCs. Geographic diversity of traffic counts is achieved by using short period counts, and this provides estimated values of AADT after applying the appropriate adjustment factors. Daily, hourly, and monthly factors (DF, HF, and MF) are developed from the PTCS data, which are applied to short term counts to estimate AADT [3].
- 4 Annual Average Daily Traffic, (AADT) values are achieved by an investigational calculation of the quantity of vehicles transiting through a definite highway segment where a weighing system is mounted. There are precise methodologies can be developed for its computation [4. 5]. The pointer is expressed in an annual average traffic value even though highways do always have a marked disparity in traffic on a periodic basis [6].
- 5 Conventionally, it means the ratio of the volume of automobile traffic on a highway or road to the total number of days in a year [7]. It is also defined as the sum total of all the automotive and vehicular traffic loads (animals inclusive) that trespass a given highway in relation to the daily ratio in one year [8]. Mathematically,
- $6 \quad \frac{AADT =}{\frac{number of automobiles navigating a highway in the year}{365 days}}$

(1)

AADT is as often as possible identified with contemplates in regards to infrastructural ability to retain traffic. High estimations of AADT legitimize the development of new infrastructure. The development of roads and the expansion of vehicles course mean an expansion of the pessimistic impacts on nature, among those who stand out: barometrical contamination, direct control of land, territories discontinuity, loss of biodiversity, a boundary impact of the foundation, an expanded danger of vehicles and people/wildlife life crash [9].

Seasonal variations in traffic volume have been affected by the environmental and

demographic factors [10]. Adverse weather conditions cause decrease in traffic volume and speed, increased accident rate and increased delayed or cancelled trips [11]. Due to unfavorable weather conditions. short-period counts are mostly conducted in spring/summer months. Pavement distress refers to the condition of a pavement surface in terms of its general appearance. A perfect pavement is level and has a continuous and unbroken surface. Contrarily, a distressed pavement may be fractured, distorted, or disintegrated. These surface defects affect the AADT of any highway. Deteriorated pavement surfaces have adverse effect on the traffic volume and speed, causes wear and tear of automobiles, thus, affects the annual average daily traffic on the highways. Consequent upon this, most highway agencies have resorted to the development of Pavement Management Systems (PMS) to investigate the life cycle of existing road infrastructures by means of pavement deterioration models [12]. Actually, most pavement deterioration models use the International Roughness Index (IRI), which is calculated considering a simulated passenger car traveling at 80 km/h, to describe pavement performance [13]. International Roughness Index (IRI) is a universally accepted index used for calculating the roughness of a road. Its variation over time is chosen as the representative variable for the continuous deterioration of a road profile. During the heavy downpours, some rivers may overflow their banks and cause surface flooding of the highways. This usually happens during the rainy seasons in the south eastern part of Nigeria. Thus, there is great reduction and fluctuation in the annual average daily traffic during this season. Flooding on a road does not necessarily prevent people from driving along it. Thus, it is important to build a more robust relationship between vehicular movement and the flood depth disruption function by improving current simulations.

Most highways traversing the commercial hubs 7 of many states in the south eastern Nigeria have been dilapidated greatly due to the sudden upsurge of traffic into the city. More so, many of the highway pavements have failed because they were initially designed to support a minimum traffic volume not considering the future growth of traffic and the effects of the environmental and demographic factors. This is due to improper or poor planning and design. There is no definite pattern of annual traffic growth on any particular highway. Many factors contribute to the anomalous traffic growth which includes but not limited to the environmental and the demographic factors



(rate of surface flooding, pavement deterioration, gross domestic products and the population density). The unpredictable nature of flooding of pavement surfaces and the resultant deteriorated highway structure induces some undesirable highway characteristics which are inimical to traffic usage. The economic viability of the teeming population residing proximally to a particular highway affects the annual traffic growth [14]. This traffic growth cannot be determined by ordinary arithmetic progression. In this thesis, a simple linear mathematical model of the annual average daily traffic in relation to the environmental and demographic factors affecting the annual average daily traffic is developed. This model can be used to forecast the future traffic when the values of the related factors are available. In the present study, significant effort has been utilized in the statistical analysis of the interaction between traffic volumes and the forecasting variables. The problem of impulsive change of traffic volume in relation to the forecasting variables on the south eastern Nigerian highways will be resolved by this model as the future annual average daily traffic can be ascertained. More significantly, the resultant model is anticipated to be easy to comprehend and to execute based the characteristic behavior of the on

environmental and the demographic factors of the highways under study.

II. MATERIALS AND METHODS 1.1 THE STUDY AREA.

The selected states from the South East Nigeria include;

- <u>Abia</u>
- <u>Anambra</u>

The region is geographically located in the <u>Southeastern</u> part Nigeria [15]. The geographic location is latitudes 4° 47' 35''N and 7° 7' 44''N and longitudes 7° 54' 26''E and 8° 27' 10''E in the tropical rain forest zone of Nigeria. The region is largely agrarian and much reliance on land resources with population density about 2000 people/km². The <u>Niger River</u> bordered it in the West with governmental and traditional boundary with the Northern Province of Nigeria. The southern coastline is alongside the <u>Gulf of Guinea</u>, while the Eastern boundaries lie in-between the borders of Nigeria and <u>Cameroon</u>. The region has a total surface area of approximately 76,000 km² (29,400 mi²).

The highways under study and their respective states of affiliation, including the data sources are listed in Table 1.

