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Mathematical Modeling on the Effects of Vaccination, Educational Campaign and Treatment on Typhoid Fever Transmission Dynamics

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ABSTRACT
This study presents a mathematical model to investigate the impact of vaccination and educational campaigns on the dynamics of typhoid fever transmission. The model incorporates key epidemiological parameters and vaccina tion coverage rates to assess the potential outcomes of implementing vaccination and educational interventions in reducing the spread of typhoid fever.
Keywords: Control reproduction number, Typhoid fever, Vaccination, Medically hygienic, Sensitivity analysis.

I. INTRODUCTION

The bacterium Salmonella typhi is the source of typhoid fever (TF), a potentially fatal illness that is one of the many regrettable outcomes of inadequate access to sanitary facilities and clean water, which is a problem in many parts of the world J. K. Nthiiri (2016). Understanding the dynamics of its transmission is crucial for designing effective prevention and control strategies. The Indian subcontinent, Africa, South and Southeast Asia, and South America all still have high rates of TF. Despite the fact that it has been mostly contained in North America and Europe National Health Service (2021). The largest prevalence is seen in children National Health Service (2021). Interestingly, humans are the only known natural host and reservoir for typhoid, and they are also the carriers who continue to spread the disease even when they are asymptomatic C. Masuet-Aumatell and J. Atouguia (2021); C. B. Acosta-Alonzo, I. V. Erovenko, A. Lancaster , H. Oh, J. Rycht´aˇr and D.Taylor (2020). Typhoid fever is thought to be the cause of approximately 16 million instances of disease and over 600,000 fatalities worldwide annually Edward, .S. (2017).

A TF infection can cause severe headaches, a high temperature that lasts for a long time, nausea, vomiting, and tiredness which are the main symptoms. The bacteria primarily target the intestine during their 7–14 day incubation period. According to estimates the annual death toll from typhoid fever between 190,000 and 216,000, and the disease global burden between 13.5 and 26.9 million episodes Edward, .S. (2017). The main preventive measures include a good personal cleanliness, proper sanitation, an educational campaign such as teaching the community to wash their hands with soap, access to healthy food and water, sensitization against open defecation, effective garbage and waste disposal, and vaccination Z. A. Bhutta & Ochiai; C. B. Acosta-A. Lancaster , H. Oh, J. Rycht´aˇr and D.Taylor (2020). Antibiotic treatment is also used. The World Health Organization has recommended that typhoid vaccination be considered for the control of outbreaks and endemic diseases, although its strategic application remains limited K. A. Date & Hyde (2014). In order to modify people's behaviors and increase public knowledge, health education is essential Edward, .S. (2017). In addition, vaccination plays a crucial role in controlling the spread of disease during pandemics and epidemics

Z. A. Bhutta & Ochiai; S. B. Omer, D. A Salmon, W. A Orenstein, M. P. Dehart and N. Halsey (2009). By vaccination, millions of lives are saved every year. By utilizing the body's antibodies to build defense, it has decreased the likelihood of getting sick S. Oztora, G. B. Gokcen and H. N. Dagdeviren (2022).

Many authors regarded Bernoulli's model as the first epidemiological mathematical model, from which he de duced that vaccinations are beneficial and might add three years to a child's life expectancy I. M. Foppa (2016); O. O. Onyejekwe, A. Tigabie, B. Ambachew and A. Alemu (2019). Since the introduction of the first heat-inactivated typhoid vaccine in 1896, vaccinations against the disease have been used to prevent illness. The vaccine was widely utilized in the 20th century due to increased reports from the English and American military. According to reports, the vaccination has decreased the prevalence of typhoid disease by more than 90% when it was first introduced. Similar outcomes were seen in Thailand following the introduction of the typhoid vaccine. Sadly, a large number of TF patients seldom receive vaccinations C. Masuet-Aumatell and J. Atouguia (2021). Additionally, the TF vaccination has shown to function for three to ten years with differing potencies (50 to 95 percent) Organization (2008); C. B. Acosta-Alonzo, I. V. Erovenko, A. Lancaster , H. Oh, J. Rycht´aˇr and D.Taylor (2020).

As a result, it is advised to vaccinate every three to seven years Organization (2019); C. B. Acosta-Alonzo, I. V. Erovenko, A. Lancaster , H. Oh, J. Rycht´aˇr and D.Taylor (2020). Therefore, in order to curb the sickness, the authorities of the infected societies must decide between treatment and/or prevention. They must also begin appropriate public education initiatives that have the potential to alter people's attitudes toward taking preventative measures and receiving medical care. If symptomatic treatment is not improving after more than three days, TF is often mistaken as other persistent feverish conditions. The remaining 10–40% of TF cases treated at hospitals include of individuals who either self-medicate or get outpatient care Z. A. Bhutta & Ochiai.

The intricate and significant of mathematical modeling of the impact of vaccination and educational campaigns efforts on the dynamics of typhoid fever transmission can not be over emphasized. The circulation of disease in the environment will be impacted by vaccination campaigns directed at high-risk groups such as food handlers and school-aged children, as well as by improvements in public hygiene, the use of antibiotics, the availability of clean water, safe food handling practices, and public health education. Vaccination of school-aged children will significantly reduce infection and the pathogen spread and could stop the disease from spreading farther than it can be cured Z. A. Bhutta & Ochiai; Organization (2019). Vaccination campaigns may provide a temporary alternative to ongoing campaigns for the provision of potable water and a clean, sanitary environment. Regardless of the effectiveness of other management approaches, the World Health Organization (WHO) advises vaccination since it can reduce the need for antibiotics, halt the development of resistant S. typhi strains, and foster community. The importance of education, both direct and indirect, in preventing typhoid fever cannot be overstated. Education also has a more profound and long-lasting impact on managing the illness. Thus, education should focus on both the ingestion of pathogen material and human-to-human interaction Edward, .S. (2017).

Since the main barrier to widespread adoption of TF vaccination has been a lack of funding, E. Cheng, N. Gambhirrao, R. Patel, A. Zhowandai, J. Rycht´aˇr and D. Taylor (2020) concluded that vaccination costs should be taken into account when determining the level of vaccination provided to any population, rather than the social benefit C. B. Acosta-Alonzo, I. V. Erovenko, A. Lancaster , H. Oh, J. Rycht´aˇr and D.Taylor (2020). Mathematical models provide a powerful tool for simulating the impact of vaccination, treatment and educational interventions on the spread of typhoid fever within populations. By using mathematical modeling, researchers and public health professionals can assess the potential effectiveness of different vaccination, treatment and educational campaign strategies, predict the long-term impact of these interventions, and make informed decisions about resource allocation and public health policies. Numerous mathematical models pertaining to the dynamics of infectious diseases have been created and examined in order to study the effects of different vaccination, treatment, and isolation policies on the spread of specific infectious diseases T. T. Yusuf and F. Benyah (2012); A. Afolabi and A. Sobowale (2017); T. D. Awoke (2019); M. Rabiu, R. Willie and N. Parumasur (2020); A. Abidemi, M. I Abd Aziz and R. Ahmad (2020); H. M. Yang, L. P Lombardi Junior, F. F. M Castro and A. C. Yang (2021); S. Moore, E. M Hill, M. J. Tildesley, L. Dyson and M. J. Keeling

(2021); V. M. Crankson, O. Olotu, N. Amegbey and A.S. Afolabi (2021); E. Kanyi, A. S Afolabi, and N. O. Onyango (2021); O. I. Idisi and T. T. Yusuf (2021); T. T. Yusuf, A. Abidemi, A. S. Afolabi and E. J. Dansu (2022); T. T Yusuf and A. Abidemi (2023); S. Olaniyi, K. O. Okosun, S. O. Adesanya and R. S. Lebelo (2020); C. B. Acosta-Alonzo, I. V. Erovenko, A. Lancaster , H. Oh, J. Rycht´aˇr and D.Taylor (2020). To investigate the effects of vaccination alone and in combination with treatment and adulticide controls on the dynamics of dengue in Johor, Malaysia, Abidemi and Aziz A. Abidemi and N. A. B. Aziz (2022) developed a deterministic model with vaccination as a constant control rate. According to the scientists, reducing dengue transmission in the interacting populations of humans and mosquitoes can be achieved by using any of the control combination tactics that were investigated. It was determined that a combination of vaccination, treatment, and adulticide control is the most effective approach for preventing and controlling dengue in Johor, Malaysia.

In order to evaluate the effect of government-provided incentives on the dynamics of infectious disease trans mission, K. M. Pal, R. K. Rai, P. K. Tiwari and Y. Kang (2023) built a deterministic compartmental model. These diseases spread through direct contact between susceptible individuals and infected individuals, as well as through environmental contamination. Similar to this, R. K. Gupta, R. K. Rai, P. K. Tiwari, A. K. Misra and M. Martcheva (2023) concentrated on building and carefully analyzing a suitable deterministic compartmental model in order to assess mathematically the effects of disinfectants on the dynamics of disease transmission in the population that arises from both environmental bacteria and direct contacts between susceptible individuals and infected individuals.

In order to assist policy-makers in developing robust optimal non-pharmaceutical strategies to combat the COVID-19 pandemic waves, the work of P. Scarabaggio, R. Carli, G. Cavone, N. Epicoco and M. Dotoli (2021b) proposed a stochastic non-linear model predictive controller. The authors tested the model on the network of Italian regions using real data. The authors in R. Carli, G. Cavone, N. Epicoco, P. Scarabaggio and M. Dotoli (2020) considered an approach involving the joint use of a non-linear model predictive control scheme and a modified epidemiological model based on the susceptible-infected-recovered (SIR) modelling framework with the aim of minimizing the implementation cost of non-pharmaceutical interventions for COVID-19 prevention and control without compromising the capacity of the network of regional healthcare systems ability to function. Moreover, P. Scarabaggio, R. Carli, G. Cavone, N. Epicoco and M. Dotoli (2021a) introduced a unique non-linear time-dependent model that aids in the prediction and analysis of COVID-19 dynamics by decision-makers in the presence of both partially and completely immune community members.