States	Name of Highways	Annual Average Daily Traffic AADT, T, (pcu/day)	Gross domestic products, G (N)	Population density (people/km ²)	Vol. of Potholes, V(m ³)	Flood Depth, D (m)	
Abia	Umuahia –	Federal	Ministry of	National	FMW&H,	National	
	Ikot-Ekpene	Ministry of	Budget and	Population	Umuahia	Emergency	
	Expressway	Works and	Planning	Commission		Management	
	(UIE)	Housing	(MBP),	(NPC),		Agency	
		(FMW&H),	Umuahia	Umuahia		(NEMA),	
		Umuahia				Umuahia	
Anambra	Onitsha –	FMW&H,	MBP,	NPC, Awka	FMW&H,	NEMA,	
	Owerri Road	Awka	Awka		Awka	Awka	
	(OOR)						

Table 1: Highways, states of affiliation and sources of data

1.2 DATA COLLECTION 1.2.1 Annual Average Daily Traffic

This is expressed as the passenger car units (pcu/day). It is a factor used to state the number of cars needed to theoretically replace a non-passenger vehicle to simulate the same effect on a road or intersection. Traffic volume counts were conducted along the various selected highways under study using an Automatic Traffic Recorder (ATR), and Automatic Vehicle Classifier (AVC) in order to

collect the categorized vehicle axle loading. The ATR is a fixed type which automatically records the number of vehicles crossing a section of the road in a desired period. The working is caused by the effect of impulses caused by the traffic movements on a pneumatic hose placed across the roadway. In the electrical recorder, the count is actuated by the closing of the electric circuit by the passage of a vehicle. The photocell recorder and the automatic recorder were actuated by the interruptions of light



beam falling on the photocell on the road side as the vehicle passes. In the case of pneumatic method, the counter is actuated by an air switch attached to a flexible hose stretched across the road over which the vehicle passes.

1.2.2 Environmental Factors

The environmental factors considered in this research include the pavement condition index (PCI) and the flood depth (D). The major variable obtained and used in this research from pavement condition indices (PCI) includes – volume of potholes (V), and the flood depth (D). An automated technique for pavement evaluation was used to record the flood depth and the volume of potholes. Film and video devices were also used to record the

Pavement surface conditions which were later analysed manually and the volumetric dimensions of the volume of potholes and the flood depths obtained. The value of the pothole was computed as a volume parameter. The length, width and depth of each pothole on the highway were measured and the volume (in m³) obtained. The value obtained from all the potholes are summed up cumulatively to arrive at a total volume in each year. The flood depth is obtained by measuring the linear depth (in meter) of water on the pavement surface along the length of the highway under study. This was done on daily or weekly basis depending on the rate of flooding in a particular season. At the end of the year, the total flood depth is recorded. The environmental factor data were obtained from the states' ministries of works and transport. These data are used to calculate maintenance feasibility and cost evaluation by the Federal Roads Maintenance Agency (FERMA).

2.2.3. Gross Domestic Product

These data were obtained from the respective states' Ministry of Budget and Planning (MBP) for a consecutive period of twenty (20) years. The information about economic budget and planning of the states are stored in this ministry.

2.2.4. Population Density

The data are obtained from the respective states' National Population Commission (NPC) for a consecutive period of twenty (20) years. This commission keeps the data of all the human population in each year.

2.3. MODEL FORMULATION AND CALIBRATION

2.3.1 Assumptions Used In the Model

The following assumptions are considered:

1. The origin and destination of all the vehicles traversing the highway under study emanate from

2. Each traffic stream forecast is a representative of the highway under investigation.

3. The volume of traffic on each highway is dependent on the pavement condition index, flood depth, gross domestic product and the population density of the associating area.

4. The potholes are regular in shape and the dimensions obtained linearly.

5. The flood water is assumed to be stationary or at a very low velocity as at the time of measuring flood depth.

2.3.2.Boundary Conditions.

1. When the value of any regression constants equals zero, the linearized form of the equations is applied.

2.3.3. Calibration of the Model

average daily traffic and the environmental factors The linear empirical regression model is applied in calibration of the relationship among the annual average daily traffic, the economic and demographic factors (gross domestic products and population density), while the multiple (non-linear) empirical regression model is used to calibrate the relationship among the annual (volume of potholes and the flood depth).

Let;

T = Annual Average Daily Traffic, AADT (pcu/day)

- G = Gross Domestic Product, GDP(N)
- P = Population Density, (people/km²)
- V = volume of potholes (m³)
- D = flood depth(m)

Using T = f(G, P, V, D) (2)

The modeling is done using the two types of regression viz;

- 1. Linear Regression: The linear relationship involving T, the economic and demographic factors (G and P as the independent variables).
- 2. Multiple (Non-Linear) Regression: The multiple (non linear) relationship involving T and the pavement environmental condition indices (V and D as the independent variables).

Case 1: Linear Regression.

Let T = f(G, P) (3)

Let $T = a_1 + a_2G + a_3P$ (4) Applying the method of least square sum minimization to estimate the value of the regression constants, a_1, a_2, a_3 . [16]. Let,

$$\overline{v_1} = \lambda_0 + \lambda_1 x_1 + \lambda_2 x_2 \quad (5)$$

Comparing equations 3.3 and 3.4



 $\overline{y_1} = T$ $\lambda_0 = a_1$ $\lambda_1 = a_2$ $x_1 = G$ $\lambda_2 = a_3$ $x_2 = P$ Using the minimum value analysis; $S^2 = \sum (y - \overline{y_1})^2,$ (6)

$$S^{2} = \sum (y - \lambda_{0} - \lambda_{1} x_{1} - \lambda_{2} x_{2})^{2} = 0$$
 (7)

Using the partial derivatives of the sum of the minimized equation;

$$\frac{\partial S^2}{\partial \lambda_0} = -\sum (y - \lambda_0 - \lambda_1 x_1 - \lambda_2 x_2) = 0 \quad (8)$$

$$\frac{\partial S^2}{\partial \lambda_0} = -\sum y + \sum \lambda_0 + \sum \lambda_1 x_1 + \sum \lambda_2 x_2 = 0 \quad (9)$$
Where

 $S^2 =$ sum of squares of deviation of points $\sum y_1 = n\lambda_0 + \lambda_1 \sum x_1 + \lambda_2 \sum x_2$ (10)

Where

$$n = number of data points, in this study, n = 20years (n = 20)$$

$$\frac{\partial S^2}{\partial \lambda_1} = -\sum (y - \lambda_0 - \lambda_1 x_1 - \lambda_2 x_2)(x_1)(11) \quad (11)$$

$$\frac{\partial S^2}{\partial \lambda_1} = -\sum y x_1 + \sum \lambda_0 x_1 + \sum \lambda_1 x_1^2 + \sum \lambda_2 x_2 = 0$$

$$0(12) \quad (12)$$

$$\sum x_1 y_1 = \lambda_0 \sum x_1 + \lambda_1 \sum x_1^2 + \lambda_2 \sum x_1 x_2 = 0 \quad (13)$$

$$(13)$$
Similarly;

$$\frac{\partial S^2}{\partial \lambda_2} = -\sum (y - \lambda_0 - \lambda_1 x_1 - \lambda_2 x_2 - \lambda_3 x_3 - \lambda_1 x_2 x_2 + \lambda_1 \sum x_1 x_2 + \lambda_2 \sum x_2^2 = 0 \quad (15)$$

Solving the system of the simultaneous equations 10, 13, 15 Using Cramer's Rule:

 $n\lambda_0 + \lambda_1 \sum x_1 + \lambda_2 \sum x_2 = \sum y_1$ $\lambda_0 \sum x_1 + \lambda_1 \sum x_1^2 + \lambda_2 \sum x_1 x_2 = \sum x_1 y_1$ $\lambda_0 \sum x_2 + \lambda_1 \sum x_1 x_2 + \lambda_2 \sum x_2^2 = \sum x_2 y_1$

Case 2: Multiple (Non-Linear) Regressions. Let T = f(V, D)(16)(16) $\text{Let}T = a_4 V^{a_5} D^{a_6}(17)$ (17)Where a_4 , a_5 , and a_6 are the regression constants. Linearizing equation 3.16 by taking logarithms of both sides; $lnT = lna_4 V^{a_5} D^{a_6}$ (18) (18) $\ln T = \ln a_4 + a_5 \ln V + a_6 \ln D \ (19)$ Let, $\overline{y_2} = \varDelta_0 + \varDelta_1 x_3 + \varDelta_2 x_4 \tag{20}$ (20)Comparing equations 3.18 and 3.19 $\overline{y_2} = lnT$ $\Delta_0 = ln a_4$ $\Delta_1 = a_5$ $x_3 = ln V$

$$\Delta_2 = a_6$$
$$x_4 = \ln D$$

Similarly, using the method of least square sum minimization to solve equation 20, We get;

$$n\Delta_{0} + \Delta_{1}\sum x_{3} + \Delta_{2}\sum x_{4} = \sum y_{2} (21)$$
(21)

$$\Delta_{0}\sum x_{3} + \Delta_{1}\sum x_{3}^{2} + \Delta_{2}\sum x_{3}x_{4} = \sum x_{3}y_{2} (22) (22)$$

$$\Delta_{0}\sum x_{4} + \Delta_{1}\sum x_{3}x_{4} + \Delta_{2}\sum x_{4}^{2} = \sum x_{4} y_{2} (23)$$
(23)

Apply Cramer's Rule in solving the systems of the simultaneous linear equations 21, 22 and 23 The values of x_1, x_2, x_3, x_4 and y were obtained from the values of the AADT corresponding to the environmental equivalent and demographic characteristics of the respective highways.

III. **RESULTS AND DISCUSSION**

The traffic, environmental and demographic characterization of the highways under study is shown below.

3.1. Umuahia – Ikot Ekpene Expressway

(15)

Table 2: Characterization of traffic, environmental and demographic factors for Umuahia – Ikot Ekpene

YEAR	Total Traffic, T =y ₁ (pcu/day)	Gross Domestic Product, $G = x_1(\mathbf{N})$	Population Density, P = x ₂ (people/sq.k m)	Vol. of Potholes, V (m ³⁾	Flood Depth , D (m)	x ₃ = lnV	x ₄ = lnD	$\ln T = y_2$
2001	973,080	14,000,000	310,122	16.01	1.03	0.03	7.69	13.79
2002	1,052,074	16,000,000	376,019	17.31	1.08	0.08	8.13	13.87
2003	1,016,245	18,500,000	393,172	16.72	1.06	0.06	7.93	13.83
2004	1,062,358	17,800,000	411,009	17.48	1.08	0.08	8.19	13.88
2005	1,120,621	21,000,000	439,327	18.43	1.11	0.10	8.49	13.93