In order to simulate the dynamics of typhoid disease transmission in the human population, Abboubakar and Racke H. Abboubakar and R. Racke (2021) developed a mathematical model that uses the incubation period, an imperfect vaccine incorporating protection, or environmental sanitation and treatment as control mechanisms. The model concentrated on the environment of bacteria and the human population, analyzing the effects of different control combinations through numerical simulation and efficiency analysis. The findings show that treatment is a necessary component of all control plans in order to significantly slow the spread of typhoid disease.They recommended that in order to get a realistic result, the model should contain characteristics such as the logistic growth rate of environmental bacteria and climatic conditions, and that the study of the complete model should be repeated.

Similar to this, Rabiu et al. M. Rabiu, R. Willie and N. Parumasur (2020) constructed and analyzed a new model describing the dynamics of disease resistance by taking into account an autonomous non-linear system of ordinary differential equations (ODEs) that incorporates treatment, quarantine, and incomplete vaccination. According to the study, there is a bifurcation that can be removed if the vaccination is perfect or if the mass action incidence is used in place of the conventional incidence. The model analysis clearly shows that, in order to guarantee that individuals acquired disease resistance under suitable quarantine settings, a great deal of work is needed. Okolo and Abu P. Okolo and O.Abu (2020) developed an optimal control model of TF transmission dynamics that includes vaccination, diagnosis and treatment, and sanitation and hygiene as time-dependent control functions to determine the most cost-effective approach for certain interventions.

An ODE-based mathematical model was developed by J. K. Nthiiri (2016) to investigate the dynamics of typhoid illness spread and include vaccination as a control measure. they found that a decrease in protection led to an increase in the disease's prevalence in a community, as shown by the model's numerical simulation. However, as far as we are aware, there have been no previous papers on the mathematical models of TF disease that consider the vaccine and medically hygienic compartments and uses vaccination, educational campaign and treatment as the control strategies. Therefore, the purpose of this paper is to investigate how vaccination and medical hygiene compartments with vaccination, education campaign and intreatment interventions impact the management and transmission of TF in communities. Typhoid fever transmission in the community can be reduced by any measures, including vaccine campaigns, education campaigns, and treatment, according to a careful analysis of the effective reproduction number. The remaining sections of this work are organized as follows: The model formulation and it's essential properties are covered in Section 2. A thorough qualitative examination of the model is provided in Section ??. Furthermore, quantitative analysis of the model is carried out in Section 4. The section also includes the presentation of results and their detailed discussion. Conclusion is drawn is Section 5.

II. MODEL FORMULATION AND ITS BASIC PROPERTIES

This section develops a TF model that takes into account both the population of humans and the amount of bacteria present in the environment. The ideas presented by the authors in H. Abboubakar and R. Racke (2021);

O. J. Peter, M. O. Ibrahim, H.O. Edogbanya, F.A. Oguntolu, K. Oshinubi, A. A. Ibrahim, T.A. Ayoola and J. O. Lawal (2021); A. Abidemi, and O. J. Peter (2023); S. Olaniyi, K. O. Okosun, S. O. Adesanya and R. S. Lebelo (2020); Lawal et al. (2023) serve as the foundation for the model that is presented in this paper. The variable $B_c(t)$ describes the population of bacteria. In addition, the entire human population at any given time t, represented as N(t), is split into seven epidemiological classes that are mutually exclusive: susceptible humans, S(t); vaccinated humans, V (t); exposed humans, E(t); symptomatic infectious humans, I(t); treated humans, T(t); recovered individuals, R(t); and medically hygienic and conscious humans, M(t). Consequently,

$$N(t) = S(t) + V(t) + E(t) + I(t) + T(t) + R(t) + M(t). \quad (1)$$

The non-linear mathematical model governing the transmission dynamics of TF is given by the following system of ordinary differential equations:

$$\begin{aligned}
\frac{dS}{dt} &= (1-\phi)\Lambda + \kappa V + \varepsilon R - \beta S(B_c + \eta I) - (\pi + \mu)S,\\
\frac{dV}{dt} &= \pi S - (1-m)\beta V(B_c + \eta I) - (\kappa + \mu)V,\\
\frac{dE}{dt} &= \beta(B_c + \eta I)(S + (1-m)V) - (\alpha + \mu)E,\\
\frac{dI}{dt} &= \alpha E + \xi T - (\gamma + \sigma_1 + \mu + \delta_1)I,\\
\frac{dT}{dt} &= \gamma I - (\xi + \sigma_2 + \mu + \delta_2)T,\\
\frac{dR}{dt} &= \sigma_1 I + (1-\varphi)\sigma_2 T - (\theta + \varepsilon + \mu)R,\\
\frac{dM}{dt} &= \phi\Lambda + \varphi\sigma_2 T + \theta R - \mu M,\\
\frac{dB_c}{dt} &= \rho I - \mu_1 B_c,
\end{aligned} \quad (2)$$

with initial conditions:

$S(0) = S_0 > 0,\ V(0) = V_0 \geq 0,\ E(0) = E_0 \geq 0,\ I(0) = I_0 \geq 0,\ T(0) = T_0 \geq 0,$ (3)
$R(0) = R_0 \geq 0,\ M(0) = M_0 \geq 0,\ B_c(0) = B_{0c} \geq 0.$

Model (2)'s schematic diagram is shown in Fig. 1, while Table 1 and 2 describe the state variables and parameters utilized in the model, respectively.

2.1. Optimal control problem formulation
We also take into consideration the following three time-dependent control variables in order to construct the best control model of the dynamics of typhoid fever:
1. Assuming that only susceptible persons receive the vaccine, the initial control $0 \leq u_1(t) \leq 1$ reflects the vaccination rate at time t.
2. By contaminating food and water with the germs, the second control, $0 \leq u_2(t) \leq 1$, represents educational campaign efforts to prevent typhoid sickness. A modified incidence function is produced by adding educational campaign control to the original incidence function.

$$\beta S(B_c + \eta I) = (1 - u_2(t))\beta S(B_c + \eta I)$$

Table 1: Description of the model's variables

Variable	Description
S	Population of susceptible human
V	Population of vaccinated human
E	Population of exposed human
I	Population of symptomatic human
T	Population of treated (including drug complaint and non-drug complaint) human
R	Population of recovered human
M	Population of medically hygienic or conscious human
N	Total population of human
B_c	Concentration of bacteria in the environment

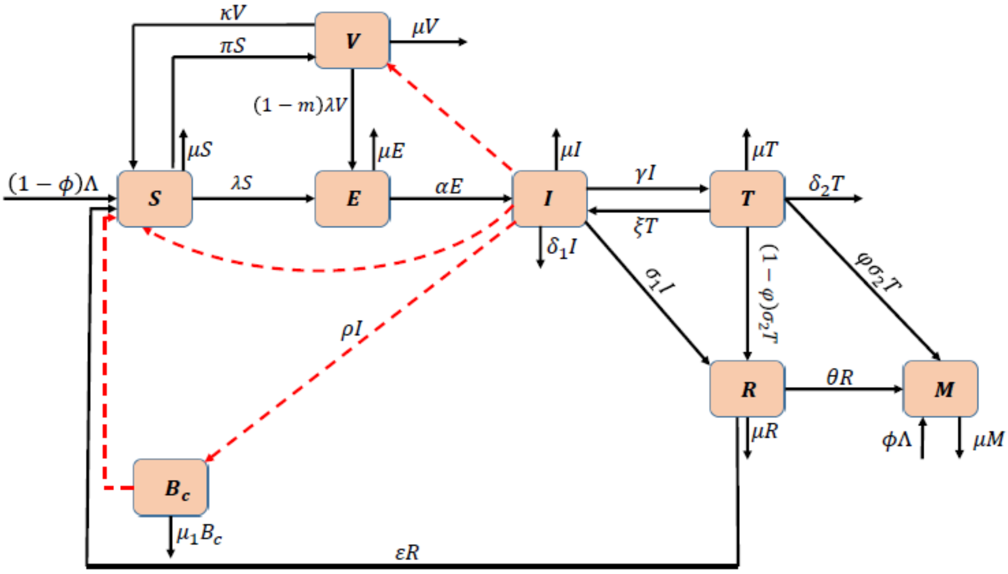

Figure 1: Schematic diagram of the TF model (2), where $\lambda = \beta(Bc + \eta I)$

It's also important to remember that this control will accelerate the rate of bacterial destruction. Therefore,
the rate of bacterial decay becomes

$$\mu 1 = \mu 1 + d1u2(t),$$

where d1 is the extra bacterial mortality rate brought on by chemical intervention.

Table 2: Description of the model's parameters

Parameter	Description
Parameter	**Description**
Λ	Rate of human population recruitment
μ	The natural death rate among humans
$1-\phi$	Proportion of susceptible humans recruitment rate
ϕ	Proportion of the recruitment rate for medically vigilant humans
ε	Rate of immunity loss per person
β	Rate of effective transmission among humans
η	Humans with symptoms and their relative transmissibility
α	Rate of transition from an exposed state to an infectious state
γ	Uptake rate of the mass administration of drug therapy (treatment) on symptomatic infectious humans
σ_1	Human symptomatic recovery rate
δ_1	Death by disease in people with symptoms of infection
ξ	Relapse rate in non-drug complaints
δ_2	Disease-related mortality in noncompliant drug users
σ_2	The rate at which drug-compliant humans recover
φ	The rate at which drug-compliant humans join vigilant class
ρ	Rate at which symptomatic humans shed bacteria
μ_1	Decay rate of bacteria from the environment
m	Vaccine efficacy
π	Vaccination rate
κ	Vaccine waning rate
θ	Progression rate of recovered individuals to medically hygienic class
d_1	Additional mortality rate of bacteria induced by the chemical intervention
d_2	Proportion of effective treatment for symptomatic infectious individuals

3. $0 \le u_3(t) \le 1$, the third control variable, reflects the treatment efforts, which include patient care and appropriate administration. The patient's immunological response affects the treatment's effectiveness and length. As a result, the recovery rate of symptomatic infectious individuals $u_3(t)$ and treated individuals is adjusted based on treatment, while the constant treatment rate of symptomatic infectious individuals, designated as γ, is considered as $u_3(t)$. Recovery rate is therefore computed as

$$\sigma_1 = \sigma_1 + d_2u_3(t),\ \sigma_2 = \sigma_2 + d_2u_3(t),$$

d_2 represents the percentage of patients in class I(t) who receive effective treatment. In line with the concepts presented in H. Abboubakar and R. Racke (2021), the equation $(1-d_2u_3(t))\delta_1$ is used to compute the fraction of successful treatment that reduces disease-induced death of infected persons exhibiting clinical symptoms. $\delta_1 = (1 - d_2u_3(t))\delta_1$, $\delta_2 = (1 - d_2u_3(t))\delta_2$.