2006	1,075,159	18,300,000	478,005	17.69	1.09	0.09	8.25	13.89
2007	1,143,409	22,000,000	490,119	18.81	1.13	0.12	8.61	13.95
2008	1,217,677	26,700,000	509,837	20.03	1.17	0.16	8.98	14.01
2009	1,071,906	21,100,000	508,911	17.63	1.09	0.09	8.23	13.88
2010	1,340,265	20,350,000	510,201	22.05	1.23	0.21	9.57	14.11
2011	1,102,079	25,700,000	499,012	18.13	1.10	0.10	8.40	13.91
2012	1,336,435	27,100,000	510,010	21.99	1.23	0.21	9.55	14.11
2013	1,312,804	24,110,000	518,911	21.60	1.22	0.20	9.44	14.09
2014	1,512,527	21,302,000	529,712	24.89	1.33	0.29	10.33	14.23
2015	1,555,769	18,500,000	525,017	25.60	1.35	0.30	10.51	14.26
2016	1,365,946	19,019,900	524,917	22.47	1.25	0.22	9.69	14.13
2017	1,296,429	25,199,000	570,301	21.33	1.21	0.19	9.36	14.08
2018	1,530,156	23,010,100	510,989	25.17	1.34	0.29	10.40	14.24
2019	1,541,130	24,100,000	597,328	25.35	1.35	0.30	10.45	14.25
2020	1,295,135	21,092,803	571,900	21.31	1.21	0.19	9.36	14.07

Table 2 shows the variation of traffic composition of Umuahia - Ikot Ekpene expressway for a period of twenty (20) years. The gross domestic products and the population density from the highway are also shown as indicated. The GDP was maximum in 2012 at a value of $N27.1 \times 10^6$, and lowest in 2001 at a value of $N14x10^6$. The population density of 597,328people/km² was the maximum recorded in 2019, while the minimum was 310,122people/km² in 2001. The values of the environmental factors are indicated by the pavement condition index (factored by the volume of pot holes, and the flood depth). The traffic on the highway is composed mostly of motor cars, buses/minibuses, trucks and motorcycles. Maximum total traffic was recorded in 2015 with a total of 1,555,797pcu/day, while the lowest traffic volume was 973,080pcu/day in 2001. The volume of pot holes were maximum in 2015, measured at 25.6m³, while the lowest volume of pot hole was 16.01m³ in 2001. This is an indication that the high volume of traffic in 2015 contributed to the degradation of the pavement surface and the sub base, thus creating more pot holes.

Decoding the values of X (x_1, x_2, x_3, x_4) , y, from Tables 2

Case 1: Linear Regression

$$\begin{array}{rl} 20\lambda_0 + \ 4.25x10^8\lambda_1 + \ 9.79x10^6\lambda_2 = 2.49x10^7 \\ 4.25x10^8\lambda_0 + \ 5.74x10^{15}\lambda_1 + \ 1.33x10^{14}\lambda_2 \\ &= 3.43x10^{14} \\ 9.79x10^6\lambda_0 + \ 1.33x10^{14}\lambda_1 + \ 3.14x10^{12}\lambda_2 \\ &= 8.11x10^{12} \end{array}$$

Note: The coefficients of λ_0 , λ_1 , λ_2 , Δ_0 , Δ_1 , and Δ_2 are written in MatLab coding format in multiples of 10^{15} , while the column vector C is written in multiples of 10^{14} A = 1.0e+015 *

0.0000 0.0000 0.0000

0.0000	3.4300	1.3300		
0.0000	0.0811	0.0314		
A3 = 1.0e-	+015 * (0	Obtained	by replacing	column 3

by C) 0.0000 0.0000 0.0000

0.0000 5.7400 0.3430

 $0.0000 \quad 0.1330 \quad 0.0081$

Computation of the determinants: A, A1, A2, A3; ans = A = -3.8561e + 027ans = A1 = -1.1241e + 032

$$ans = A1 = -1.1241e + 0.052$$
$$ans = A2 = -2.6591e + 0.032$$

ans = A3 = -1.0857e + 028

% apply Cramer's Rule;

$$\lambda_0 = \frac{\det \mathbb{Q}(A)}{\det \mathbb{Q}(A)} \tag{24}$$

$$\lambda_1 = \frac{\det \mathbb{Q}A2}{\det \mathbb{Q}A}$$
(25)

$$\lambda_2 = \frac{\det \mathbb{Q}A3}{\det \mathbb{Q}A}$$
(26)



$\lambda_0 = 2.92 \times 10^4$
$\lambda_1 = 689.57$
$\lambda_2 = 2.82$

$$f_{\lambda} = (\lambda_0, \lambda_1, \lambda_2) = (2.92 x 10^4 \,, \, 689.57, \, 2.82)$$

From calibrated equation 4

But;

 $\lambda_0 = a_1$ $\lambda_1 = a_2$ $\lambda_2 = a_3$

 $T = a_1 + a_2 G + a_3 P$

Therefore, the multiple (nonlinear regression model equation for Umuahia - Ikot Ekpene Expressway becomes;

$$T = 2.92x10^4 + 689.57G + 2.82P \tag{27}$$

Case 2: Multiple (Non-Linear) Regressions

 $20\varDelta_0 + 60.19\varDelta_1 + 3.29\varDelta_2 = 155.47$ $60.19\varDelta_0 + 181.58\varDelta_1 + 10.15\varDelta_2 = 844.60$ $3.29\varDelta_0 + 10.15\varDelta_1 + 0.69\varDelta_2 = 46.37$ A =20.0000 60.1900 3.2900 60.1900 181.5800 10.1500 3.2900 10.1500 0.6900 C = 155.4700 844.6000 46.3700 A1 = (Obtained by replacing column 1 by C)155.4700 60.1900 3.2900 844.6000 181.5800 10.1500 46.3700 10.1500 0.6900 A2 = (Obtained by replacing column 2 by C)20.0000 155.4700 3.2900 60.1900 844.6000 10.1500 3.2900 46.3700 0.6900 A3 = (Obtained by replacing column 3 by C)20.0000 60.1900 155.4700 60.1900 181.5800 844.6000 3.2900 10.1500 46.3700