It also makes it feasible to lessen the quantity of microorganisms that ill individuals who exhibit clinical symptoms expel.Thus,

$$\rho = (1 - d_2u_3(t))\rho.$$

Based on the existing model (2) and the aforementioned description and assumptions, the optimal control model for the dynamics of typhoid disease is produced as

$$\begin{aligned}
\frac{dS}{dt} &= (1-\phi)\Lambda + \kappa V + \varepsilon R - (1-u_2(t))\beta S(B_c + \eta I) - (u_1(t) + \mu)S, \\
\frac{dV}{dt} &= u_1(t)S - (1-u_2(t))(1-m)\beta V(B_c + \eta I) - (\kappa + \mu)V, \\
\frac{dE}{dt} &= (1-u_2(t))\beta(B_c + \eta I)(S + (1-m)V) - (\alpha + \mu)E, \\
\frac{dI}{dt} &= \alpha E + \xi T - (u_3(t) + \sigma_1 + a_2u_3(t) + \mu + (1 - a_2u_3(t))\delta_1)I, \\
\frac{dT}{dt} &= u_3(t)I - (\xi + \sigma_2 + a_2u_3(t) + \mu + (1 - a_2u_3(t))\delta_2)T, \\
\frac{dR}{dt} &= (\sigma_1 + a_2u_3(t))I + (1-\varphi)(\sigma_2 + a_2u_3(t))T - (\theta + \varepsilon + \mu)R, \\
\frac{dM}{dt} &= \phi\Lambda + \varphi(\sigma_2 + a_2u_3(t))T + \theta R - \mu M, \\
\frac{dB_c}{dt} &= (1 - a_2u_3(t))\rho I - (\mu_1 + a_1u_2(t))B_c,
\end{aligned} \tag{4}$$

with initial conditions:

$S(0) = S_0$, $V(0) = V_0$, $E(0) = E_0$, $I(0) = I_0$, $T(0) = T_0$, $R(0) = R_0$, $M(0) = M_0$, $B_c(0) = B_{0c}$. (5)

Our primary objectives are to maximize the size of the medically vigilant and hygienic human sub-population ($M(t)$) and minimize the size of the symptomatic infectious human sub-population ($I(t)$) and the number of bacteria ($B_c(t)$) in the community, as well as the costs associated with the implementation of vaccination control ($u_1(t)$), educational campaign control ($u_2(t)$), and treatment control ($u_3(t)$). Consequently, we view the functional definition of the cost as

$$J(u_1,u_2,u_3) = \int_0^{t_f}\left(A_1 I - A_2 M + A_3 B_c + \frac{1}{2}D_1u_1^2 + \frac{1}{2}D_2u_2^2 + \frac{1}{2}D_3u_3^2\right)dt \tag{6}$$

subject to the state system (4), where D_i, $i = 1, 2, 3$, represents the positive weight constants for the optimal control variables, u_i, and A_1, A_2, and A_3 represent the positive weight constraints for symptomatic infectious human, medically hygienic individuals, and bacteria population. Over the interval $[0, t_f]$, where the final time interval is indicated by tf, the optimal control intervention is put into practice. The triple control functions define the control intervention's nonlinearity. Consequently, the costs function related to immunization, sanitation and good hygiene, and treatment control techniques are represented by the nonlinear terms $\frac{1}{2}D_1u^2_1$, $\frac{1}{2}D_2u^2_2$, and $\frac{1}{2}D_3u^2_3$. Finding a triple control that satisfies $u^* = (u^*_1, u^*_2, u^*_3)$

$$J(u^*) = \min\{J(u_1, u_2, u_3) : (u_1, u_2, u_3) \in U\} \tag{7}$$

is therefore the overall interest, where $t \in [0, t_f]$, and U is a non-empty Lesbesgue measurable set for the controls $0 \le u_1(t) \le 1$, $0 \le u_2(t) \le 1$, and $0 \le u_3(t) \le 1$.

III. ANALYSIS OF THE OPTIMAL CONTROL MODEL

Based on a first-order condition developed by Pontryagin's maximum principle ? for identifying optimal solutions to diverse control problems, the non-autonomous system of model (4) is studied.

3.1. Optimal control characterisation

In order to describe the three optimal controls, Pontryagin's maximum principle is used to define the essential conditions that the controls and their corresponding states must satisfy. ?.

The Hamiltonian is given by

$$
\begin{aligned}
H = {} & A_1 I - A_2 M + A_3 B_c + \frac{1}{2}B_1 u_1^2(t) + \frac{1}{2}B_2 u_2^2 + \frac{1}{2}B_3 u_3^2 \\
& + \lambda_1\Big\{(1-\phi)\Lambda + \kappa V + \epsilon R - (1-u_2)\beta S(B_c+\eta I) - (\mu_1+\mu)S\Big\} \\
& + \lambda_2\Big\{\mu_1 S - (1-u_2)(1-m)\beta V(B_c+\eta I) - (\kappa+\mu)V\Big\} \\
& + \lambda_3\Big\{(1-u_2)\beta(B_c+\eta I)(S+(1-m)V) - (\alpha+\mu)E\Big\} \\
& + \lambda_4\Big\{\alpha E + \xi T - (u_3+\sigma_1+a_2u_3+\mu+(1-a_2u_3)\delta_1)I\Big\} \\
& + \lambda_5\Big\{u_3 I - (\xi+\sigma_2+a_2u_3+\mu+(1-a_2u_3)\delta_2)T\Big\} \\
& + \lambda_6\Big\{(\sigma_1+a_2u_3)I + (1-\varphi)(\sigma_2+a_2u_3)T - (\theta+\epsilon+\mu)R\Big\} \\
& + \lambda_7\Big\{\phi\Lambda + \varphi(\sigma_2+a_2u_3)T + \theta R - \mu M\Big\} \\
& + \lambda_8\Big\{(1-a_2u_3)\rho I - (\mu_1+a_1u_2)B_c\Big\}
\end{aligned} \tag{8}
$$

where, λ_1, λ_2, λ_3, λ_4, λ_5, λ_6, λ_7, λ_8 are adjoint variable corresponding to the state variables of the model (4).

Theorem 1. The adjoint variables λ_1, λ_2, λ_3, λ_4, λ_5, λ_6, λ_7, λ_8 exist and satisfy the adjoint system for an optimal control triple u^* with corresponding solutions S, V, E, I, T, R, M, B_c of the associated state system satisfying (7) such that,

$$
\begin{aligned}
\frac{d\lambda_1}{dt} &= \lambda_1(u_1+\mu) + (\lambda_1-\lambda_3)(1-u_2)\beta(B_c+\eta I) - \lambda_2 u_1 \\
\frac{d\lambda_2}{dt} &= -\lambda_1\kappa + \lambda_2(\kappa+\mu) + (\lambda_2-\lambda_3)(1-u_2)(1-m)\beta(B_c+\eta I) \\
\frac{d\lambda_3}{dt} &= \lambda_3(\alpha+\mu) - \lambda_4\alpha \\
\frac{d\lambda_4}{dt} &= -A_1 + (\lambda_1-\lambda_3)\beta\eta S + (\lambda_2-\lambda_3)(1-u_2)(1-m)\beta\eta V \\
&\quad + \lambda_4\Big(u_3+\sigma_1+a_2u_3+\mu+(1-a_2u_3)\delta_1\Big) - \lambda_5u_3 - \lambda_6(\sigma_1+a_2u_3) - \lambda_8(1-a_2u_3)\rho \\
\frac{d\lambda_5}{dt} &= -\lambda_4\xi + \lambda_5(\xi+\sigma_2+a_2u_3+\mu+(1-a_2u_3)\delta_2) - \lambda_6(1-\varphi)(\sigma_2+a_2u_3) - \lambda_7\varphi(\sigma_2+a_2u_3) \\
\frac{d\lambda_6}{dt} &= -\lambda_1\varepsilon + \lambda_6(\theta+\varepsilon+\mu) - \lambda_7\theta \\
\frac{d\lambda_7}{dt} &= A_2 + \lambda_7\mu \\
\frac{d\lambda_8}{dt} &= -A_3 + (\lambda_1-\lambda_3)(1-u_2)\beta S + (\lambda_2-\lambda_3)(1-u_2)(1-m)\beta V + \lambda_8(\mu_1+a_1u_2))
\end{aligned} \tag{9}
$$

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with transversality conditions

$\lambda_i(t_f) = 0$, i = 1, 2, ..., 8 (10)

and optimal control characterizations given by,

$$u_1^* = \min\left\{0, \max\left\{1, \frac{(\lambda_1-\lambda_2)S}{C_1}\right\}\right\},$$
$$u_2^* = \min\left\{0, \max\left\{1, \frac{(\lambda_3-\lambda_1)\beta S(B_c+\eta I)+(\lambda_3-\lambda_2)(1-m)\beta V(B_c+\eta I)+\lambda_8 a_1 B_c}{C_2}\right\}\right\}, \quad (11)$$
$$u_3^* = \min\left\{0, \max\left\{1, \frac{\lambda_4(1+a_2-a_2\delta_1)I+\lambda_5(a_2-a_2\delta_2)T-\lambda_5 I - \lambda_6 a_2 I-\lambda_6(1-\varphi)a_2 T-\lambda_7\varphi a_2 T+\lambda_8 a_2\rho I}{C_3}\right\}\right\}.$$

Proof. The adjoint system of equations (9)-(9) is produced by calculating the partial derivative of the Hamiltonian in (8) with respect to each of the state variables, S, V, E, I, T, R, M, B_c.

$$\frac{d\lambda_1}{dt} = \frac{-\partial H}{\partial S}, \frac{d\lambda_2}{dt} = \frac{-\partial H}{\partial V}, \frac{d\lambda_3}{dt} = \frac{-\partial H}{\partial E},$$
$$\frac{d\lambda_4}{dt} = \frac{-\partial H}{\partial I}, \frac{d\lambda_5}{dt} = \frac{-\partial H}{\partial T}, \frac{d\lambda_6}{dt} = \frac{-\partial H}{\partial R}, \frac{d\lambda_7}{dt} = \frac{-\partial H}{\partial M}, \frac{d\lambda_8}{dt} = \frac{-\partial H}{\partial B_c}.$$

Use the partial differential equation $\frac{\partial H}{\partial u_i} = 0$ as an additional tool, i = 1, 2, 3 to get the three control variables' ideal levels of control as

$$\frac{\partial H}{\partial u_1} = C_1 u_1 - \lambda_1 S + \lambda_2 S = 0$$
$$\Longrightarrow C_1 u_1 = (\lambda_1-\lambda_2)S \quad (12)$$
$$u_2^* = \frac{(\lambda_3-\lambda_1)\beta S(B_c+\eta I)+(\lambda_3-\lambda_2)(1-m)\beta V(B_c+\eta I)+\lambda_8 a_1 B_c}{C_2}$$
$$\Longrightarrow u_3^* = \frac{\lambda_4(1+a_2-a_2\delta_1)I+\lambda_5(a_2-a_2\delta_2)T-\lambda_5 I - \lambda_6 a_2 I-\lambda_6(1-\varphi)a_2 T-\lambda_7\varphi a_2 T+\lambda_8 a_2\rho I}{C_3}\}\} \quad (13)$$

As a result, setting limits on the controls produces

$$u_1^* = \min\left\{0, \max\left(1, \frac{(\lambda_1-\lambda_2)S}{C_1}\right)\right\}$$
$$u_2^* = \min\left\{0, \max\left(1, \frac{(\lambda_3-\lambda_1)\beta S(B_c+\eta I)+(\lambda_3-\lambda_2)(1-m)\beta V(B_c+\eta I)+\lambda_8 a_1 B_c}{C_2}\right)\right\} \quad (14)$$
$$u_3^* = \min\left\{0, \max\left(1, \frac{\lambda_4(1+a_2-a_2\delta_1)I+\lambda_5(a_2-a_2\delta_2)T-\lambda_5 I - \lambda_6 a_2 I-\lambda_6(1-\varphi)a_2 T-\lambda_7\varphi a_2 T+\lambda_8 a_2\rho I}{C_3}\right)\right\}$$

This completes the proof

IV. NUMERICAL SIMULATIONS, RESULTS AND EFFICIENCY ANALYSIS

4.1. Numerical simulation

In accordance with the recommendations given by Lawal et al. (2023), the study used numerical simulations to examine the dynamic behavior of the TF illness model (2) and the effects of vaccination-related and control parameters on the population dynamics of TF disease. The forward-backward sweep approach is implemented in MATLAB, and the optimum control problem, which combines system (4) and adjoint system (9)

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− (9), is solved using the Runge-Kutta fourth order scheme. The state system (4) is solved forward in time, whereas the adjoint system (9) − (9) is solved backward in time. The model parameter values found in Table 3 and the beginning conditions of the model variables are used to simulate the model for different disease outbreak scenarios. These values are determined in (5). Using at least one of the three time-dependent control functions, the optimality system is implemented under seven distinct control combination strategies. The definition of these tactics is as follows: The first strategy, S1, uses vaccination alone (u_1); the second, S2, uses an educational campaign alone (u_2); the third, S3, uses treatment alone (u_3); the fourth, S4, combines vaccination and educational campaign alone (u_1, u_2); the fifth, S5, combines vaccination and treatment alone (u_1, u_3); the sixth, S6, combines educational campaign and treatment alone (u_2, u_3); and the seventh, S7, combines all three control interventions (u_1, u_2, u_3).

Table 3: Parameter values of the model

Parameter	**BaselineValue**	**Source**
γ	0.002	A.Alhassan,andA.A.Momoh,S.A.AbdullahiandA.Audu(2021)
ϕ	0.15	Assumed
Λ	10726.44506419313	estimatedworldometer(2022)
β	0.00000001	assumed
μ	$\frac{1}{55.75\times365}$	estimatedworldometer(2022)
ε	0.000904	H.AbboubakarandR.Racke(2021)
ξ	0.000009	Assumed
η	0.00001	Assumed
σ_2	0.1	A.Alhassan,andA.A.Momoh,S.A.AbdullahiandA.Audu(2021)
α	0.03	H.AbboubakarandR.Racke(2021)
ρ	0.50	O. J. Peter, M. O. Ibrahim, H.O. Edogbanya, F.A.Oguntolu, K. Oshinubi, A. A. Ibrahi
σ_1	0.75	O. J. Peter, M. O. Ibrahim, H.O. Edogbanya, F.A.Oguntolu, K. Oshinubi, A. A. Ibrahi
π	0.50	H.AbboubakarandR.Racke(2021)
δ_1	0.2	O. J. Peter,M. O.Ibrahim, H.O. Edogbanya,F.A. Oguntolu, K.Oshinubi, A.A. Ibrahi
κ	0.0009041	H.AbboubakarandR.Racke(2021)
δ_2	0.001	A.Alhassan,andA.A.Momoh,S.A.AbdullahiandA.Audu(2021)
d_2	0.7	H.AbboubakarandR.Racke(2021)
φ	0.005	Assumed
d_1	0.3	H.AbboubakarandR.Racke(2021)
μ_1	0.4	A.Alhassan,andA.A.Momoh,S.A.AbdullahiandA.Audu(2021)
m	0.95	H.AbboubakarandR.Racke(2021)

4.2. Results

The spread of TF is not being attempted to be controlled when the control is zero; maximum control is being employed when the control is one; and a control is being implemented when the control is not equal to zero. The population

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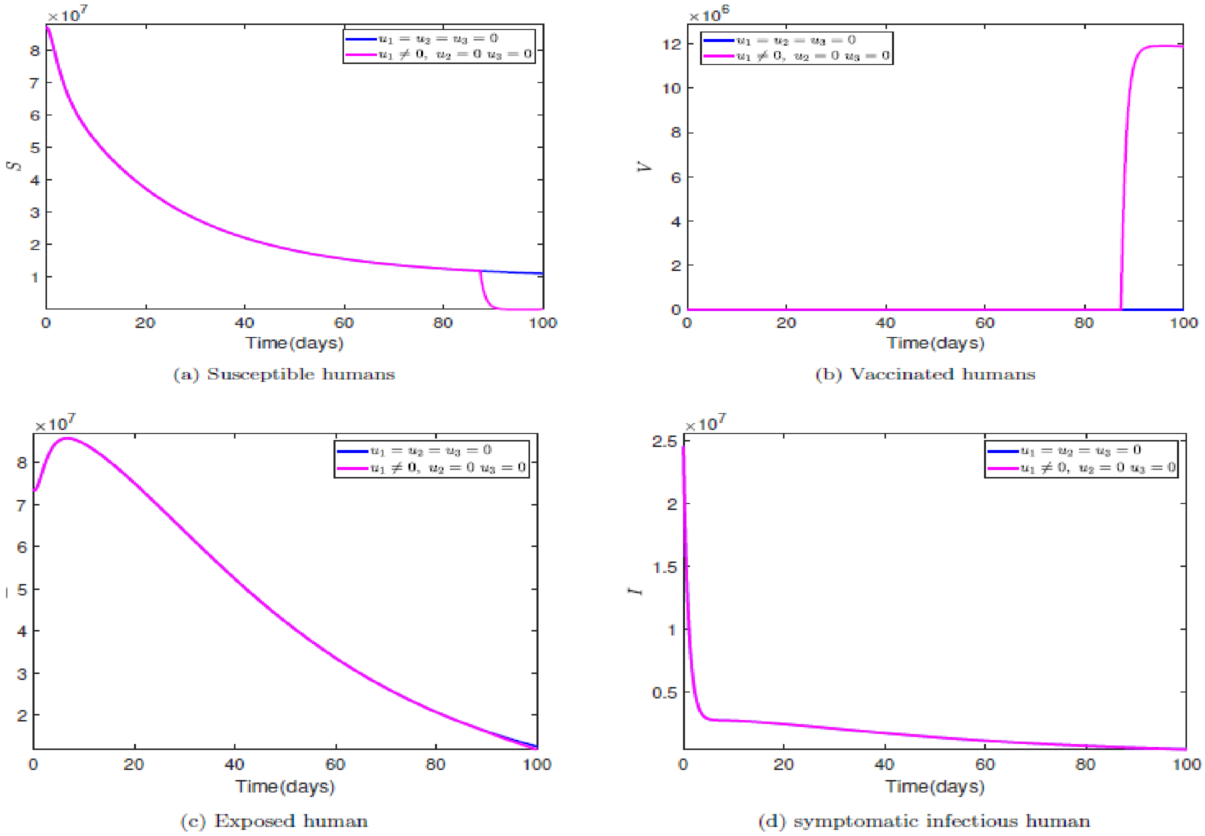
exposure to TF may not be effectively stopped by a single control, which is why the following seven options represent potential optimal control interventions:

1. Strategy 1: S1 vaccination only u_1
2. Strategy 2: S2 educational campaign u_2
3. Strategy 3: S3 treatment only u_3
4. Strategy 4: S4 combination of vaccination, educational campaign only u_1, u_2
5. Strategy 5: S5 combination of vaccination and treatment only u_1, u_3
6. Strategy 6: S6 combination of educational campaign with treatment only u_2, u_3
7. Strategy 4: S7 combination of all the three interventions u_1, u_2, u_3

4.2.1. Strategy 1

The effect of vaccination as the sole control measure ($u_1 \neq 0$) on the dynamics of typhoid fever transmission in a population is depicted in Figure 2. The effectiveness of the control is demonstrated by minimizing the objective functional J while setting the values of the other two controls, u_2 and u_3, to zero, i.e., ($u_1 \neq 0$, $u_2 = u_3 = 0$). Figure 2a illustrates how the susceptible population is affected by the presence of control, with a decrease observed at 85 days and a near doubling of susceptible individuals at 90–100 days.Similarly, Figure 2b shows that until around 87 days after applying the control, there was no noticeable effect of the vaccination on the vaccinated compartment. Following that, there was a noticeable rise that peaked at 92 days and continued for 100 days. Likewise, In Figure 2c The exposed human population declined due to vaccination control on the susceptible and vaccinated compartments, reducing the risk of spreading TF diseases. Additionally, Figures 2d and 2e demonstrate how this technique lowered the number of infected humans to almost zero at 80 days and kept them there for the duration of the interval. It also had a reducing influence on the population's symptomatic infectious and bacteria. However, as Figure 2a illustrates, immunization as a preventive measure has some beneficial effects on the susceptible; hence, vaccination is necessary, particularly in areas that are prone to typhoid fever.



(a) Susceptible humans

(b) Vaccinated humans

(c) Exposed human

(d) symptomatic infectious human

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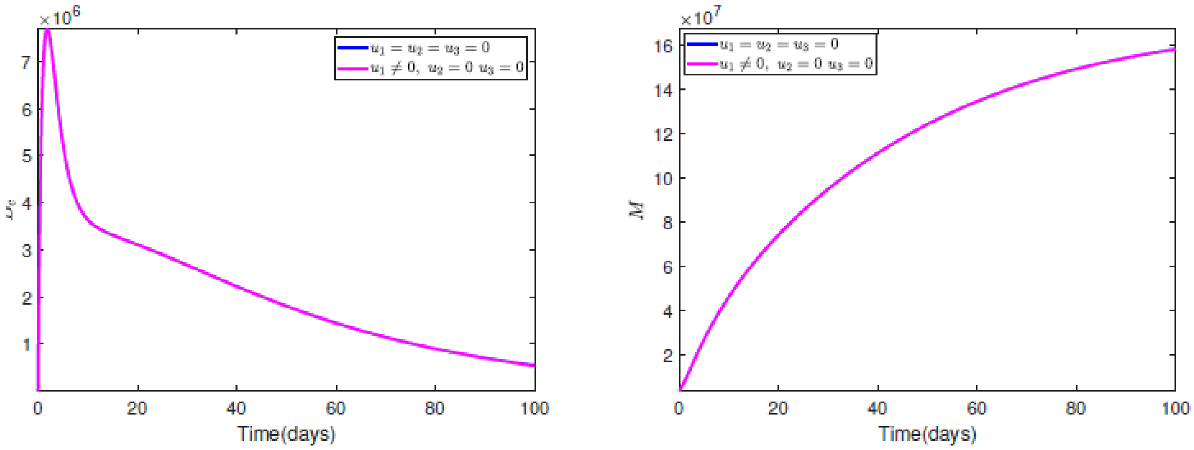


(e) Bacteria population (f) Medically hygienic human

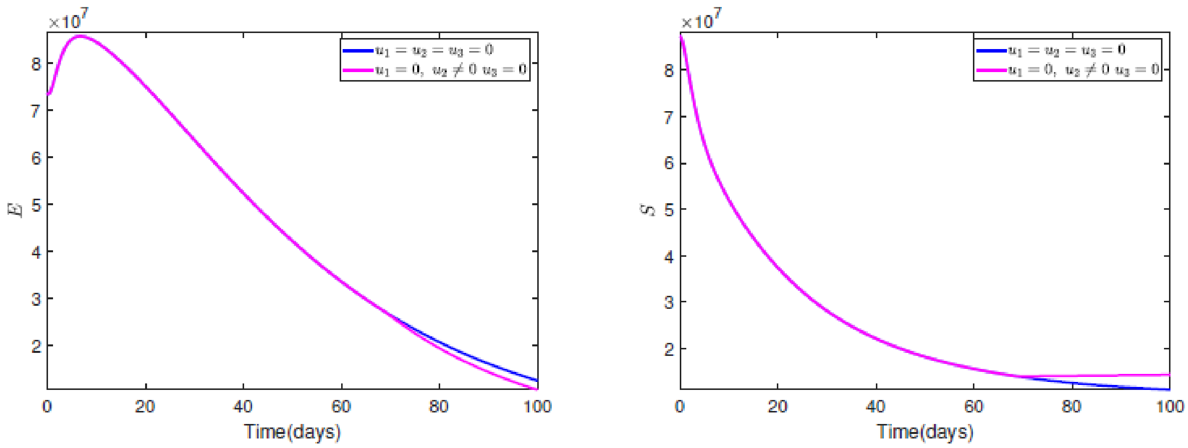
Figure 2: Simulation showing the dynamics of the states variables of model (2) for optimal vaccination only 14

4.2.2. Strategy 2

4.2.3. Implementing Sanitation and Proper hygiene as the only control u_2

Education campaigns must be implemented as a preventative approach to stop the TF sickness from spreading among a population. u_2, which consists of educating the public about the dangers of typhoid fever, portable drinking water, improved restroom facilities, and a healthy atmosphere, is one of the preventive measures that must be followed. u_1 and u_3 are set to zero ($u_2 \neq 0$, $u_1 = u_3 = 0$) in order to optimize the objective functional J and show how successfully the control stops the spread of sickness.

Figures 3b, 3a, and ?? illustrate the fast population decreases of Susceptible, Exposed, and Symptomatic Infectious throughout the duration of the intervention, with a rise in the susceptible population from the 70th to the 100th day. Furthermore, Figure 3d shows that, while receiving the control intervention, there was an increase in incidence in the human population until the 80th or 100th day, at which point the infection rate decreased. This suggests that people have started putting the educational campaign into practice at the 80th day. Conversely, Figure 3e shows that the presence of the control intervention reduces the amount of bacteria in the environment. As a result, an effective tactic for stopping the spread of typhoid fever infections in the community is the implementation of educational campaign.



(a) Exposed (b) Susceptible

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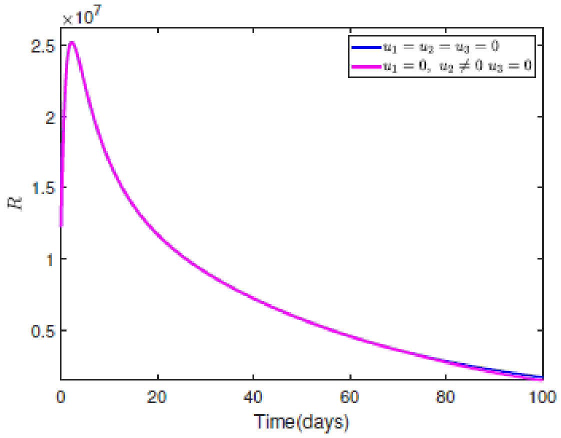
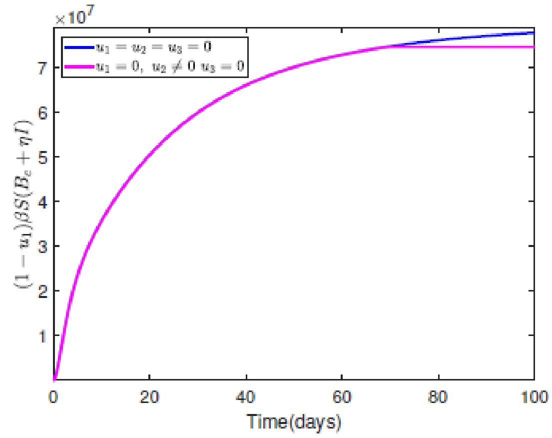
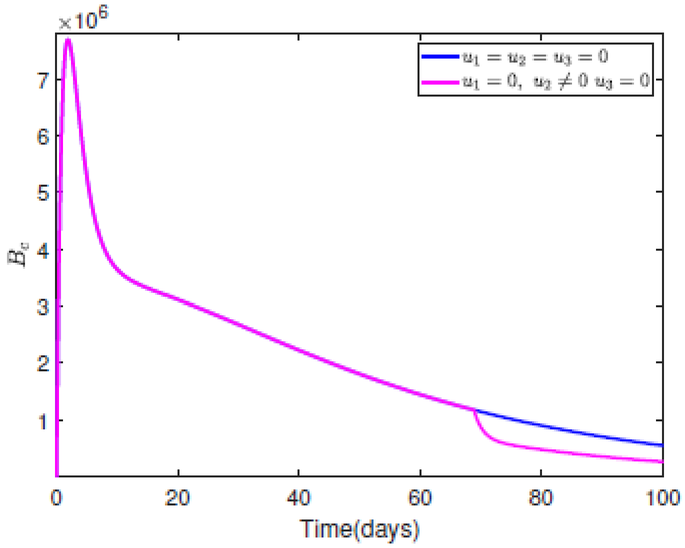


(c) Recovered



(d) Incidence in human population



(e) Bacteria population

Figure 3: Simulation showing the dynamics of the states variables of model (2) for optimal educational campaign 16