Computation of the determinants: A, A1, A2, A3 ans = A = 0.0665

ans = A1 = -2.7837e + 003ans = A2 = 1.0176e + 003ans = A3 = -1.6919e + 003

% apply Cramer's Rule;

$$\begin{array}{lll} \Delta_{0} &= \frac{\det{(A1)}}{\det{(A)}} & (28) \\ \Delta_{1} &= \frac{\det{(A2)}}{\det{(A)}} & (29) \\ \Delta_{2} &= \frac{\det{(A3)}}{\det{(A)}} & (30) \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & \\ & & & & \\ & & &$$

From calibrated equation 17

$$T = a_4 V^{a_5} D^{a_6}$$

Recall;

9

$$\Delta_0 = \ln a_4$$

$$\Delta_1 = a_5$$

$$\Delta_2 = a_6$$

3.2. Onitsha – Owerri Road

$$\ln a_4 = \Delta_0 = -4.18 \times 10^4$$

$$\ln a_4 = \log_e a_4 = -4.18 \times 10^4$$

$$a_4 = exp(-4.18x10^4) = 0.$$

Since the regression constant, $a_4 = 0$, we apply the inearized equation.
Thus, from equation 19, $\ln T = \ln a_4 + a_5 \ln V + a_6 \ln D$, $(a4 \rightarrow 0)$
Substitute the values of a_4 , a_5 , and a_6 into equation 19, we have;

$$\ln T = 0 + 1.53x10^4 \ln V + (-2.54x10^4 \ln D)$$

the

V +

(31)

Therefore, the multiple non - linear regression model equation for Umuahia - Ikot Ekpene Expressway becomes;

$$ln T = 1.53x 10^4 ln V - 2.54x 10^4 ln D$$
 (32)

YEAR	Total Traffic, T =y ₁ (pcu/day)	Gross Domestic Product, $G = x_1(\mathbb{N})$	Populati on Density, P = x ₂ (people/s q.km)	Vol. of Potholes, V (m ³⁾	Floo d Dept h, D (m)	x ₃ = lnV	x ₄ = lnD	In T = y ₂
2001	1,343,494	26,000,000	290,121	13.00	2.33	2.56	0.85	14.11
2002	1,370,042	21,000,000	409,233	10.32	1.99	2.33	0.69	14.13
2003	1,200,085	24,000,000	301,023	9.11	2.03	2.21	0.71	14.00
2004	1,284,762	25,000,000	390,410	12.01	2.67	2.49	0.98	14.07

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2005	1,149,506	23,200,000	472,001	14.90	2.33	2.70	0.85	13.95
2006	1,046,327	26,340,000	423,197	10.01	1.68	2.30	0.52	13.86
2007	1,052,028	24,113,000	419,333	11.73	2.11	2.46	0.75	13.87
2008	897,216	30,910,000	488,001	9.02	1.78	2.20	0.58	13.71
2009	1,179,681	29,990,000	530,745	8.21	2.61	2.11	0.96	13.98
2010	1,265,230	32,102,300	700,271	6.03	2.44	1.80	0.89	14.05
2011	1,222,022	30,287,300	630,712	4.02	1.90	1.39	0.64	14.02
2012	1,127,240	34,288,000	701,491	3.97	2.01	1.38	0.70	13.94
2013	1,444,400	37,290,000	579,340	3.01	1.88	1.10	0.63	14.18
2014	1,590,931	41,209,000	633,020	4.08	1.31	1.41	0.27	14.28
2015	1,431,409	29,090,000	649,099	2.11	1.70	0.75	0.53	14.17
2016	1,684,165	31,230,000	500,239	2.78	1.93	1.02	0.66	14.34
2017	1,287,679	37,980,000	672,022	2.40	1.91	0.88	0.65	14.07
2018	1,405,773	41,098,000	711,038	1.92	1.47	0.65	0.39	14.16
2019	1,589,349	44,220,000	830,922	1.87	1.88	0.63	0.63	14.28
2020	1,602,498	47,900,000	790,641	2.03	1.92	0.71	0.65	14.29

Table 3 shows the variation of traffic composition of Umuahia - Ikot Ekpene expressway for a period of twenty (20) years. The gross domestic products and the population density from the highway are also shown as indicated. The GDP was maximum in 2012 at a value of $N27.1 \times 10^6$, and lowest in 2001 at a value of $N14x10^6$. The population density of 597,328people/km² was the maximum recorded in 2019, while the minimum was 310,122 people/km² in 2001. The values of the environmental factors are indicated by the pavement condition index (factored by the volume of pot holes, and the flood depth). The traffic on the highway is composed mostly of motor cars, buses/minibuses, trucks and motorcycles. Maximum total traffic was recorded in 2015 with a total of 1,555,797pcu/day, while the lowest traffic volume was 973,080pcu/day in 2001. The volume of pot holes were maximum in 2015, measured at 25.6m³, while the lowest volume of pot hole was 16.01m³ in 2001. This is an indication that the high volume of traffic in 2015 contributed to the degradation of the pavement surface and the sub base, thus creating more pot holes.