4.2.4. Strategy 3

To control the progression of the TF diseases $u_3 \neq 0$, treatment is used as the sole strategy. Setting u_1 and u_2 to zero results in the minimization of the objective function J . Figures 4a–4d and 4e illustrate how this method yields better results by decreasing the overall number of bacteria in the environment as well as the total number of exposed, infectious, treated, incidence while increasing the number of individuals who are medically hygienic 4f. The exposed group decreases during the course of the time period, as seen in Figure 4a. A dramatic drop in the infectious population is seen in Figure 4b, particularly after day 20. The disease had almost totally disappeared from the population by day 40, and it had completely vanished by days 60 to 100 following the application of the control. Similarly, Figure 4c shows that the control intervention led to a drop in the number of treated individuals. The data shown in Figure 4d indicates a considerable decrease in the number of new instances of infection (Incidence). Furthermore, Figure 4e shows that when the control was applied, the bacteria population reached its lowest point between days 60 and 100. This tendency started within the first 10 days and lasted until the bacterium population, which had peaked, started to drastically fall. Typhoid fever transmission in the general

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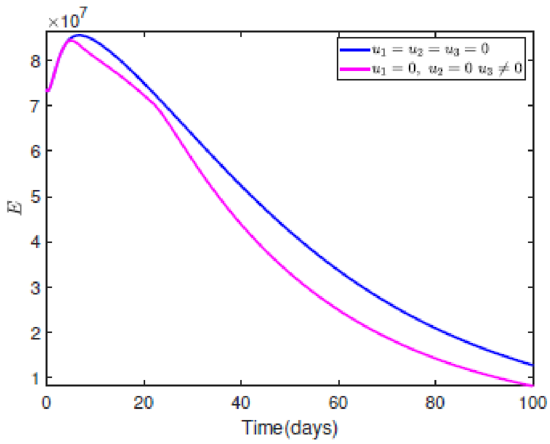
population can be effectively stopped by treatment as control.

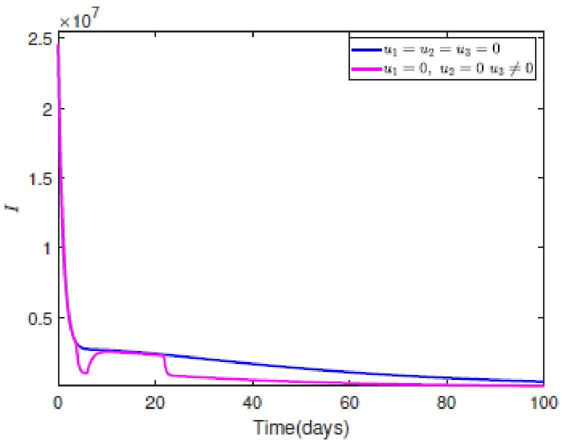
4.2.5. Strategy 4: S4 Combination of Vaccination and educational campaign only u_1, u_2 Utilized in these strategies are vaccinations u_1, proper hygiene practices and sanitation u_2. The results of these controls are shown in Figures 5a- 5e. Figure 5a demonstrates that the controls have minimal impact on susceptible until 70 days, when the susceptible population increased. Moreover, Figure 5b depicts that the vaccination and proper hygiene combined with sanitation used as the control did not impact on the vaccinated compartment until around 90 days after the control had been administered which followed with a considerable uptick that peaked and lasted for 100 days. Figure 5c shows this strategy has a decreasing effect on infectious individuals, which started to show at the 5th day and declines throughout the interval after the control intervention was applied. Comparably, in Figure 5d demonstrates that the controls had no significant impact initially until 70 days, when the incidence population is decreased by the presence of the control, particularly between 70 and 100 days. Moreover, Figure 5e Show how the control's presence significantly affects the bacteria compartment, which initially peaks and then decreases considerably throughout the time period. Vaccination is an important technique in stopping the transmission of infectious diseases and will aid in stopping the development of typhoid fever when combined with sanitation and proper hygiene.

4.2.6. Strategy 5: S5 Combination of Vaccination and Treatment only u_1, u_3

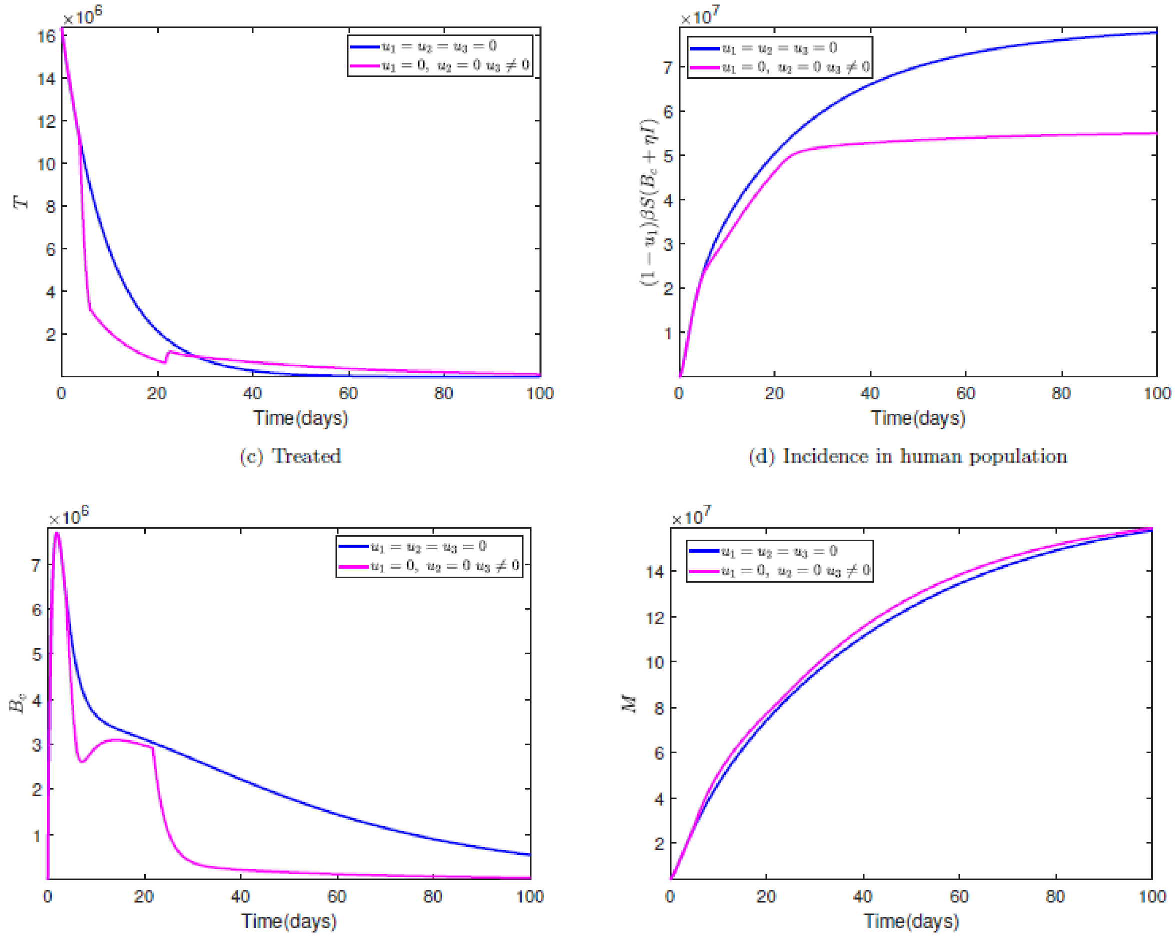
Figures 6a-6e demonstrate how vaccination and treatment work. In this case we set ($u_1 \neq 0$, $u_2 = 0$,) and $u_3 \neq 0$ as a preventative measure. It further illustrates how vaccination affects susceptible, vaccinated, infectious, incidence and bacteria population and how typhoid contacts (infection) can be treated to stop the disease from spreading. Figure 6a demonstrates that the control has an increasing influence on susceptible in the community, Figure 6b reveals an increase in the vaccinated population especially around the 95-100days, likewise Figure 6c, depicts a reducing effect on infectious, and Figure 6d demonstrates reduction in the number of new cases of infection in the population as a result of control intervention. Additionally, Figure 6e shows a sizeable decrease in the population of bacteria. From the simulation, It has been noted that vaccination combined with treatment enables the reduction



(a) Exposed



(b) Infectious population



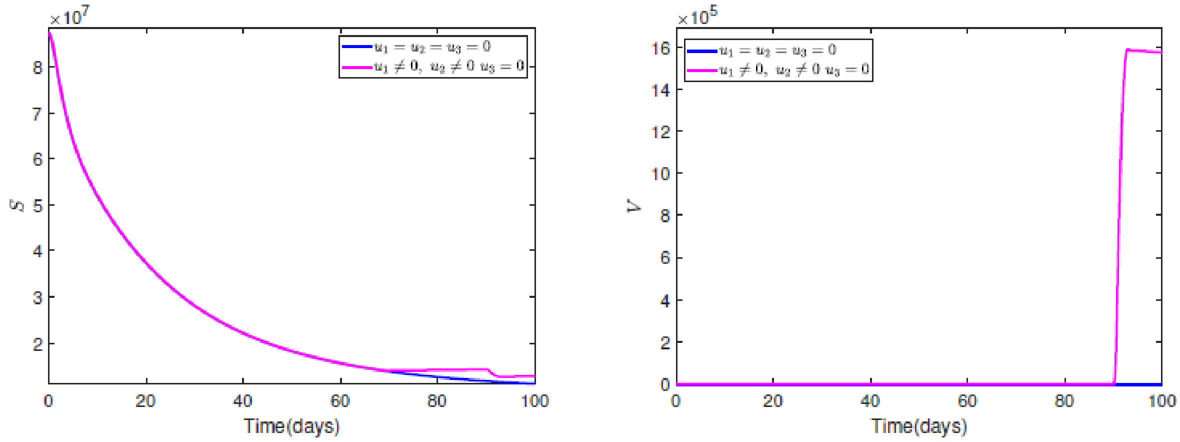
(c) Treated

(d) Incidence in human population

(e) Bacteria population

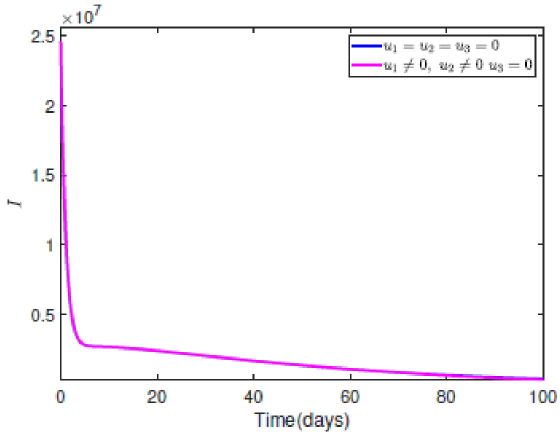
(f) Medically hygienic

Figure 4: Simulation showing the dynamics of the states variables of model (2) for optimal treatment only 18