Decoding the values of X (x_1, x_2, x_3, x_4) , y, from Tables 3

Case 1: Linear Regression

 $\begin{aligned} 20\lambda_0 + 6.37x10^8\lambda_1 + 1.11x10^7\lambda_2 &= 2.62x10^7\\ 6.37x10^8\lambda_0 + 2.14x10^{16}\lambda_1 + 3.74x10^{14}\lambda_2\\ &= 8.51x10^{14}\\ 1.11x10^7\lambda_0 + 3.74x10^{14}\lambda_1 + 6.65x10^{12}\lambda_2\\ &= 1.48x10^{13} \end{aligned}$

Note: The coefficients of λ_0 , λ_1 , and λ_2 are written in MatLab coding format in multiples of 10^{16} , while the column vector C is written in multiples of 10^{14} A = 1.0e+016 *

0.0000	0.0000	0.0000				
0.0000	2.1400	0.0374				
0.0000	0.0374	0.0007				
C = 1.0e + 0)14 *					
0.0000						
8.5100						
0.1480						
A1 = 1.0e	+016 *(0	Obtained	by	replacing	column	1
by C)			•			
0.0000	0.0000	0.0000				
0.0851	2.1400	0.0374				
0.0015	0.0374	0.0007				
A2 = 1.0e	+014 *(0	Obtained	by	replacing	column	2
by C)	,		2	1 0		
0.0000	0.0000	0.0000				
0.0000	8.5100	3.7400				
0.0000 0	.1480 0	.0665				

A3 = 1.0e+016 *(Obtained by replacing column 3 by C)

0.0000 0.0000 0.0000 0.0000 2.1400 0.0851 0.0000 0.0374 0.0015 Computation of the determinants; A, A1, A2, A3; ans = A = 2.5058e + 027 ans = A1 = 2.0640e + 033 ans = A2 = 5.5820e + 025ans = A3 = -1.0079e + 027

% apply Cramer's Rule;

$$\lambda_0 = \frac{\det \left(A1\right)}{\det \left(A\right)} \tag{33}$$

 $\lambda_1 = \frac{\det (A2)}{\det (A)} \tag{34}$



$\lambda_{2} = \frac{det \mathbb{Q}A3}{det \mathbb{Q}A} $ (35) $\lambda_{0} = 8.2373e+005$ $\lambda_{1} = 0.0223$ $\lambda_{2} = -0.4022$ $f_{\lambda} = (\lambda_{0}, \lambda_{1}, \lambda_{2}) = (8.23x10^{5}, 0.02, -0.40)$ From calibrated equation 4 $T = a_{1} + a_{2}G + a_{3}P$ But:	A3 = 20.0000 33.0700 281.0000 33.0700 64.9700 463.9300 13.5100 23.6600 189.9600 Computation of the determinants; A, A1, A2, and A3 ans = A = 87.3172 ans = A1 = $1.2143e+003$ ans = A2 = -12.1758
$\lambda_{2} = a_{1}$	ans = A3 = 48.3240
$\lambda_0 = \alpha_1$ $\lambda_1 = \alpha_2$	% apply Cramer's Rule;
$\lambda_1 = \alpha_2$	$\Delta_0 = \frac{det(A)}{det(A)} \tag{37}$
Therefore, the multiple linear regression model	
equation for Onitsha – Owerri road becomes:	$\Lambda_1 = \frac{\det \left[(A2) \right]}{(A2)} \tag{38}$
$T = 8.23x10^5 + 0.02G - 0.4P \tag{36}$	$= 1 \qquad \text{det } (A) \tag{30}$
Case 2: Multiple (Non-Linear) Regressions	$\Delta_2 = \frac{\det \mathbb{Q}(3)}{\det \mathbb{Q}(4)} \tag{39}$
$20\Delta_0 + 33.0/\Delta_1 + 13.51\Delta_2 = 281$	$\Delta_0 = 13.9067$
$33.0/\Delta_0 + 64.9/\Delta_1 + 23.66\Delta_2 = 463.93$	$\Delta_1 = -0.1394$
$13.51\Delta_0 + 23.66\Delta_1 + 9.72\Delta_2 = 189.96$	$\Delta_2 = 0.5534$
A = 20,0000,22,0700,12,5100	$f_{\Delta} = (\Delta_0, \Delta_1, \Delta_2) = (13.91, -0.14, 0.55)$
20.0000 55.0700 15.5100	From calibrated equation 17
13 5100 23 6600 9 7200	$I = a_4 V^{\omega_5} D^{\omega_6}$
C -	Recall; $A = lmg$
281,0000	$\Delta_0 = i \hbar a_4$
463.9300	$\Delta_1 - a_5$
189.9600	$u_2 - u_6$
A1 =	$ln a_4 = a_0 = 13.91$
281.0000 33.0700 13.5100	$u_{4} - u_{6} u_{4} - 15.91$ $a_{1} - arm(12.91) - 1.1 \times 10^{6}$
463.9300 64.9700 23.6600	$u_4 - e_x p(13.51) - 1.1 \times 10$ Therefore the non linear calibrated equation for
189.9600 23.6600 9.7200	Onitsha – Owerri road becomes:
A2 =	$T = 1.1 r 10^{6} V^{-0.14} D^{0.55} $ (40)
20.0000 281.0000 13.5100	$n = 1.1 \times 10^{6} V$ D (40)
33.0700 463.9300 23.6600	$T = \frac{1}{V^{0.14}} \tag{41}$
13.5100 189.9600 9.7200	

S/N	Name of Highway	Model Equations				
		Linear	Multiple or Non-Linear			
1.	Umuahia – Ikot Ekpene Expressway	$T = 2.92 \times 10^4 + 689.57G + 2.82P$	$ln T = 1.53 \times 10^4 ln V - 2.54 \times 10^4 ln D$			
2.	Onitsha – Owerri Road	$T = 8.23 \times 10^5 + 0.02G - 0.4P$	$T = \frac{1.1 \times 10^6 D^{0.55}}{V^{0.14}}$			

Table 4: Summary of the multiple regression model equations as computed using MatLab.

Multiple and linear regression prediction models relate the annual average daily traffic (T) for a given highway section to several factors that affect the traffic movement such as economic, demographic and environmental indices. The models can be applied to predict the value of the total traffic when the demographic, economic and the environmental characteristics of the highway are known. Contrarily, [1], forecasted the projected traffic stipulation based on the Vehicles Registrations Data (VRD). In this regard, the VRD



is an important parameter for traffic growths and for resultant supportive data.

From the mathematical models of the equations in table 4, it implies that the volume of pot holes and the flood depth have a non linear relationship with the AADT, while the gross domestic products and the population density have a linear relationship with the AADT. Consequent upon this, their combined and unilateral effects contributes to the variation of AADT on the highways under study. [1] used the collected data to formulate the correlation linking the logarithmic value of the quantity of automobiles on the inventory.

[14] discovered that the elasticity of transportation is the evaluation of the percentage variation in transportation demand with regard to percentage variation in the factors affecting the demand. They presented the forecaster formula as:

Elastic coefficient =

 $\begin{array}{l} \frac{\text{Percentage variation in trasport indicators}}{\text{Percentage variation in economic indicators}} \\ \text{(42)} \\ \text{The predicting formula of growth rates is given as;} \\ \text{G} = \text{e.G}_{\text{GDP}} \\ \text{Where } \text{e} = \text{elasticity of transport demand,} \\ \end{array}$

 G_{GDP} = gross domestic product growth rate (%) G = growth rates (%)

In contrast, one image system by method of automated identification of satellite imagery applicable to a big district was used to approximate AADT [17]. Consequently, a ground-fixed information source is required for the improvement and approximation execution since sound related effect with the imagery can cause obstruction with this method.

From their findings, the AADT can be computed with the following equation;

$$AADT_{ij} = \frac{N_{ij}}{L_i/S_i} x \frac{1}{7R_w} x \frac{1}{365R_m}$$
(44)

Where; i = road segments,

j = type of vehicle,

 $R_{\rm w}$ = transforming ratio of traffic in an hour to week after week, contingent upon weekly distribution and the particular hour in the week,

 R_m = transforming ratio of periodic traffic (from daily to annual), dependence on the monthly distribution and the particular day in a year

3.4. THE GRAPHICAL REPRESENTATION OF THE EFFECTS ENVIRONMENTAL AND DEMOGRAPHIC DATA ON ANNUAL AVERAGE DAILY TRAFFIC. 3.4.1. Umuahia – Ikot Ekpene Expressway



Figure 1: The relationship of annual average daily traffic with gross domestic product for Umuahia – Ikot Ekpene expressway





Figure 2: The relationship of annual average daily traffic with population density for Umuahia – Ikot Ekpene



expresswa

Figure 3: The relationship of annual average daily traffic with volume of potholes for Umuahia – Ikot Ekpene expressway.





Figure 4: The relationship of annual average daily traffic with flood depth, D, for Umuahia – Ikot Ekpene expressway.



3.4.2.Onitsha – Owerri Road

Figure 5: The relationship of annual average daily traffic with gross domestic product for Onitsha – Owerri road.









Figure 7: The relationship of annual average daily traffic with volume of potholes for Onitsha – Owerri road.





Figure 8: '	The relationship of annua	average daily traffic	with flood depth for	Onitsha - Owerri road.
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Region (State)	Name of highway	Total Traffic correlation with;	\mathbb{R}^2	R	Best fit
Abia	Umuahia - Ikot Ekpene expressway	Gross domestic products, G	0.1530	0.3912	0.9975
		Population density, P	0.6276	0.7922	
		Volume of potholes, V	0.9717	0.9857	
		Flood depth, D	0.9950	0.9975	
Anambra	Onitsha - Owerri road	Gross domestic products, G	0.3166	0.5627	0.6573
		Population density, P	0.2339	0.4836	
		Volume of potholes, V	0.4321	0.6573	
		Flood depth, D	0.1729	0.4158	

Table 5: Discrete values of correlation coefficients, R, for the highways

The values of R indicate the degree of goodness of fit between the model and the measured parameters. When R is close to 1, it means that the equation represents the measured AADT values very well, and forecast of future values may be good as well. From table 4.12, the correlation between the annual average daily traffic indicates a very good fit between the model, the flood depth and the volume of potholes for Umuahia – Ikot Ekpene expressway.

This is demonstrated by the value of the coefficient of correlation, R = 0.9975 and R = 0.9857 for flood depth and volume of potholes. There is also a comparatively good fit between the model and the parameters for the correlation among AADT, population density and the gross domestic products. The R-values for the population density and gross domestic products are 0.7922 and 0.3922 respectively. The potholes along the Umuahia – Ikot



Ekpene expressway are not always allowed to persist in its existence. There are always highway maintenance interventions by several government agencies on periodic basis. Likewise, the volume of flood along the highway does not have a definite pattern of existence over the years under study. Heavy rainfall may cause a temporary flooded surface on the pavement, but may not deter motorist to ply the road for a long period of time. This made the correlation between the model and the flood depth to have a better fit than the gross domestic products and population density.