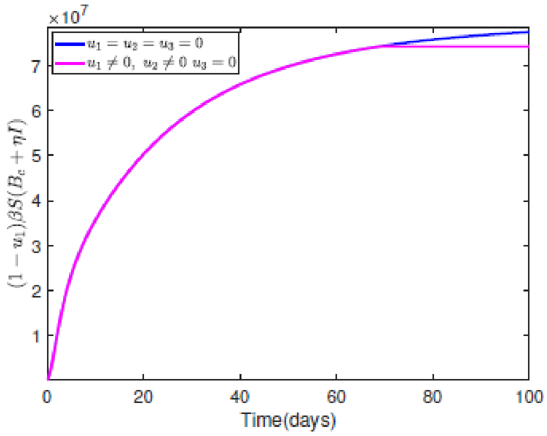


(a) Susceptible humans

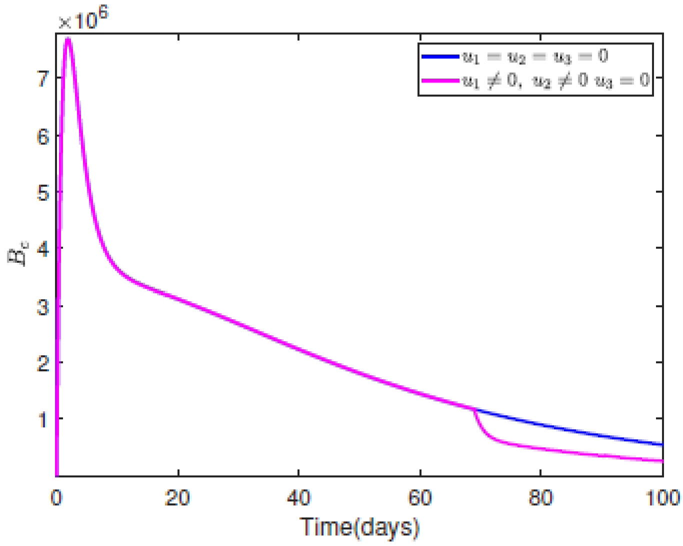
(b) Vaccinated



(c) Symptomatic infectious humans



(d) Incidence in human population



(e) Bacteria population

Figure 5: Simulation on the dynamics of model (2) states variables for optimal Combination of Vaccination and educational campaign only of symptomatic infectious, incidence in human and bacteria in the population and an improvement in the number of susceptible population. This is consistent with the idea that individual treatments can help to lower the number of bacteria and infected compartments.

4.2.7. Strategy 6: S6 Combination of Sanitation and Proper hygiene with Treatment only u_2, u_3 Figure 7a- 7e illustrates the impact of implementing sanitation and proper hygiene along with treatment to maximize the objective functional J and setting $u_2 = 0$ as the control intervention, that is ($u_1 = 0$, $u_2 \neq 0$, $u_3 \neq 0$). It demonstrates the efficiency of simultaneously using both controls and the steady reduction in typhiod fever transmission when compared to no controls. As shown in Figures 7a-7d, when sanitation, proper hygiene, and treatment are adopted, would reduce the number of exposed, infectious, treated and incidence human in the population. Additionally, the simultaneous application of two controls, as depicted in Figure 7e, results in a decrease in the bacterial population. In conclusion, implementing sanitation and proper hygiene along with treatment on the exposed, symptomatic infectious and treated individuals will reduce the infectious population as shown in Figures 7a- 7e.

4.2.8. Strategy 7: S7 Combination of all the three interventions u_1, u_2, u_3
4.2.9. Control with Vaccination, educational campaign with Treatment $u_1 \neq 0$, $u_2 \neq 0$, $u_3 \neq 0$ This approach uses all three controls that is control with vaccination u_1, control with sanitation and proper

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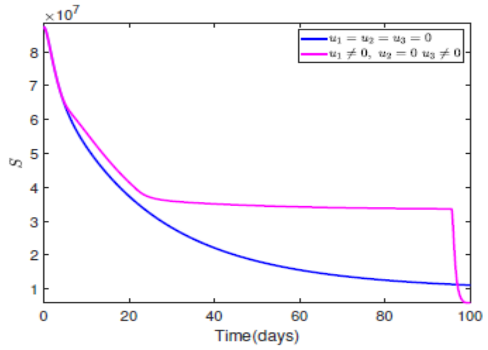
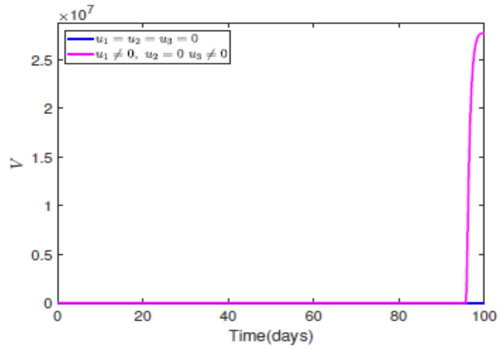
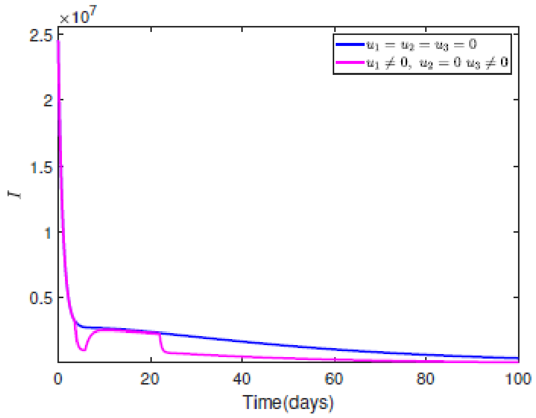
hygiene u_2 and control with treatment u_3, concurrently as an intervention to reduce typhoid fever disease. Figures 8a-8e demonstrate a better outcome following the application of all the controls, with a sharp decline in the population of Susceptible, Vaccinated, Infectious, Incidence in human population as well as Bacteria population. In comparison to the case when there is no control, Figures 8c and 8d illustrates how the infection transmission and new cases of infection decreases steadily. Therefore, we draw the conclusion that using all three measures is more efficient in containing and controlling typhoid fever in communities that are susceptible to it throughout a given period of time.



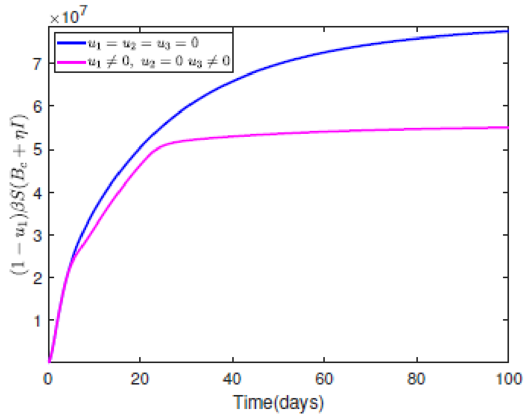
(a) Susceptible humans



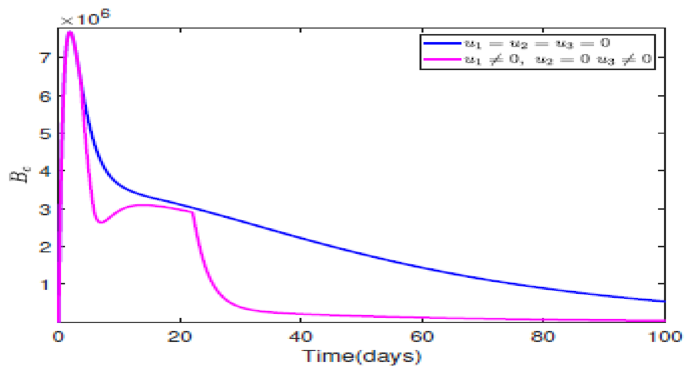
(b) Vaccinated



(c) Symptomatic infectious humans



(d) Incidence in human population



(e) Bacteria population

Figure 6: Simulation on the dynamics of the states variables of model (2) for optimal vaccination combined with treatment only 22

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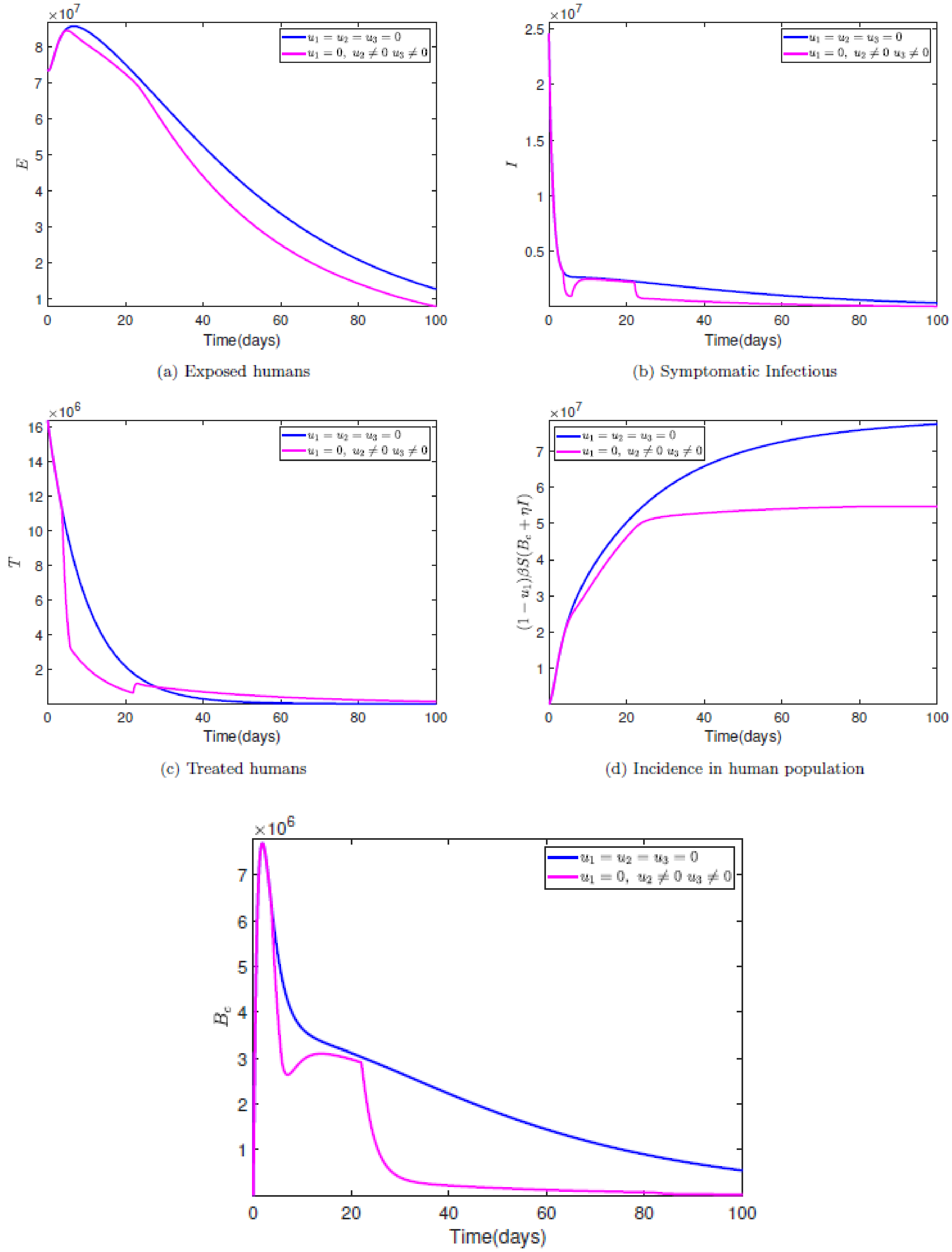

(a) Exposed humans

(b) Symptomatic Infectious

(c) Treated humans

(d) Incidence in human population

(e) Bacteria population

Figure 7: Model (2) simulation on the dynamics of the states variables for optimal combination of educational campaign with Treatment only

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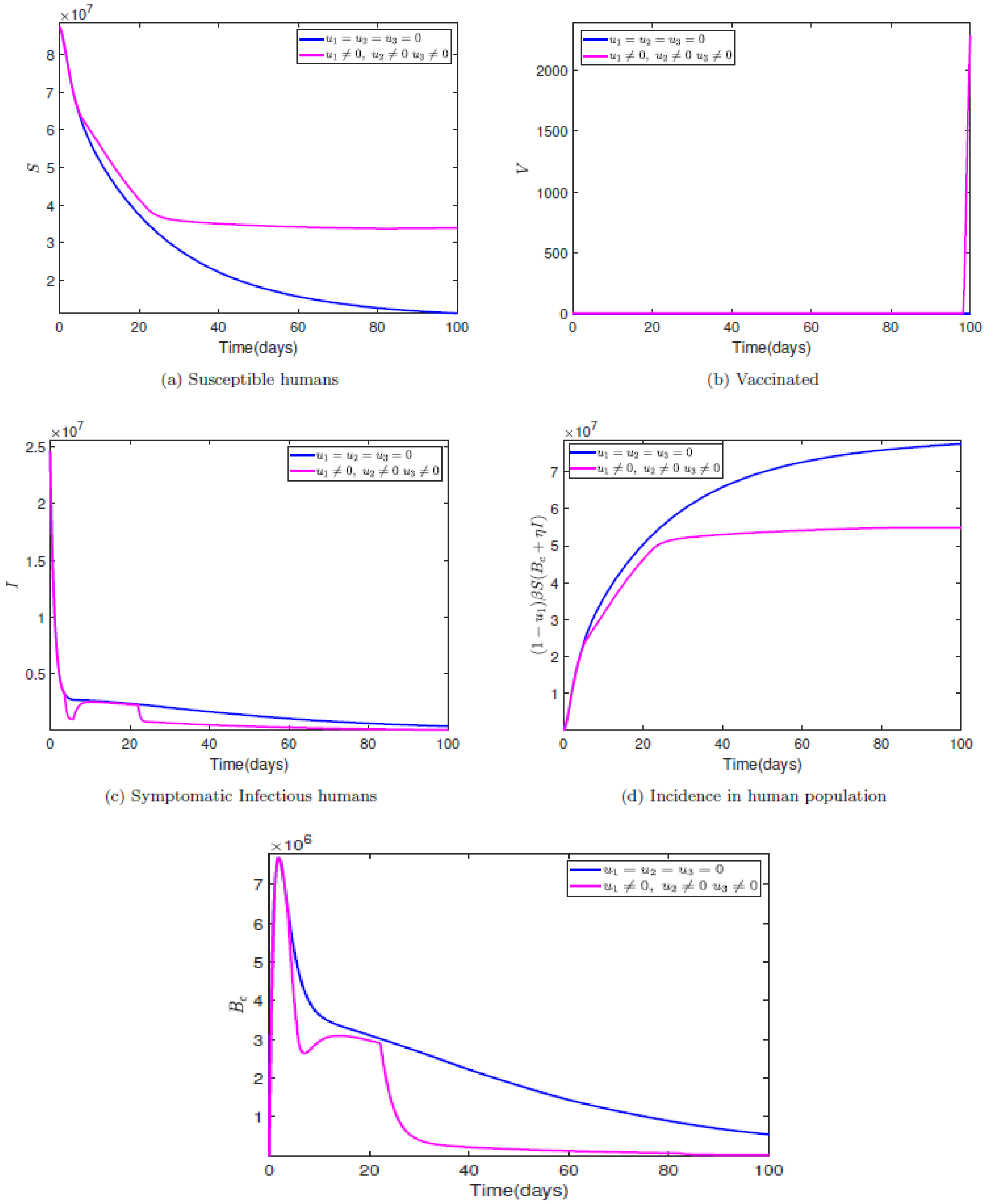

(a) Susceptible humans

(b) Vaccinated

(c) Symptomatic Infectious humans

(d) Incidence in human population

(e) Bacteria population

Figure 8: Simulation of the state variables dynamics in model (2) for optimal combination of all the three interventions 24

4.3. Efficiency analysis

The most effective intervention that can be used to prevent the greatest number of illnesses in the community is determined by calculating the efficiency indices of the three distinct control combination strategies in this subsection. Therefore, the mathematical definition of the efficiency index (EI) according to ?T. T Yusuf and A. Abidemi (2023) is.

$$\mathrm{EI} = \frac{\text{Total infection averted by the control intervention}}{\text{Total infection without any control intervention}} \times 100$$

Table 4: Efficiency indices of Si, i = 1, 2, . . . , 7

Strategy	Total infection averted	EI
S1	1.212631×10^6	0.0703
S2	1.284769×10^7	0.7453
S4	1.283052×10^7	0.7443
S7	6.986808×10^8	40.5326
S5	7.000095×10^8	40.6097
S6	7.003138×10^8	40.6273
S3	7.045366×10^8	40.8723

As can be seen in Table 4, Strategy 1 has the lowest Efficiency Index (0.0703) and Strategy 3 the greatest (40.8723). However, because method 3 employs fewer control, it is the most effective technique to stop the spread of typhoid disease in the community.

V. CONCLUSION

This study extends a mathematical model to an optimal control problem for the dynamics of typhoid fever transmission in the community. The findings have demonstrated that various factors influence the dynamics of typhoid disease control. Typhoid epidemics are extremely difficult to contain or even eradicate without concerted effort. The optimal control model was also analyzed, and the optimal control's characterization was derived using the Pontryagin maximum principle. The goal of this analysis was to identify the conditions under which the spread of typhoid fever can be stopped, as well as to investigate the effects of three time-dependent control variables, including vaccination use, educational campaign and treatment, as well as the impact of a potential combination of these three controls. Furthermore, efficiency analysis and numerical simulation were employed to identify the strategy that avert the most number of cases of typhoid fever. As a result, strategy 3 (treatment alone) is the most effective control method, while strategy 1 (control with vaccine only) is the least effective. The findings of this study recommend that, in order to prevent the spread of typhoid fever infection, any control strategy should take treatment of those who show symptoms of the disease into account. To prevent a typhoid outbreak in the populace, we advise that many sectors, including the health, sanitation, and water supply groups, collaborate with one another. further more, we advise that any program to prevent typhoid be developeded in conjunction with culturally appropriate population-level education for those who are susceptible to the disease.

Statements and declarations

The authors declare that they do not have financial or personal conflicts of interest that would seem to have impacted the research described in this study

Competing interests and funding
None.

Data availability

All data generated or analysed during this study are included in this article.

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