On Onitsha – Owerri road, the best coefficient of correlation, R, is obtained when the total traffic is correlated with volume of potholes (V), the value being 0.6573. This is an indication of the fact that the absence of alternative routes to substitute for the dilapidated highway, motorists continue to ply the road despite the expansion of the pot holes on the pavement surface. The minimum value of R (0.4158), obtained when AADT is correlated with flood depth (D), shows that the traffic intensity of the Onitsha – Owerri road is responsive to the volume of annual flooding on the

highway pavement. There is an anomalous relationship between AADT and the GDP. Political and commercial disorderliness and disagreement has contributed to the deficit in the correlation value due to the fact that residences seek to take refuge elsewhere when the socio-political or communal conditions become unhealthy. This is an indication of the fact that the highway under responds positively with the rate of flooding caused by the seasonal changes and the overflow of the proximal water bodies. This contributed to the synchronized growth of AADT and the flood depth.

3.5. SUMMARY OF THE REGRESSION MODEL EQUATIONS OBTAINED FROM THE GRAPHICAL ANALYSIS.

The graphical analysis displayed the modeled equations relating the independent variable, T with the dependent variables, G, P, V and D. The coefficients of the various dependent variables are approximated to two (2) decimal places.

AADT (T) relationship with:		
	Abia.	Anambra.
	Umuahia – Ikot Ekpene Expressway	Onitsha – Owerri Road
Gross domestic products, G	T = 0.02G + 804111	T = 0.02G + 814188
Population density, P	T = 3.29P + 425865	$T = 0.77P + 1x10^6$
Volume of pot holes, V	$T = 76111e^{0.05V}$	$T = 2x10^6 e^{-0.03V}$
Flood depth, D	$T = 366947e^{1.44D}$	$T = 2x10^{6}e^{-0.2D}$

Table 6. Regression model equations for the selected highways under study

From table 4.3, it can be observed that the annual average daily traffic (AADT), T, is the dependent variable, and is related linearly with the gross domestic products, G, and the population density, P. Whereas the relationship between among the independent variables V and D with the dependent variable T, is non linear.

IV. CONCLUSION AND RECOMMENDATIONS

4.1. CONCLUSION

From the experience gained in the course of the research, the following conclusion may be drawn:

- i. Annual average daily traffic is a non steady system that changes without a definite pattern.
- ii. Mathematical modeling of engineering systems (traffic engineering inclusive) minimizes

engineering time spent, reduces prototype productions, and optimizes product quality.

- iii. Field collection of data is a sine qua non in validating the results obtained from the computer simulation of the developed models of the physical systems.
- iv. Accurate modeling of traffic engineering with physical quantities associated with human enterprise is vital for highway design and national development.
- v. Knowledge of the traffic forecast on highways helps to mitigate road conflicts and congestions.
- vi. Adequate exposure of students to the various traffic modeling and software packages is very vital in understanding the complex nature of engineering systems.
- vii. The environmental and demographic data were used in the formulation of the mathematical



model for the forecasting of the annual average daily traffic for the five highways using the MatLab computational software

- viii. The annual average daily traffic for the highways under study was determined using various traffic compositions. This was obtained by using a mechanical and automated method of traffic counting.
- ix. Automatic Traffic Recorder (ATR) and Automatic Vehicle Classifier (AVC) were the machines applied during the traffic counting.
- x. The traffic characteristics of the highways were vividly stated for the period of twenty (20) years. The data were obtained from the various commands of the Federal Road Safety Corps in the various designated states
- xi. The effects of the environmental and demographic factors were demonstrated using the graphical analogy.
- xii. The traffic responsiveness to the variation of gross domestic products as occasioned by changes in government policies were noticed in the data.

4.2. RECOMMENDATIONS

- This research is highly recommended for various highway design and construction firms. It can be used to determine the past and future traffic composition of a particular highway which will help in the rehabilitation design and construction programmes.
- ii. Real estate developers can apply the research in their housing development projects as the population patterns of the proximal environment of the highways indicate the density of human settlement.
- iii. The characteristic traffic compositions of the various highways are very important to the state and national transportation agencies as it aids AADT estimation.
- iv. The traffic counts obtained is very imperative for planners and designers as a useful tool in conducting mechanistic empirical pavement design analysis.
- v. It is also recommended for the national emergency rescue workers as the flooding characteristics of the various highways are clearly stated.
- vi. Road users who are not familiar with the highways will find the research useful. It will help to determine the effects of various economic and human factors to traffic volume.
- vii. Highway and transportation engineering researchers can apply the research as a useful tool in carrying out further studies on mathematical modeling.

- viii. It can also be recommended for government at levels to apply the research to determine areas with high productivity and commercial zones, hence, select the best and suitable policies to safeguard the economic incentives of the masses.
- ix. The government can use the research to determine the highways that needs emergency attentions in terms of rehabilitation and maintenance. The flood depth and volume of potholes can be the factorials of the maintenance programmes.

This paper has evaluated several experimental and modeling data which can be used by other researchers to conduct a more precise modeling study on the impact of the economic, environmental and demographic changes on the Nigerian highway and transportation systems.

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